

Frontiers in Applied Dynamical Systems:
Reviews and Tutorials 4

Mason A. Porter
James P. Gleeson

Dynamical Systems on Networks

A Tutorial

 Springer

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Volume 4

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Frontiers in Applied Dynamical Systems: Reviews and Tutorials

The Frontiers in Applied Dynamical Systems (FIADS) covers emerging topics and significant developments in the field of applied dynamical systems. It is a collection of invited review articles by leading researchers in dynamical systems, their applications and related areas. Contributions in this series should be seen as a portal for a broad audience of researchers in dynamical systems at all levels and can serve as advanced teaching aids for graduate students. Each contribution provides an informal outline of a specific area, an interesting application, a recent technique, or a “how-to” for analytical methods and for computational algorithms, and a list of key references. All articles will be refereed.

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*To our teachers, students, postdocs, and
collaborators. (We are aware that these
are overlapping communities.)*

Preface to the Series

The subject of dynamical systems has matured over a period of more than a century. It began with Poincaré's investigation into the motion of the celestial bodies, and he pioneered a new direction by looking at the equations of motion from a qualitative viewpoint. For different motivation, statistical physics was being developed and had led to the idea of ergodic motion. Together, these presaged an area that was to have significant impact on both pure and applied mathematics. This perspective of dynamical systems was refined and developed in the second half of the twentieth century and now provides a commonly accepted way of channeling mathematical ideas into applications. These applications now reach from biology and social behavior to optics and microphysics.

There is still a lot we do not understand and the mathematical area of dynamical systems remains vibrant. This is particularly true as researchers come to grips with spatially distributed systems and those affected by stochastic effects that interact with complex deterministic dynamics. Much of current progress is being driven by questions that come from the applications of dynamical systems. To truly appreciate and engage in this work then requires us to understand more than just the mathematical theory of the subject. But to invest the time it takes to learn a new subarea of applied dynamics without a guide is often impossible. This is especially true if the reach of its novelty extends from new mathematical ideas to the motivating questions and issues of the domain science.

It was from this challenge facing us that the idea for the *Frontiers in Applied Dynamics* was born. Our hope is that through the editions of this series, both new and seasoned dynamicists will be able to get into the applied areas that are defining modern dynamical systems. Each article will expose an area of current interest and excitement and provide a portal for learning and entering the area. Occasionally, we will combine more than one paper in a volume if we see a related audience as we have done in the first few volumes. Any given paper may contain new ideas

and results. But more importantly, the papers will provide a survey of recent activity and the necessary background to understand its significance, open questions, and mathematical challenges.

Editors-in-Chief
Christopher K.R.T Jones, Björn Sandstede, Lai-Sang Young

Preface

Origin

Traditionally, much of the study of networks has focused on structural features. Indeed, mathematical subjects such as graph theory have a rich history of investigating network structure, and most early work by physicists, sociologists, and other scholars also focused predominantly on structural features. The beginnings of the field of “network science,” which one can characterize as the science of connectivity, also started out by focusing on network structure (i.e., literal connectivity). Although some scholars (e.g., many control theorists) have traditionally stressed the importance of dynamics in their study of networks, many network-science practitioners who were trained in fields like dynamical systems and nonequilibrium statistical mechanics (which are both concerned very deeply with dynamical processes) have written myriad papers that seem to focus predominantly or even exclusively on structure. This is valuable and we ourselves have written papers on network structure, but one also needs to consider dynamics, and it is good to wear a dynamical hat even for investigations whose primary explicit focus is on structure. Indeed, a major purpose for studying network structure is as a necessary prerequisite for attaining a deep understanding of dynamical processes that occur on networks. How do social contacts affect disease and rumor propagation? How does connectivity affect the collective behavior of oscillators? The purpose of our monograph is to provide a tutorial for conducting investigations that explore (and try to answer) those types of questions. We will occasionally discuss network structure in our tutorial, but we are wearing our dynamical-systems hats.

Scope, Purpose, and Intended Audience

The purpose of our monograph is to give a tutorial for studying dynamical systems on networks. We focus on “simple” situations that are analytically tractable, though