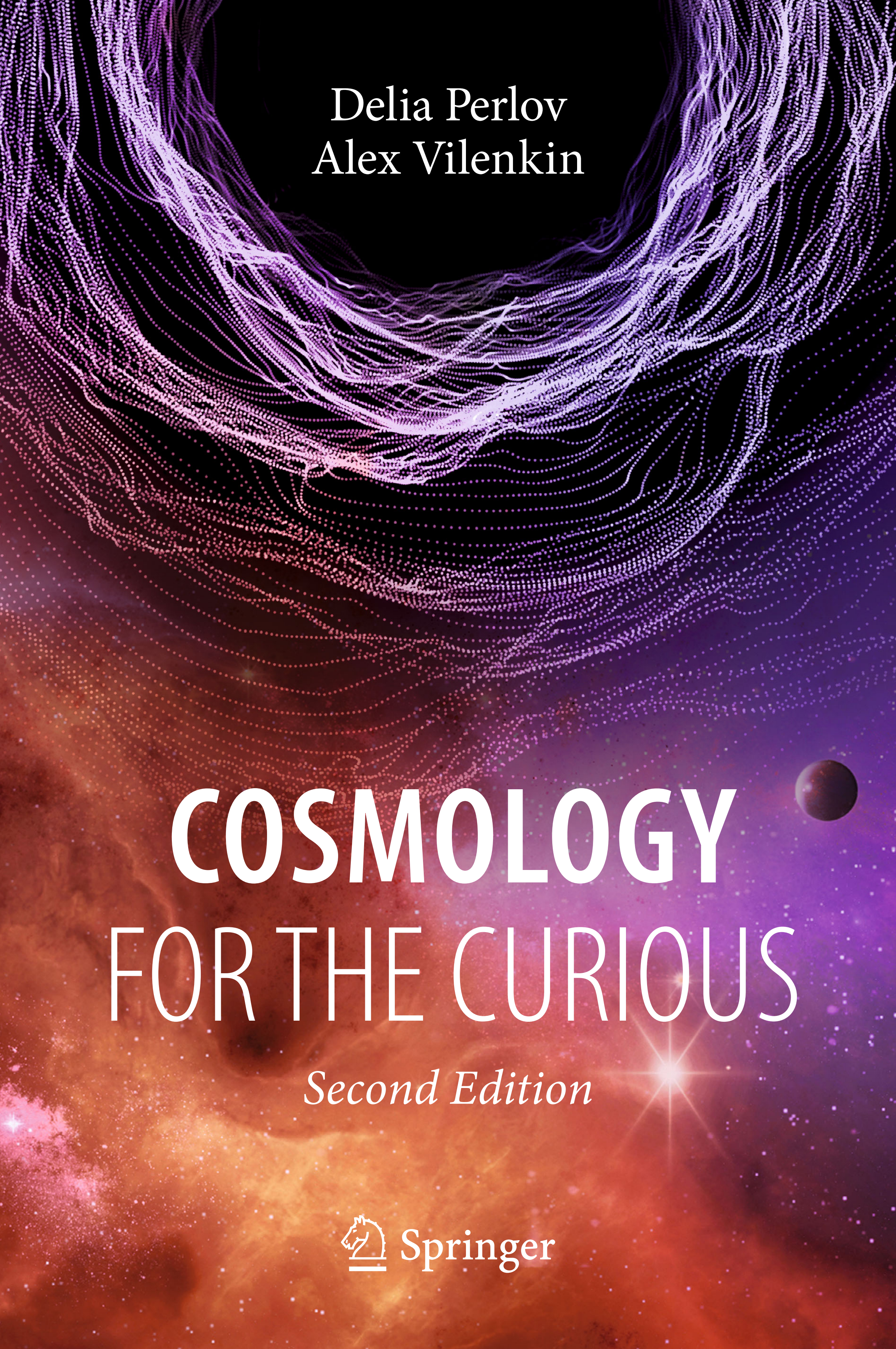

Delia Perlov
Alex Vilenkin

COSMOLOGY FOR THE CURIOUS

Second Edition

Springer

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Delia Perlov · Alex Vilenkin

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To the memory of Allen Everett and Leonard Schwartz

Preface

Since Cosmology for the Curious appeared in 2017, we have had many requests—from educators, students and general readers—to offer hints and solutions for the questions we include at the end of each chapter. In this second edition, we offer a large selection of hints that should help to surmount potential obstacles, while still allowing for the challenge of creative problem-solving. Numerical solutions are also offered where they can be used to see if a particular problem was solved correctly.

We have added a new chapter, which we included as a standalone appendix, that gives a comprehensive summary of the Standard Cosmological Model. We also expand the discussion of gravitational waves and their detections and discuss the exciting results from the NanoGrav collaboration. We added a section about the Hartle-Hawking no-boundary proposal of quantum cosmology. Finally, we discuss the challenges of searches for dark matter and of the Hubble expansion discrepancy, which may suggest unknown new phenomena.

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Alex Vilenkin

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Alex Vilenkin would like to add his sincere gratitude to Leonard and Jane Bernstein for their continuing interest and support.

We would both like to express our sincere gratitude to Ken Olum and Andrei Gruzinov for making many very useful suggestions for the second edition of Cosmology for the Curious.

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Part I

The Big Bang and the Observable Universe

1

A Historical Overview

1.1 The Big Cosmic Questions

Cosmology is the study of the origin, nature and evolution of our universe. Its practitioners strive to describe cosmic history in quantitative detail, using the language of modern physics and abstract mathematics. Yet, at its core, our cosmological knowledge is the answer to a few fundamental questions. Have you ever drifted off deep into thought, wondering: Is the universe finite or infinite? Has it existed forever? If not, when and how did it come into being? Will it ever end? How do we humans fit into the grand scheme of things? All ancient and modern cultures have developed creation stories where at least some of these questions have been addressed.

In one of the Chinese creation myths, the universe begins as a black egg containing a sleeping giant, named Pan Gu. He slept for 18,000 years and grew while he slept. Then he woke up and cracked the egg open with an ax. The light part of the egg floated up to form the sky, while the heavy part stayed down and formed the Earth. Pan Gu remained in the middle and continued to grow, pushing the sky and the Earth further apart. When Pan Gu died, his breath became the wind, his eyes the Sun and the Moon, his sweat turned into rain, and the fleas in his hair transmuted into humans.

The prospect of being a descendant of fleas may not be fully satisfying, but perhaps an even more objectionable aspect of this story is that it does not address the obvious question: "Where did the black egg come from in the first place?" Similar types of questions also arise in the context of scientific cosmology. Even if we claim to know what happened at the beginning of the

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universe, you can always ask: And what happened before that? There is also a limit to how far we can see in space, so how can we know what lies beyond?

For a long time it seemed as though we would never know the answers to the "big" cosmic questions. Thus, cosmologists focused mostly on the part of the universe that could be directly observed, leaving it to philosophers and theologians to argue about the great mysteries. We shall see, however, that due to remarkable developments in cosmology over the last few decades, we now have answers, that we have reason to believe, to at least some of the big questions.

1.2 Origins of Scientific Cosmology

The idea that the universe can be rationally understood is at the foundation of all scientific knowledge. This concept is now commonplace, but in Ancient Greece more than 20 centuries ago it was a daring hypothesis. The Greek philosopher Thales (6th century BC) suggested that all of Nature's variety could be understood from a few basic principles, without the intervention of gods. He believed that the primary element of matter was water. Two centuries later, Democritus advocated that all matter was made up of tiny, eternal, indivisible particles, called atoms, which moved and collided with one another in empty space. He stated: "Nothing exists except atoms and empty space." This line of thought was further developed by Epicurus (3rd century BC), who argued that complex order, including living organisms, evolved in a natural way, by random collisions and rearrangements of atoms, without any purpose or intelligent design. Epicurus asserted that atoms occasionally experience small random "swerves" from their rectilinear motion. He believed that these deviations from strict determinism were necessary to explain the existence of free will. Epicurus taught that the universe is infinite and that our Earth is just one of countless worlds that constantly form and decay in an infinite space (Fig. 1.1).

Another important direction of thought originated with Pythagoras (6th century BC), who believed that mathematical relations were at the heart of all physical phenomena. Pythagoras was the first to call the heavens *cosmos*, which means *order*. He suggested that the Earth, the Sun, and other celestial bodies are perfect spheres and move in perfect circles around a central fire, which cannot be seen by human eyes. Think about how different this is from the random aggregates of atoms envisioned by Epicurus!

In the 4th century BC, Plato and then Aristotle proposed more elaborate versions of this picture, placing the Earth at the center of the universe, with

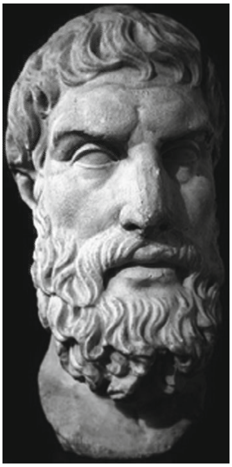

Fig. 1.1 Epicurus (341–270 BC) taught philosophy in the garden of his house in Athens, where he regularly met with a small group of followers over a simple meal. The group included women and one of his slaves. Epicurus was a prolific writer, but almost all of his writings have vanished. Epicurean philosophy flourished in ancient Greece and Rome for several centuries, but was banished in the Christian world, because of its uncompromising materialism. Its most complete exposition came to us in a magnificent poem "On the Nature of Things", written in the first century AD by the Roman poet Lucretius. The poem was lost for more than a thousand years and was rediscovered in a German monastery in 1417, just in time to influence the development of ideas during the Renaissance

the planets, the Sun and the stars attached to translucent spheres rotating about the center. This was a decidedly finite universe, where the stars were placed on the outermost sphere.

The Greeks made very accurate observations of the planets, and already in the 3rd century BC it had become evident that the simple model of concentric spheres could not adequately explain the observed motion of the planets. Further refinements of the model were getting more accurate, at the expense of becoming more complicated. First, the centers of the spheres were displaced by certain amounts from the Earth. Then came the idea of epicycles: each planet moves around a small circle, whose center rotates around a large circle, as shown in Fig. 1.2. Epicycles explained why planets seem to move backward and forward on the sky, and why they appear to be brighter during the periods of backward motion.

In some cases epicycles had to be added on top of other epicycles. All of these ideas were consolidated by Claudius Ptolemy in his book *Almagest* (The Great System), in the 2nd century AD. Ptolemy's mathematical model of the universe endured for fourteen hundred years. It accounted for all known astronomical data and also made accurate predictions.

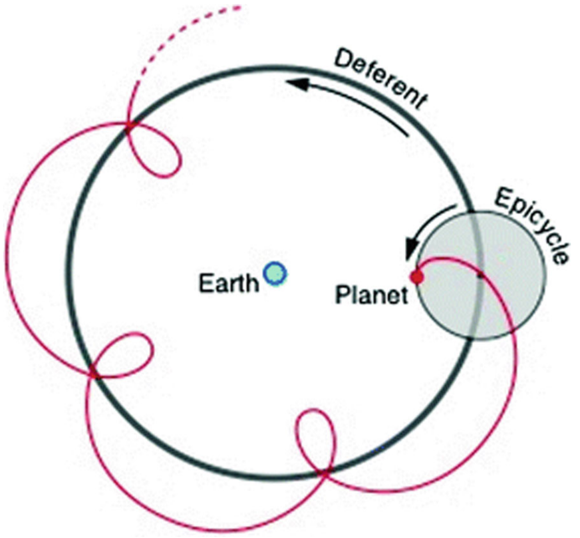

Fig. 1.2 The planet moves around a *small circle* (epicycle), whose center moves around a *large circle* (deferent) centered on Earth. The planet's resulting trajectory is shown here in *red*; most of the time the planet moves in the "forward" direction relative to the background stars, but for brief intervals, when the planet is close to the Earth, and hence is at its brightest, its direction of motion is reversed relative to the background stars. *Credit* Daniel V. Schroeder

The dismantling of the Ptolemaic worldview began in the 16th century with the work of Nicolaus Copernicus. He wanted to restore the ideal of perfect circular motion by placing the Sun at the center of the universe, and allowing the Earth to move around it in a circular orbit (this idea actually goes back to Aristarchus in the 3rd century BC). As the Earth circles around the Sun, the planets appear to move backward and forward across the sky, removing the "need" for epicycles. Copernicus devoted his life to the computation of heliocentric orbits and published his work in the book *On the revolutions of celestial spheres*, which came out in 1543, shortly before his death.

Despite its tremendous impact, it was not immediately clear that the Copernican system was superior to that of Ptolemy. Copernicus discovered that the simple model of circular orbits did not fit the data well enough. Ultimately, he also had to introduce epicycles, and even then he could not match the accuracy of Ptolemy's *Almagest*. Despite these setbacks, Copernicus still deserves to be immortalized for his greatest achievement—removing the

Earth from the center of the universe. It has been downhill for the Earth ever since then,[1] but more on that later.

The next great astronomical breakthrough was made by Johannes Kepler in the early 1600s. After nearly three decades of studying the data amassed by his eccentric mentor Tycho Brahe, Kepler discovered that planets actually move along elliptical orbits. He realized the importance of his work, but was still very disappointed, because he believed that circles are more perfect than ellipses. Kepler had other mystical beliefs—in answer to the mystery of why each planet followed its particular orbit, he suggested that the planet grasped it with its mind! (Fig. 1.3).

Then along came Isaac Newton, who had very different ideas about how the laws of Nature operate. In his seminal book *Philosophiae naturalis principia mathematica* (1687), now known as the Principia, he showed how to derive the elliptical orbits of the planets from his three laws of motion and the law of universal gravitation. He postulated that the laws of Nature apply to all bodies, in all places and at all times. Newton's laws are mathematical equations that determine how physical bodies move from one moment to the next, describing a universe which functions like a giant clockwork mechanism. To set the clockwork up, one only needs to specify the initial conditions—the positions and velocities of all physical objects at some initial moment of time. Newton believed these were provided by God. We will return to Newton and his laws in some detail, but for now we jump ahead a few hundred years to outline what we know today.

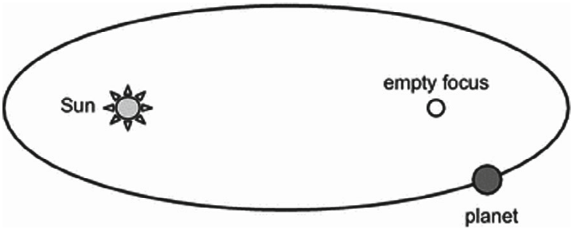

Fig. 1.3 Kepler discovered that planetary orbits are ellipses. (What is an ellipse? Consider two points, called the foci. An ellipse is the locus of points such that the sum of the distances to each focus is constant.) The Sun is located at one of the focal points of the ellipse, while the other focus is empty. For planets in the Solar System, the two foci of the ellipse are very close to one another, so the orbits are nearly circular. In this figure the ellipse is exaggerated

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[1] In fact, removing the Earth from the center of the universe was not necessarily viewed as a demotion. In those days the further out you went from the center, the closer you got to the heavenly celestial realm.

1.3 Cosmology Today

Despite its ancient roots, scientific cosmology is a relatively young science. Most of what we know about the universe has been learned within the last 100 years. In broad-brush strokes, we have discovered that our Sun belongs to a huge disk-like conglomeration of about three hundred billion stars, known as the Milky Way galaxy. Not only is the Sun merely one out of hundreds of billions of stars in our galaxy, the Milky Way is itself only one out of hundreds of billions of galaxies that are scattered throughout the observable universe. Furthermore, Edwin Hubble showed (1929) that these distant galaxies are not just suspended at rest throughout space. Rather, they are rushing away from us, and each other, at very high speeds as the entire universe expands (Fig. 1.4).

If we extrapolate this expansion backwards in time, we realize that the universe was once much denser and much hotter. In fact, we believe that the universe as we know it originated some 14 billion years ago in a great explosion called the big bang. At that time, all of space was filled with an extremely hot, dense, and rapidly expanding "fireball"—a mixture of subatomic particles and radiation. As it expanded, the fireball cooled, along the way producing nuclei and atoms, stars and galaxies, you and us! In 1965,

Fig. 1.4 Andromeda Galaxy is one of our close neighbors at some 2.5 million light years away. It is about the same size as the Milky Way. *Credit* Robert Gendler

Arno Penzias and Robert Wilson discovered a faint remnant of the primordial fireball. They found that the entire universe is bathed in a sea of low-intensity microwaves,[2] known as the Cosmic Microwave Background radiation, or CMB. Although the CMB had been predicted by theorists, Penzias and Wilson stumbled upon it serendipitously, providing the smoking gun proof for the big bang theory and earning themselves a Nobel prize in the process.

The big bang cosmology has its roots in Einstein's theory of gravity—the general theory of relativity (1915). Solutions of Einstein's equations describing an expanding universe were found by the Russian mathematician Alexander Friedmann (1922), and independently by the Belgian priest Georges Lemaitre (1927). The idea that the early universe was hot was introduced by the Russian expatriate George Gamow. Gamow wanted to explain the abundances of different chemical elements that we now observe in the universe. He argued that the hot primordial fireball was the furnace where the elements were forged by nuclear reactions. In 1948 Gamow and his colleagues Ralph Alpher and Robert Herman successfully calculated the abundances of hydrogen and helium produced during the big bang. They also tried to explain the abundances of heavier elements in the periodic table, but alas, here they were unsuccessful. It turns out that heavy elements are not synthesized during the big bang, but rather are produced in the interiors of stars. We will return to this part of our ancient history in more detail later. But suffice it to say, by the mid 1970s the major ingredients of the hot big bang picture were clearly outlined (Fig. 1.5).

Not so long ago, cosmology was not considered to be a reputable branch of science. There was very little data to test theoretical models. Two Nobel prize winning physicists, Lev Landau and Ernest Rutherford quipped, respectively, "Cosmologists are often in error, but never in doubt." and "Don't let me catch anyone talking about the universe in my lab!" Attitudes changed dramatically in the 1980s and 90s, when an abundance of data emerged. Radio and optical astronomy flourished with computerized galaxy surveys and instruments like the Very Large Array (VLA) and the Cosmic Background Explorer (COBE) satellite. A detailed map of the distribution of galaxies in space has been compiled, showing remarkable large-scale structures of filaments, sheets and voids. The Hubble Space Telescope has captured images of galaxies so far away that it took much of the age of the universe for their light to reach us. By observing these distant galaxies we can see cosmic history unfolding. The turn of the century saw the launch of the Wilkinson Microwave Anisotropy Probe

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[2] We are all familiar with x-rays, visible light and radio waves from our everyday lives. All of these are forms of electromagnetic radiation, which we will discuss later. Microwaves are a subset of radio waves.

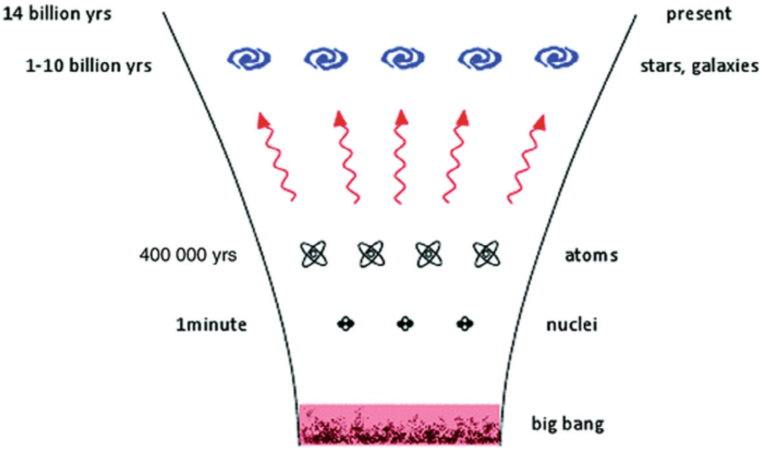

Fig. 1.5 Abridged history of the universe. Atomic nuclei were formed a few minutes after the big bang; four hundred thousand years later they combined with electrons to form atoms. At that point the universe became transparent to light, so we can see its image at that early era imprinted in the Cosmic Microwave Background radiation. Galaxies were pulled together by gravity over the course of several billion years, and we appeared on the scene in very recent cosmic time

(WMAP) satellite, to further study the image of the early universe imprinted in the Cosmic Microwave Background radiation. All these developments (and others) ushered in an era of unprecedented precision cosmology, and we are fortunate to find ourselves living during this golden age! (Fig. 1.6).

While the hot big bang theory is supported by all observations, luckily for today's cosmologists, some intriguing questions still remain. These questions bring into play a combination of studies on the largest imaginable scales, and new theoretical insights from particle physics, on the smallest imaginable scales. From the microcosm to the macrocosm, our journey has begun…

Questions

What would be your answers (or best guesses) to the following questions:

1. Is the universe infinite or finite? If it is finite, does it have a boundary? If so, what lies beyond?
2. Did the universe have a beginning? If it did, was it an absolute beginning, or did the universe exist before that in some other form?
3. If the universe did have an absolute beginning, would that require a supernatural intervention?
4. Will the universe ever end? If so, will that be an absolute end, or will the universe be transformed into some other form?

Fig. 1.6 Very Large Array radio telescopes in New Mexico. *Credit* VLA, NRAO

5. What does the universe look like in far-away regions that we cannot observe? Is it similar to our cosmic neighborhood? Is our location in the universe in any way special?
6. Do you think the universe was designed to host intelligent life?
7. Do you think we are the only life in the universe?
8. Do you find it surprising that we are able to understand the universe? Do you find it surprising that mathematics is able to explain physical phenomena (like the elliptical orbits of planets)?
9. Do you think we have free will? If so, how can it coexist with deterministic laws of physics? Do the "swerves" of atoms posited by Epicurus give a satisfactory answer?

See if your answers change after you read this book!

2

Newton's Universe

In his monumental Principia, Newton formulated the general laws of motion and the law of universal gravitation. He then applied these laws to explain the motion of planets and comets, projectile trajectories, and the marine tides, among other things. In so doing, he showed how natural phenomena could be understood using a handful of physical laws, which hold just as well for the "heavenly Moon" as for the "Earthly apple" (Fig. 2.1).

2.1 Newton's Laws of Motion

Newton's first law states that a body that is at rest will stay at rest, and a body that is moving with a constant velocity will maintain that constant velocity, unless it is acted upon by a force.

What does this mean? Let's imagine we are at an ice rink and there is a hockey puck which has been carefully placed at rest on the ice. Now we stand and watch the puck. What happens? According to Newton, the puck will stay where it is unless someone comes by and gives it a push—that is, applies a force.[1]

Now imagine we have given our little puck a push, so that it is sliding along the surface of the ice. We will assume that our ice rink has no friction.

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[1] Even a motionless puck on frictionless ice is subject to forces. Gravity pulls the puck downwards, but the surface of the ice pushes back with equal and opposite force, so the total force on the puck is zero.

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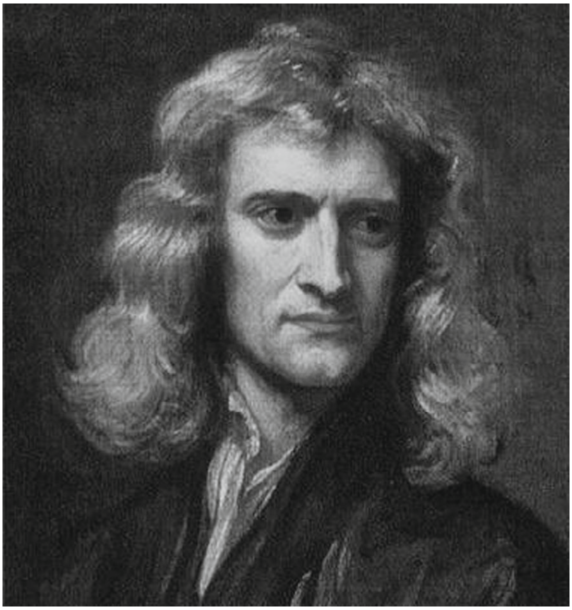

Fig. 2.1 Isaac Newton (1642–1726) made most of his major discoveries in 1665–1667, shortly after receiving his Bachelor's degree from the University of Cambridge. Although Newton earned financial support for further study, the University closed because of the plague, and he had to return to his family home in Lincolnshire for 18 months. It was during this time that he discovered his theory of colors, the law of gravitation, and calculus. In later years, apart from pursuing research in physics and mathematics, Newton devoted much effort to alchemy and to Scriptural studies. *Credit* Copy of a painting by Sir Godfrey Kneller (1689), painted by Barrington Bramley

The puck will then continue to move at a constant speed in the same direction, unless it hits the wall of the rink, or bumps into someone or something along its way. These obstructions would provide a force that would alter the puck's uniform state of motion. If our imaginary frictionless ice rink were also infinite and devoid of other obstacles, the puck would coast along at the same velocity for eternity.

Newton's first law also goes by the name of *The Law of Inertia*.[2] A spaceship traveling with its engines turned off in interstellar space glides along with a constant velocity, and provides yet another example of a body undergoing "inertial" motion.

Newton's second law tells us that if a force is applied to a body, the body accelerates—meaning its velocity changes. The law can be stated

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[2] The law of inertia was actually discovered by Galileo and was adopted by Newton as one of his laws of motion.

mathematically as

$$\vec{a} = \vec{F}/m \tag{2.1}$$

where $\vec{a}$ is the acceleration of the body, m is its mass, and $\vec{F}$ is the applied force. The acceleration is defined as the rate at which the velocity changes. For example, if in one second the velocity changes by one meter per second, then the acceleration is one meter per second per second, or one meter per second squared ($\mathrm{m/s^2}$). In general, if the velocity is in m/s, the acceleration is measured in $\mathrm{m/s^2}$.

The overhead arrows indicate that force and acceleration are vector quantities, which means they each have a magnitude and direction. Another example of a vector is velocity. The magnitude of a car's velocity is its speed, but very often we also need to know the direction in which the car is traveling. In Newton's first law, when we say that in the absence of forces a body moves at a constant velocity, this means that both the magnitude and direction of the velocity remain constant. When we want to refer only to the magnitude of a vector quantity, we drop the overhead arrow. For example, F is the magnitude of $\vec{F}$ and $a = F/m$ means that the magnitude of the acceleration is given by the magnitude of the force divided by the mass.

We can arrange an experiment in which the same force is applied to two different masses. Equation (2.1) tells us that the acceleration of the larger mass will be less than the acceleration of the smaller mass. Thus mass is a measure of a body's resistance to acceleration. More massive objects are harder to accelerate.

Force is measured in Newtons, which can be expressed in terms of other units as: $1\ \mathrm{N} = 1\ \mathrm{kg\ m/s^2}$. One Newton is the force required to accelerate a one kilogram (1 kg) mass at $1\ \mathrm{m/s^2}$. It is important to remember that physical quantities only have meaning when we specify units. For example, if someone asks you how old you are and you reply 240, they would think you're crazy. However, if you said 240 months, they would probably convert that to 20 years, and think it just a little odd that you chose to measure your age in months instead of years. It is also essential to use consistent units throughout any calculation.

A common misconception is to think that the direction of an applied force is always the same as the direction of motion. We need to remember that a net force acting on an object produces an acceleration in the same direction as the force, but the *velocity* of the object might be in a different direction. For example, suppose you are traveling in your car at a uniform speed, and then you apply the brakes. The force your brakes apply is in the opposite direction to motion, although your declining velocity is still in the original direction.

We have been discussing Newton's laws governing the motion of objects.[3] Although we are all familiar with velocities and accelerations from our everyday experience, it is important to point out that when we say an object is moving, we need to specify what it is moving with respect to. This defines a "reference frame". For example, during dinner on an airplane, your food tray is motionless relative to your lap, although relative to the ground it is traveling as fast as the plane. We can call your lap a "frame of reference" (the one in which the tray is still) and the ground is another, different frame of reference (relative to this frame the tray is moving very fast). So, a reference frame is an object relative to which we measure the locations and motions of other objects.

An *inertial frame of reference* is a frame associated with an object that is not acted upon by any net force and is moving by inertia. Once we specify one inertial frame of reference, any other frame that is moving with a constant velocity relative to the chosen frame, is also an inertial frame of reference. For example, the room you are in now is an inertial frame of reference (approximately).[4] Any train outside that is moving with a constant speed relative to the room is also an inertial reference frame. Newton's laws apply in all inertial frames of reference, thus any experiment you do in your room will yield the same results as the identical experiment performed by a friend on one of those trains.

2.2 Newtonian Gravity

Every day we experience the force of gravity. Gravity is an attractive force—it brings objects together. Every atom in our bodies is attracted to the Earth. Furthermore, every atom in the Earth is attracted to us. In fact any two objects in the Universe exert a gravitational attraction on one another. Newton realized that the same kind of force responsible for an apple falling from a tree was also responsible for the revolution of the Moon around the Earth, and the Earth around the Sun (see Fig. 2.2). Thus his law of gravity

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[3] Newton also formulated a third law, which states that in every interaction between two bodies, the force the first body exerts on the second body is equal and opposite to the force the second body exerts on the first. If you push your friend facing you on an ice-rink, she will coast backwards, but so will you.

[4] The Earth is not exactly an inertial frame because of its rotation about its axis, which can be observed with a Foucault pendulum.

is sometimes called The Law of Universal Gravitation, applying both to the Earthly and the heavenly realm.

Newton's law of gravity states that any two objects are attracted to one another with a force

$$F = \frac{GMm}{r^2} \tag{2.2}$$

where M and m are the masses of the two objects and r is the distance between them. The force acting on mass m is directed towards the mass M and vice versa (see Fig. 2.3). We have also introduced Newton's gravitational constant G, which has a measured value of $G = 6.67 \times 10^{-11}\ \mathrm{Nm^2/kg^2}$.

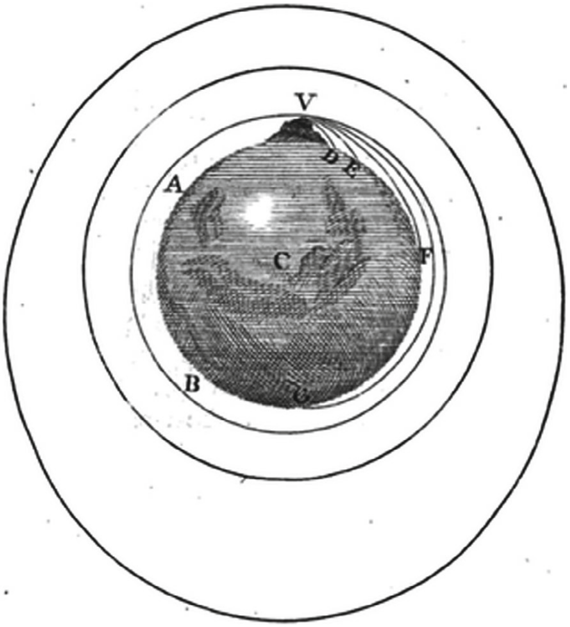

Fig. 2.2 Newton's thought experiment. Suppose a cannon is placed on top of a mountain and is fired with a moderate muzzle velocity. What will happen to the projectile? It will fall to Earth as shown at point *D*. If the muzzle velocity is increased it will fall a little farther away, as shown at points *E*, *F*, and *B*. Newton deduced that if the projectile is launched with progressively larger velocities, eventually, at just the right launch velocity, it will travel all the way around the Earth in a circular path, always falling in towards the Earth, but never reaching it, as indicated at *A*. Newton concluded that the Moon's orbit was of the same nature, with the Moon constantly falling toward the Earth. He also realized that if the launch velocity got higher, then elliptical orbits would be possible as shown. *Credit* Philosophae Naturalis Principia Mathematica

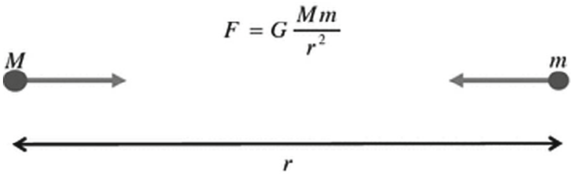

Fig. 2.3 Gravitational force of attraction between two point masses a distance r apart

Newton's law of gravity is an "inverse square law", because in Eq. (2.2) the gravitational force is inversely proportional to the square of the distance between the two objects. For example, let M be the mass of the Earth and m the mass of the Moon. If the Moon were placed twice as far away from the Earth as its actual distance, then the Earth would exert a force of gravity on the Moon that is one quarter as strong as it currently is.

The masses in Eq. (2.2) are assumed to be "point masses"; that is, we assume that their sizes can be neglected, so we can imagine that each mass is located at a point. This is a good approximation for the Earth—Moon system: the sizes of the Earth and the Moon are much smaller than the distance between them, so they can be approximated as point masses located at their centers. Then, to calculate the gravitational force of attraction, we use the distance from the center of the Earth to the center of the Moon. The same logic applies to the Earth orbiting the Sun.

Furthermore, Newton proved the "shell theorem", which states two important facts: (1) A uniform spherical shell of matter attracts an outside object as if all of the shell's mass were concentrated at its center. This applies to any uniform spherically symmetric object, like a solid sphere, since the object can be thought of as consisting of shells. (2) The gravitational force exerted on an object that is *inside* a uniform spherical shell of matter is zero. This result is surprising. The object doesn't even have to be at the center of the spherical shell—it can be anywhere inside the shell, and it will still feel no force.[5]

To find the force of gravity acting on a small object near the surface of the Earth, we can imagine that the Earth (which is nearly spherical) is composed of a large number of thin concentric shells. Each shell will act as if all its mass is localized at the center, so the overall effect will be as if the entire mass of the Earth is localized at its center. Note that we do not have to assume that the mass density is uniform throughout the volume: each individual shell must

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[5] To prove the shell theorem, Newton represented the shell as consisting of a large number of point masses and added together the forces produced by all of these masses. He had to invent calculus to perform this calculation!