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Andreas Öchsner

Plasticity Theory

Yield Conditions in the Invariant Space

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Preface

Classical engineering materials have reached in many technical applications their limits, and the demand for more performance drives the development of new materials such as composite or structured materials. This is many times the result of strict regulations, for example, in regards to fuel consumption or ecological aspects. Many of such advanced materials cannot be described by classical constitutive equations and as a result, commercial finite element packages may lack such advanced formulations. To experimentally investigate the yield condition of advanced materials, the realization of multiaxial stress states is required. Thus, a comprehensive overview of different experimental techniques is given. An advantageous description of such yield conditions is based on so-called stress invariants, which are independent of particular coordinate systems and represent the physical content of the stress matrix. This theory is also introduced and serves at the end of the book as the basis for the implementation of constitutive equations into finite element programs.

Esslingen, Germany
January 2024

Andreas Öchsner

Contents

Symbols and Abbreviations

Latin Symbols (Capital Letters)

A Area
$\boldsymbol{C}$ Elasticity matrix
F Force, yield condition
G Shear modulus
H Kinematic hardening modulus
I Principal invariant, second moment of area
I_{p} Polar second moment of area
I° Principal invariant of hydrostatic stress matrix
I' Principal invariant of deviatoric stress matrix
J Basic invariant
J° Basic invariant of hydrostatic stress matrix
J' Basic invariant of deviatoric stress matrix
$\boldsymbol{K}$ Stiffness matrix
M Moment
T_{mt} Melting temperature

Latin Symbols (Small Letters)

d Diameter
$\boldsymbol{f}$ Column matrix of external forces
h Evolution function of hardening parameter
i Index number
k Yield stress
k^{init} Initial yield stress
m Element of Jacobian matrix, slope
$\boldsymbol{m}$ Vector function
$\boldsymbol{n}$ Normal vector

p Pressure
q Internal variable (hardening)
$\boldsymbol{q}$ Column matrix of hardening variables
r Plastic flow direction, radius, residual
s Wall thickness
s_{ij} Deviatoric stress matrix
$\boldsymbol{s}$ Deviatoric stress matrix
$\boldsymbol{t}$ Traction vector
$\boldsymbol{u}$ Column matrix of displacements
v Argument matrix
w Volume-specific work or energy
w^{s} Deviatoric part of volume-specific work or energy
w° Spherical part of volume-specific work or energy
x Cartesian coordinate
y Cartesian coordinate
z Cartesian coordinate

Greek Symbols (Small Letters)

α Angle, back stress/kinematic hardening parameter, factor
η Parameter
θ Lode angle
κ Isotropic hardening parameter
λ Consistency parameter
ν Poisson's ratio
ξ Haigh-Westergaard coordinate
$\boldsymbol{\xi}$ Vector, $|\boldsymbol{\xi}| = \xi$
π Volume-specific energy
$\overline{\pi}$ Volume-specific complementary energy
ρ Haigh-Westergaard coordinate
$\boldsymbol{\rho}$ Vector, $|\boldsymbol{\rho}| = \rho$
σ Normal stress
σ_{m} Hydrostatic stress
σ_{ij} Stress matrix
σ_{ij}° Hydrostatic stress matrix
$\boldsymbol{\sigma}$ Stress matrix
τ Shear stress