

SpringerBriefs in Physics

Stéphane Peigné

Color in QCD

An Introduction Featuring
the Birdtrack Pictorial
Technique

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 Springer

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Nantes, France

ISSN 2191-5423

SpringerBriefs in Physics

ISBN 978-3-031-53680-9

<https://doi.org/10.1007/978-3-031-53681-6>

ISSN 2191-5431 (electronic)

ISBN 978-3-031-53681-6 (eBook)

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Preface

Quarks and gluons, the elementary particles that make up nucleons (protons and neutrons) and, more generally, all hadrons, interact via the strong interaction described by quantum chromodynamics (QCD). This manual focuses on the *color* of quarks and gluons, the quantum number responsible for their mutual strong interactions, and provides an elementary introduction to the birdtrack technique, which is a powerful tool for addressing the color structure of QCD in a pictorial way, drawing color graphs named *birdtracks* rather than writing mathematical symbols carrying color indices. The birdtrack technique shows how quark and gluon colors are combined and mixed in QCD. We will learn the basic rules, discuss color conservation and infinitesimal color rotations, learn how to project on the color states of systems of quarks, antiquarks, and gluons (called partons), derive their Casimir charges... and at the same time learn a little bit of representation theory.

This manual is primarily intended for particle physics students, graduates, and researchers working in the field of QCD, but may also be of interest to students of mathematics, as an illustration of the use of the birdtrack technique in representation theory. It does not require any specific prerequisite (except perhaps a few notions of linear algebra, which will be recalled anyway) and includes very simple exercises to keep the focus and learn by doing. Doing these exercises is mandatory to avoid superficial reading and to learn how to use birdtracks in practice.

We will consider $SU(N)$ with $N \geq 3$ as the symmetry group of QCD, i.e., N quark colors, $N = 3$ corresponding to real-world QCD. Indeed, for general N the birdtrack technique is not more complicated than for $N = 3$. On the contrary, working with a general parameter N for the number of quark colors usually brings a deeper understanding than working with fixed $N = 3$. The birdtrack technique can be used as a handy tool in virtually any $SU(N)$ color calculation, whether it is evaluating simple color factors or addressing complex color structures that may otherwise seem out of reach.

This introductory manual should provide a basic toolbox for drawing birdtracks, which I hope will be useful for acquiring more in-depth knowledge to tackle advanced problems on the subject.

The content of Chaps. 1–4 is that of the lectures given at the *6th Chilean School of High Energy Physics* (Valparaiso, January 2023). These lectures inspired this manual, and I'd like to thank the organizers of the Chilean school for their invitation, and all the participants for their enthusiastic involvement.

May the present manual give you a taste of the unspeakable satisfaction that can come from drawing birdtracks!

Nantes, France
December 2023

Stéphane Peigné

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