

Michał Stosiak · Mykola Karpenko

Dynamics of Machines and Hydraulic Systems **Mechanical Vibrations and Pressure Pulsations**



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Dynamics of Machines and Hydraulic Systems

Mechanical Vibrations and Pressure Pulsations



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Preface

The subject of this book is to examine the influence of mechanical vibration on the changes in the pressure pulsation spectrum of hydraulic systems. In book displayed that machines and equipment equipped with hydraulic systems are a source of vibration with a wide frequency spectrum. Additionally, hydraulic valves are also exposed to vibration. Vibrations of the substrate on which the hydraulic valve is installed force the control element of the hydraulic valve to vibrate. The control element's vibration produced in this way causes changes in the pressure pulsation spectrum of the hydraulic system. A friction model modified using mixed friction theory can be used for the oscillating motion of the hydraulic directional control spool. Passive vibration isolation methods are proposed to reduce valve vibration. The biomimetic approach can be implemented in hydraulic systems (for pipeline) to reduce mechanical vibration and fluid pulsation. Numerical methods are employed to analyze the effect of changes in the pressure pulsation spectrum on the hydraulic efficiency of the pipelines. Examples are provided for the implementation of numerical methods in the calculation of hydraulic components and systems. Additionally, the effects of energy-saving in hydraulic systems by applying proposed results overview in the current book. The current book will be interesting for both-scientifical and manufacturing stuff—since the implementation of knowledge can help to design more substantial construction of machine hydraulic systems with avoiding vibration problems.

Chapter 1 is a concise classification of the term vibration, as well as its sources and selected effects. We emphasize that mechanical vibration that occurs in the working environment of the machine adversely affects the hydraulic valves, the machine, and its human operator. The frequency spectrum of the acting vibrations has been highlighted, with a particular focus on low frequencies, infrasonic frequencies, and frequencies close to the natural frequency of the hydraulic valve controls. Based on reported experiments, Chap. 1 also addresses the excitation of mechanical vibration of hydraulic lines caused by the pulsating flow of the working fluid. To demonstrate the coincidence between pressure pulsation in the hydraulic system and mechanical vibrations of the system's components, a constructed hydraulic pulsator stand is presented based on the latest generation proportional valve. The experimental results prove the occurrence of vibration of the body of the tested proportional directional valve. The vibration is induced by the pulsation of

pressure in the hydraulic system, where reports focus on the frequency range <100 Hz. This problem is referred to in the literature as flow-induced vibration (FIV). Additionally, Chap. 1 explores energy-saving strategies in hydraulic systems through the mitigation of dynamic loads, focusing on mechanical and hydraulic vibrations. It delves into the realm of mechanical vibrations in machinery equipped with hydraulic systems, offering insights into their classification.

Chapter 2 identifies the sources of pressure pulsations in hydraulic systems. Four basic sources are specified: pulsation of displacement pumps efficiency, transient states, the influence of external mechanical vibrations on hydraulic system elements, and the wave phenomena in hydraulic lines of a long hydraulic line (hydraulic transmission line). Based on literature analysis, the instantaneous efficiency of the basic types of positive displacement pumps (single-acting vane and external tooth meshing) has been determined using analytical relationships. Computer simulation models have been created using the available modules of computer software. To identify the frequency spectrum of the capacity pulsation, the time waveforms and the amplitude-frequency spectra are presented. This chapter shows that an inseparable stage of the hydraulic system's operation is its start-up or braking (sometimes temporary), which is classified by transient processes. Furthermore, unlike pressure pulsation caused by the pulsation of the capacity of positive displacement pumps, the spectrum of pressure pulsations during the system start-up also includes lowfrequency harmonics. We provide an example of the coincidence of external mechanical vibration and pressure pulsations for a micro-relief valve and a conventional one-stage relief valve. The occurrence of resonance phenomena in the hydraulic lines at quasisteady flows is also described: the conditions for the occurrence of such phenomena and their effects are given. The quasi-steady friction model is combined with the concept of a hydraulic cross involving four items, Laplace transforms of pressure p_1 , flow rate q_1 at the beginning of the hydraulic transmission line, and Laplace transforms of pressure and flow rate at the end of the hydraulic transmission lines p_2 and q_2 , respectively. These tools are utilized to determine the transmittance maxima in the hydraulic system, and for the selected and parameterized hydraulic system, the resonance lengths of the lines are determined that increase the amplitude of pressure pulsation due to quasi-steady flow.

Chapter 3 is dedicated to employing numerical techniques for the assessment of the influence of fluid flow pulsations on vibrations in hydraulic lines. Throughout this chapter, we explore the mathematical representation of heightened pulsations in hydraulic system lines, employing the axial-piston pump model. Additionally, we elaborate on the mathematical depiction of high-pressure hose behavior, contingent upon fluid flow. This chapter introduces a fluid-solid coupling mathematical framework, integrating mechanical equations resolved through the finite elements method (FEM) and hydrodynamic equations addressed via the method of characteristics (MoC). These two sets of equations are concurrently solved, resulting in a comprehensive model designed to enhance our understanding of the complexities associated with fluid pulsations within hydraulic drive lines.

Chapter 4 examines the transmission pathways of external mechanical vibrations to the control element of the hydraulic valve (e.g., lift valve poppet or directional spool). For this purpose, we present common mathematical models of friction in motion pairs occurring in hydraulic elements. The experimental results of work conducted on the value related to friction force in the spool pair are shown. Then, based on our own theoretical and experimental considerations, we attempt to identify a significant damping parameter in the transmission of mechanical vibration to the valve control element. Our results demonstrate the excitation of vibrations of the valve control element by the external mechanical vibration and the related changes in the amplitude-frequency spectrum of pressure pulsations of the system. The influence of the direction of mechanical vibration on the excitation of the control element of the hydraulic directional valve is presented. Correlation is shown between the external mechanical vibrations of the valve and changes in the amplitudefrequency spectrum of pressure pulsations in the hydraulic system with the valve exposed to vibration, depending on the direction of external vibration. This problem is referred to as fluid-structure interaction (FSI) in the international nomenclature and covers more complex issues than the previously mentioned FIV. Mathematical models of cooperation of the hydraulic spool pair of the hydraulic proportional valve are refined using elements of mixed friction theory. A mathematical description closest to the experimental data has been determined based on the experimental verification studies. The parameters of the mathematical models are estimated using special computer tools.

Chapter 5 focuses on the possibilities of reducing the impact of mechanical vibration on hydraulic values: on their body and the control element. This is examined theoretically and experimentally. Passive methods of isolating the valve body from vibration are considered using a specially designed and manufactured holder that simulates the machine frame, in which the hydraulic valve is mounted in a flexible manner. Flexible mounting of the valve consists of inserting a spring pack or a set of elastomer washers (or made of oil-resistant rubber) between the ground (special holder) and the valve body, on one or both sides of the valve's body. The elastomer washers are introduced by placing the washers between a special handle and the valve body, or specially profiled washers are inserted inside the valve, between the springs centering the spool and the body. Conclusions from the experimental testing were extended by the theoretical analysis of the possibilities of vibration reduction carried out in the further part of Chap. 7. Additionally, the present chapter unveils the research findings on a suggested energy-saving method in hydraulic drives rooted in a biomimetic approach. The study involves experimental measurements aimed at investigating energy consumption across various types of highpressure hoses (pipelines) and assessing its impact on internal fluid flow dynamics. In the chapter, approximate analytical methods for the analysis of non-linear vibration reduction systems are provided, which can be applied to hydraulic valves.

Chapter 6 presents the research results of the proposed energy-saving way in hydraulic drives based on a reducing vibration. Additionally, the effects of energy-saving in

hydraulic systems by applying proposed results overview in the current chapter. The current book chapter held present how implementation of knowledge can help to design more substantiable construction of machine hydraulic systems with avoiding vibration problems.

Chapter 7 shows the general observations on transmission paths, as well as the impact and the possibility of reducing the impact of external mechanical vibration on hydraulic valves. A simplified, practical scheme for selecting the hydraulic valve's vibration isolation is also provided.

Appendix provides extended results of testing the effects of external mechanical vibrations on a single-stage pressure relief valve and a single-stage conventional 4/3 spool valve. The results are presented in the form of amplitude-frequency spectra of pressure pulsations in the hydraulic system, in which the tested valve vibrates harmonically with a specific frequency and amplitude.

Wrocław, Poland Vilnius, Lithuania Michał Stosiak Mykola Karpenko

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Vibrations in Machines Fitted with Hydraulic Systems

1.1 Introduction

The utilization of fluid power in hydraulic systems has witnessed a transformative evolution over the past century. In 1906, oil emerged as a substitute for water as the pressurized fluid in hydraulic systems, opening up a new possibilities for industrial applications. The introduction of servo valves during World War II further revolutionized the field by enabling electrical control of fluid power, paving the way for its integration into the commercial sector. As a result, the fluid power industry has grown into a thriving multibillion-dollar sector today. In the extraction industries, such as mining, road construction, logging, farming, and transportation, fluid power circuits have become an integral part of most mobile machines. This widespread adoption has rendered fluid power a crucial element in the global economy's raw materials collection process. Moreover, hydraulic drives have proven indispensable in the transportation industry due to their ability to generate substantial force and torque within systems.

However, hydraulic systems encounter challenges during operation, particularly in dealing with various types of vibrations. These vibrations range from fluid flow-induced vibrations to internal and external machine-related vibrations. Additionally, hydraulic drives employed in construction and mobile machines suffer from high-energy consumption and low efficiency, typically achieving only around 60–70% efficiency. To address these issues, the European Union has adopted strategies like "Transport 2050" [1] to improve energy efficiency and sustainability. As a response to the growing importance of energy-saving measures in European strategies, extensive research is currently directed towards increasing the efficiency of hydraulic drives. One promising approach involves mitigating different types of vibrations in hydraulic systems to enhance overall performance. Consequently, in-depth research into the types and potential methods of vibration damping in hydraulic drives has become a prominent and contemporary in pursuit of

energy-saving goals. By investigating these aspects, researchers aim to make substantial contributions to enhancing the efficiency and reliability of hydraulic drives, aligning with the objectives set forth by energy-conscious European initiatives.

This book chapter explores energy-saving strategies in hydraulic systems through the mitigation of dynamic loads, focusing on mechanical and hydraulic vibrations. It delves into the realm of mechanical vibrations in machinery equipped with hydraulic systems, offering insights into their classification. The chapter also examines how machines can serve as both generators and recipients of mechanical vibrations, emphasizing their dual role. Furthermore, the text delves into fluid pulsations (vibrations) within hydraulic systems of machines. It discusses the impact of fluid pulsation on pipelines and highlights the consequential influence on hydraulic system performance.

1.1.1 Energy-Saving in Hydraulic Systems by Reduce Dynamic Loads—Mechanical and Hydraulic Vibrations

Fluid-based power technology involves the process of converting mechanical energy into liquid energy, transmitting this energy to a designated point of use, and subsequently converting it back into mechanical energy. A fluid power circuit incorporates all three essential elements: the conversion of mechanical energy to liquid energy, the delivery of this fluid energy, and the reconversion of the fluid energy back into mechanical energy. This process occurs simultaneously, even when subject to varying types of vibration (refer to Fig. 1.1).

Various construction, building, transport, and road machines have been developed using diverse design concepts and approaches. According to Helbig [2], these machines utilize a hydraulic drive system for powering their working equipment. This hydraulic drive is achieved through power transmission, which can consume a significant portion of the energy generated by the primary combustion engine or electric motor during the machine's working cycle. The energy consumption for the power transmission can range from 30% to as high as 90% of the total energy output from the combustion engine or electric motor. The hydraulic drive remains relevant in three main areas of energy conservation, which can be further divided into several sub-directions: Regulation and control of hydraulic drives; Increasing the efficiency of hydraulic drives through fluid recovery; Reducing



Fig. 1.1 Concept of hydraulic drive fluid power circulation and vibration types



Fig. 1.2 Energy study in hydraulic systems: a energy-saving methods compering; b reducing dynamic loads (vibration)

dynamic loads in the system. Karpenko et al. [3] conducted a detailed review of energysaving research in hydraulic drives, revealing the following popular research directions and their respective percentages:

- Usage of energy recovery systems (ERS) (37%);
- Methods for reducing dynamic loads in a system (RDL) (33%);
- Control of systems (CS) (30%).

Each of these reviewed methods is nearly equivalent in importance, as depicted in Fig. 1.2a. According to Borghia et al. [4] only RDL method can be implemented in nearly any hydraulic drive with minimal modifications.

In the research works by Adachi et al. [5] and Wang, Huang [6] have suggested that an effective approach to lower power consumption in hydraulic drives is by minimizing dynamic effects within the system. Stosiak [7] also highlights the adverse consequences of periodic pressure fluctuations in hydraulic systems, such as excessive noise emission, reduced component lifespan, disruptions in control loops, power loss (pressure) within the system, and more. To address this type of problem, one common approach involves incorporating elastic-damping elements into the hydraulic drive, as proposed by Ortwig [8]. Additionally, hydraulic accumulators and various fluid flow dampers, as indicated in the works of Han and Wang [9], and Amirante et al. [10], can be utilized to reduce dynamic loads in hydraulic drives.

For instance, Makaryants et al. [11] focused on mitigating fluid pressure pulsations and hydraulic resistance in a punching machine equipped with a high-pressure hose near the pump. By installing a damping element, they achieved a remarkable reduction of hydraulic pulsations in the system, resulting in about 38...40% decrease from the initial levels. This reduction in pulsations led to a decrease in pressure loss and contributed to

approximately 13% energy savings in the hydraulic drive system. In test benches with harmonic vibrations, proposed by Yakushev [12], the feed pump's supply can be reduced by 30% with reducing fluid fluctuations by installing in the pressure line air-hydraulic accumulator for saving energy more than 23%.

Certainly! The utilization of a hydraulic accumulator in power cylinders of a manipulator, with a payload capacity of 0.8 t, has proven to be effective in reducing energy costs by 7...12%. This reduction is attributed to the alleviation of peak pressure spikes within the hydraulic system during transient regimes, resulting in a diminished dynamic effect on the hydraulic pump. This research was conducted by Nesmiyanov and Khavronin in 2007 [13]. Furthermore, when short-term high-speed mechanisms (ejectors) are present in the hydraulic system, installing damping elements directly near the cylinders has shown promising energy-saving results. This installation approach eliminates vibrations in long high-pressure hoses, contributing to enhanced energy efficiency within the hydraulic system.

According to Shen et al. [14], one of the drawbacks of active vibration compensation systems is that the energy introduced into the hydraulic system through the actuator can negatively impact the overall system stability. In cases where the regulator is not precisely tuned, there is a risk of deteriorating the system's characteristics, and under certain conditions, the amplitude of pressure fluctuations may actually increase instead of being reduced. Hence, careful and accurate tuning of the regulator is crucial to ensure the effectiveness and stability of active vibration compensation systems in hydraulic applications.

The comperes of using different types of reducing dynamic loads in the system for energy-saving shown in Fig. 1.2b, where: DE—Damping element; DCV—Damping of the cylinder and valves; RHP—Reduce hydraulic pulsation; DEHA—Damping effect by use hydraulic accumulator.

From another point of view, pressure fluctuations can occur either in the hydraulic systems themselves (fluid vibration) or due to external causes (mechanical vibration), for example, due to periodic fluctuations in load on hydraulic cylinders, valves, pipeline etc. [15]. Also known that in hydraulic systems with high dynamics (for example, proportional valve or servo valve) and with hydraulic cylinder or motor, there maybe a too high-pressure level of fluctuation what leads to energy losses mentioned by Stosiak [7]. In this case the detail research on the mechanical and fluid vibration characteristics in hydraulic systems and possible of its reducing currently one of the commom problem for solving in line obtain energy-saving by minimal modifications of hydraulic system.

1.1.2 Mechanical Vibrations in Machines Fitted with Hydraulic Systems

In this section, the main focus is provided on free (natural) vibrations when no force is applied to the system (machine), and when the pre-existing state of equilibrium is disturbed in another way. The section includes classification of mechanical vibration in machines fitted with hydraulic drive and introduction to machine as a source and receiver of mechanical vibration.

1.1.2.1 General Remarks on the Classification of Mechanical Vibrations

Operating machines and devices fitted with hydraulic systems are a source of mechanical vibrations over a wide spectrum of frequencies [16, 17]. These vibrations can generally be divided into free, forced, and self-excited categories. In this chapter, the mathematical description related to vibrations induced by a harmonic force is examined. This type of vibration will be analyzed in further chapters.

In this section, we focus on free (natural) vibrations when no force is applied to the system (machine), and when the pre-existing state of equilibrium is disturbed in another way. In working machines, such vibrations can be caused by a sudden change in the movement conditions of the machine (e.g., start-up, braking—during a sudden override of the hydraulic valves controlling the operating parameters of hydraulic systems and a sudden change in the speed of the receiver or tooling), and a sudden change in load conditions (e.g., change in the resistance of the excavated soil, the machining tool blade entering and going out of the material, and change in the direction of the load). These vibrations (adamping forces). Free vibrations depend on the properties of the system and its initial state [18], i.e., on its velocity and position at the initial moment t = 0.

In the case of forced vibrations when the external impacts on the system (machine) are variable in time. When the external forces are periodic, the vibration forces will also be periodic. Forced vibrations can be induced kinematically, in which case we speak of kinematic forcing. In addition, vibrations can be forcefully induced, in which case we speak of force forcing. Kinematic excitations are created, for example, by transferring vibrations generated by other machines through the foundation during the movement of a mobile machine on uneven ground [19]. The causes of forced vibrations also include factors such as unbalance of rotating parts, the eccentricity of the machined surface in relation to the machined one [20] (for machine tools), and the intermittent nature of the machine operation. Some mechanical vibration impact and their applying point in machine (including hydraulic system), on the example of excavator displayed on Fig. 1.3.

<u>Vibrations as effects of a harmonic force applied</u>. There is an abundance of literature related to the analytical study of this type of vibration [18, 21–23]. In the case of vibration isolation, harmonic vibrations and forces are of particular importance, hence, we briefly