SpringerBriefs on PDEs and Data Science

Mikko Parviainen

Notes on Tug-of-War Games and the p-Laplace Equation

Springer

SpringerBriefs on PDEs and Data Science

Editor-in-Chief

Enrique Zuazua, Department of Mathematics, University of Erlangen-Nuremberg, Erlangen, Bayern, Germany

Series Editors

Irene Fonseca, Department of Mathematical Sciences, Carnegie Mellon University, Pittsburgh, PA, USA

Franca Hoffmann, Hausdorff Center for Mathematics, University of Bonn, Bonn, Germany

Shi Jin, Institute of Natural Sciences, Shanghai Jiao Tong University, Shanghai, Shanghai, China

Juan J. Manfredi, Department of Mathematics, University Pittsburgh, Pittsburgh, PA, USA

Emmanuel Trélat, CNRS, Laboratoire Jacques-Louis Lions, Sorbonne University, PARIS CEDEX 05, Paris, France

Xu Zhang, School of Mathematics, Sichuan University, Chengdu, Sichuan, China

SpringerBriefs on PDEs and Data Science targets contributions that will impact the understanding of partial differential equations (PDEs), and the emerging research of the mathematical treatment of data science.

The series will accept high-quality original research and survey manuscripts covering a broad range of topics including analytical methods for PDEs, numerical and algorithmic developments, control, optimization, calculus of variations, optimal design, data driven modelling, and machine learning. Submissions addressing relevant contemporary applications such as industrial processes, signal and image processing, mathematical biology, materials science, and computer vision will also be considered.

The series is the continuation of a former editorial cooperation with BCAM, which resulted in the publication of 28 titles as listed here: https://www.springer.com/gp/mathematics/bcam-springerbriefs

Mikko Parviainen

Notes on Tug-of-War Games and the p-Laplace Equation

Springer

Mikko Parviainen
Department of Mathematics and Statistics
University of Jyväskylä
Jyväskylä, Finland

ISSN 2731-7595 ISSN 2731-7609 (electronic)
SpringerBriefs on PDEs and Data Science
ISBN 978-981-99-7878-6 ISBN 978-981-99-7879-3 (eBook)
https://doi.org/10.1007/978-981-99-7879-3

© The Editor(s) (if applicable) and The Author(s), under exclusive license to Springer Nature Singapore Pte Ltd. 2024
This work is subject to copyright. All rights are solely and exclusively licensed by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.
The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.
The publisher, the authors, and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, expressed or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

This Springer imprint is published by the registered company Springer Nature Singapore Pte Ltd.
The registered company address is: 152 Beach Road, #21-01/04 Gateway East, Singapore 189721, Singapore

Paper in this product is recyclable.

Preface

These notes were written up after my lectures at the Beijing Normal University in January 2022. I am grateful to Yuan Zhou for organizing such an interesting intensive period. I would like to thank Ángel Arroyo and Jeongmin Han for reading the original manuscript and contributing valuable comments; as well as Janne Taipalus for some of the illustrations.

The objective is the interplay between stochastic processes and partial differential equations. To be more precise, we focus on the connection between the nonlinear p-Laplace equation and the stochastic game called tug-of-war with noise. The connection in this context was discovered roughly 15 years ago, and has provided novel insight and approaches ever since. These lecture notes provide a short introduction to the topic and to more research-oriented literature, as well as to the recent monographs of Lewicka, *A Course on Tug-of-War Games with Random Noise*, and Blanc-Rossi, *Game Theory and Partial Differential Equations*. We also introduce the parabolic case side by side with the elliptic one, and cover some parts of the regularity theory.

Jyväskylä, Finland — Mikko Parviainen
July 30, 2023

Contents