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Introduction to Modified Gravity

Second Edition

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Preface

In this book, we review various modifications of the Einstein gravity, First, we consider theories where only the purely geometric sector is changed. Second, we review scalar-tensor gravities. Third, we examine vector-tensor gravity models and the problem of Lorentz symmetry breaking in a curved space-time. Fourth, we present some results for the Horava-Lifshitz gravity. Fifth, we consider nonlocal extensions for gravity. Also, we give some comments on non-Riemannian gravity theories. We close the book with the discussion of perspectives of modified gravity.

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Chapter 1
Einstein Gravity and the Need for Its Modification

General relativity (GR) is clearly one of the most successful physical theories. Being formulated as a natural development of special relativity, it has made a number of fundamental physical predictions which have been confirmed experimentally with a very high degree of precision. Among these predictions, a special role is played by expansion of the Universe and precession of Mercure perihelion, which have been proved many years ago, while other important claims of GR, such as gravitational waves and black holes, have been confirmed through direct observations only recently (an excellent review of various experimental tests of gravity can be found in [1]).

By its concept, GR is an essentially geometric theory. Its key idea consists in the fact that the gravitational field manifests itself through modifications of the space-time geometry. Thus, one can develop a general theory of gravity where the fields characterizing geometry, that is, metric and connection, become dynamical variables so that a nontrivial space can be described in terms of curvature, torsion and non-metricity. It has been argued in [2] that there are eight types of geometry characterized by possibilities of zero or non-zero curvature tensor, torsion and so-called homothetic curvature tensor, with all these objects constructed on the base of metric and connection. Nevertheless, the most used formulation of gravity is based on the (pseudo-)Riemannian approach where the connection is symmetric and completely characterized by the metric. Within this book, we concentrate namely on the (pseudo-)Riemannian description of gravity where the action is characterized by functions of geometric invariants constructed on the basis of the metric (i.e. various contractions of Riemann curvature tensor, its covariant derivatives and a metric), and possibly some extra fields, scalar or vector ones, and only in Chap. 7 we discuss theories of gravity defined on a non-Riemannian manifold. So, let us introduce some basic definitions of quantities used within the Riemannian approach.

By definition, the infinitesimal line element in curved spaces is defined as $ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu$. The metric tensor $g_{\mu\nu}(x)$ is considered as the only independent dynamical variable in our theory. As usual, the action must be a (Riemannian) scalar, and for the first step, it is assumed to involve no more than the second derivatives of the metric tensor, in the whole analogy with other field theory models where the action involves only up to the second derivatives. The unique scalar involving

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only second derivatives is a scalar curvature R (throughout the book, we follow the definitions and conventions from the book [3] except for special cases):

$$R = g^{\mu\nu} R_{\mu\nu}; \quad R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu};$$
$$R^{\kappa}{}_{\lambda\mu\nu} = \partial_{\mu}\Gamma^{\kappa}_{\lambda\nu} - \partial_{\mu}\Gamma^{\kappa}_{\lambda\nu} + \Gamma^{\kappa}_{\rho\mu}\Gamma^{\rho}_{\lambda\nu} - \Gamma^{\kappa}_{\rho\nu}\Gamma^{\rho}_{\lambda\mu}, \tag{1.1}$$

where $\Gamma^{\mu}_{\nu\lambda}$ are the Christoffel symbols, that is, affine connections expressed in terms of the metric tensor as

$$\Gamma^{\mu}_{\nu\lambda} = \frac{1}{2} g^{\mu\rho}(\partial_{\nu} g_{\rho\lambda} + \partial_{\lambda} g_{\rho\nu} - \partial_{\rho} g_{\nu\lambda}). \tag{1.2}$$

The Einstein–Hilbert action is obtained as an integral from the scalar curvature over the D-dimensional space-time:

$$S = \int d^D x \sqrt{|g|} (\frac{1}{2\kappa^2} R + \mathcal{L}_m), \tag{1.3}$$

where g is the determinant of the metric. We assume the signature to be $(+ - - -)$. The $\kappa^2 = 8\pi G$ is the gravitational constant (it is important to note that its mass dimension in D-dimensional space-time is equal to $2 - D$, but within this book we concentrate on the usual case $D = 4$); nevertheless, in some cases we will define it to be equal to 1. The $\mathcal{L}_m$ is the matter Lagrangian.

Varying the action with respect to the metric tensor, we obtain the Einstein equations:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa^2 T_{\mu\nu}, \tag{1.4}$$

where $T_{\mu\nu}$ is the energy-momentum tensor of the matter. The conservation of the energy-momentum tensor presented by the condition $\nabla_{\mu} T^{\mu\nu} = 0$ is clearly consistent with the Bianchi identities $\nabla_{\mu} G^{\mu\nu} = 0$.

One should emphasize several most important solutions of these equations for the four-dimensional space-time. The first one is the Schwarzschild metric, which solves the vacuum Einstein equations, $T_{\mu\nu} = 0$, and describes the simplest black hole with mass M. The corresponding space-time line element looks like

$$ds^2 = (1 - \frac{2M}{r}) c^2 dt^2 - (1 - \frac{2M}{r})^{-1} dr^2 - r^2 (d\theta^2 + \sin^2\theta d\phi^2). \tag{1.5}$$

Actually, this metric is a particular case of the more generic static spherically symmetric metric (SSSM).

The second one is the Friedmann-Robertson-Walker (FRW) metric describing the simplest (homogeneous and isotropic) cosmological solution whose line element is