

SpringerBriefs in Physics

Slava Rychkov

Lectures on the Random Field Ising Model

From Parisi-Sourlas
Supersymmetry
to Dimensional Reduction

SpringerBriefs in Physics

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Preface

This book is about the Random Field Ising Model (RFIM)—a paradigmatic spin model featuring a frozen disordering field. The focus is on the second-order phase transition between the paramagnetic and ferromagnetic phases, and the associated critical exponents. The book starts by summarizing the current knowledge about the RFIM from experiments, numerical simulations and rigorous mathematical results. It then reviews the classic theoretical works from the 1970s which suggested a property of dimensional reduction—that the RFIM critical exponents should be the same as for the ordinary, non-disordered, Ising model of lower dimensionality, and related this to an emergent Parisi-Sourlas supersymmetry. As is now known, these remarkable properties only hold when the spatial dimensionality of the model d is larger than a critical dimension d_c . The book presents a new theory to estimate d_c , using standard tools such as the replica trick and perturbative renormalization group. The results of this theory are in agreement with the numerical simulations. The last chapter of the book gives a panorama of related questions which are still open.

This book will be of interest to researchers, Ph.D. students and advanced master students specializing in statistical field theory. Some more elementary steps in the derivations are left as exercises for the readers.

I am grateful to Apratim Kaviraj and Emilio Trevisani for the collaboration on the theory presented in this book, and for the valuable suggestions concerning the exposition. The book is mostly based on our papers, as well as on lectures I gave in Cortona, in Montreal, and at the IHES. I am grateful to the organizers of the Cortona Ph.D. school “Mathematics Meets Physics on Disordered Systems” (2022) and of the Montreal meeting “Conformal field theory and quantum many-body physics” (2022) for the invitation. In particular Sects. 3.2 and 4.4 contain previously unpublished material presented in Montreal and at the IHES.

Bures-sur-Yvette, France
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Slava Rychkov

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