Synthesis Lectures on Signal Processing

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William A. Pearlman

Mathematical Transformations and Wavelet Filters for Source Coding and Signal Processing Systems

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Synthesis Lectures on Signal Processing

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Summary

This lecture teaches the mathematical transforms prevalent in modern source coding and signal processing systems. We start with showing that the transform coefficients of a block of source samples perfectly represent the source with respect to a new basis of the same dimension. The objective of the transformation, implemented through matrix operations, is to produce statistically uncorrelated coefficients. We derive the optimal Karhunen-Loeve transform and present in detail other commonly used transforms with approximately uncorrelated coefficients. An efficiency measure of a transform is *coding gain*, which expresses the reduction in distortion achieved by coding the transform instead of the source data. Therefore we derive the allocation of the bit budget to transform coefficients that minimize the distortion. Coding gain calculations are carried out for a few transforms and source correlation models. While matrix operations produced the aforementioned transformations, we prove that a bank of linear filters can yield the same result. That leads to subband transforms and, in particular, wavelet transforms. Again, we derive the optimal allocation of rate to subbands and calculate coding gains using both unrealizable ideal and realizable filters. We proceed by deriving the conditions for perfect reconstruction using realizable filters that actualize the orthogonal and biorthogonal wavelet transforms. We introduce the *lifting* scheme that creates wavelet filters from stages of *lifting* starting from simple low and high pass filters. We also present wavelet filters with integer input and output. Although such filters show a small performance loss, they suffer no precision loss in computer processing. We conclude with tables showing filters and tap values of several wavelet filters in current use.

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