The Mathematics Education for the Future Project

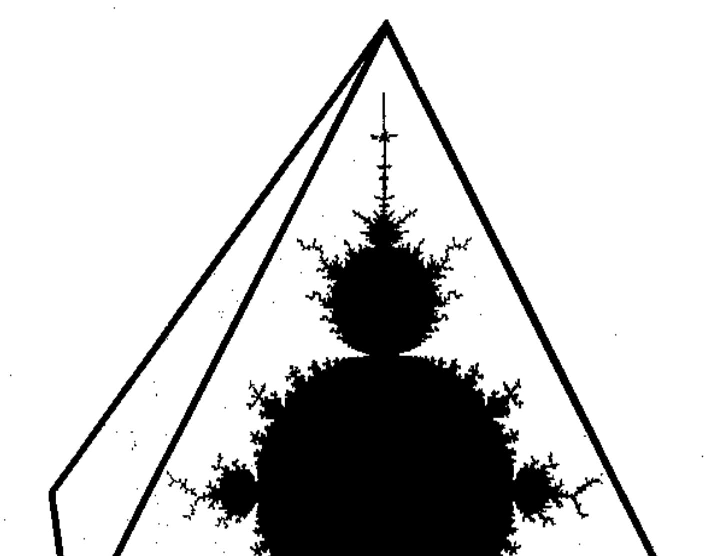
Symposium Proceedings

Innovative Teaching Practices

Held at The Queen's College
Oxford University, UK

August 14-18, 2023

Edited by Janina Morska and Alan Rogerson



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Foreword

This Symposium was organized by The Mathematics Education for the Future Project, founded in 1986 and dedicated to the dissemination of innovative ideas in mathematics, science, statistics and computer education. Please contact rogersona8@gmail.com for further information.

This volume contains all papers and workshop summaries, in alphabetical order of the principal authors, which were presented at the Symposium and also papers accepted for our Deserving Scholars and Researchers Program (DSR) by authors who were unable to attend the Symposium.

In 2022 our DSR program published papers by 15 scholars, including one by a very gifted 15 years old student from India: Tanishq Sah. In 2023, after two years of contact with Mayo College Girls School in India, papers by 8 of their students were published in the DSR Program. We wish to thank the school management and authorities and especially their teacher Mrs Deepti. This year altogether 12 DSR scholars have their work published as follows: Drishana Bachhawat, Ruhani Gulati, Saumya Gupta, Ishikka Ladia, Mrigshira Mehra, Diva Moriani, Nishika Shah, Pari Shah from India, İpek Saralar-aras and Bengi Bİrgİlİ from Turkey, Stefanides Panagiotis from Greece and Clifford Singer from the USA.

We thank all these authors for their time and creative effort. It is clear from the variety and quality of the papers that the symposium has attracted many innovative mathematics educators from all over the world.

We are especially grateful to Professor Dr. Martin Stein of Münster University, the Owner and Manager of the company that publishes these proceedings: WTM-Verlag (Wissenschaftliche Texte und Medien – scientific texts & media).

These Proceedings contain the papers & workshop summaries in alphabetical order of the principal authors.

Dr. Alan Rogerson
D.Phil (Oxon), M.Sc., B.Sc., B.A. (Lon), Dip.Ed., Cert. Ed. (Cantab).
Coordinator of the Mathematics Education for the Future Project

Contents List of Papers and Workshop Summaries

Ethnomathematics of Bedouin Culture (Geometry in Bedouin Embroidery)

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Abstract

In this essay we will present the ethnomathematics of the Bedouin society in the south of Israel, and in particular the ethnomathematics manifested in the folkloric embroidery of Bedouin women. The purpose of this essay is to show how Bedouin women knew mathematics intuitively and used unique cultural values and elements, which contain and reflect a variety of didactical mathematical aspects, concepts, and attributes. Also, we will show how Bedouin women knew mathematics through their embroidery work. The data presented based on research we conducted among the Bedouin population, with the aim of searching for collecting ethnomathematics knowledge in Bedouin society. This goal was derived from our need to make the subject easier and more accessible to our students, and to increase their desire to study it. **Keywords:** Bedouin embroidery, Geometry, Ethnomathematics.

Introduction

In light of the numerous changes that have occurred in the structure of Bedouin society in Israel and the cultural customs of this population as a result of the process of modernization, many of the traditional socio-cultural characteristics and values have disappeared. For example, one of the basic characteristics of traditional Bedouin life was living in a tent and roaming the land. One can argue that today, in Israel, such a way of life no longer exists, as the Bedouins now reside in permanent homes. Nevertheless, despite the numerous changes in their way of life, the Bedouins have managed to preserve to some degree several of their traditional social and cultural values and practices. One of these practices is the sewing and embroidery skills of Bedouin women, which were passed informally from mother to daughter, from one generation to another, until very recently. The embroidery traditionally practiced by Bedouin women included numerous shapes and objects, such as flowers, plants, geometric shapes, numbers, round letters, line types, and animals all of which were hand-embroidered very precisely, using various methods and colors, several types of cloths, needles, and a special white net. This essay details these tools and the way they embodied ethno-mathematical knowledge that was used to consistently embroider precise geometric shapes. In light of the numerous changes that have occurred in the structure of Bedouin society in Israel and the cultural customs of this population as a result of the process of modernization, many of the traditional socio-cultural

Abu Qouder, Fouze & Amit, Miriam (2023). Ethnomathematics of Bedouin Culture (Geometry in Bedouin Embroidery), in Morska, Janina & Rogerson, Alan (Eds) Innovative Teaching Practices, Proceedings of the First International Symposium of The Mathematics Education for the Future Project, Oxford University, Aug 14-18, 2023 (pp. 1 - 6). Münster: WTM.
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characteristics and values have disappeared. For example, one of the basic characteristics of traditional Bedouin life was living in a tent and roaming the land. One can argue that today, in Israel, such a way of life no longer exists, as the Bedouins now reside in permanent homes. Nevertheless, despite the numerous changes in their way of life, the Bedouins have managed to preserve to some degree several of their traditional social and cultural values and practices. One of these practices is the sewing and embroidery skills of Bedouin women, which were passed informally from mother to daughter, from one generation to another, until very recently. The embroidery traditionally practiced by Bedouin women included numerous shapes and objects, such as flowers, plants, geometric shapes, numbers, round letters, line types, and animals all of which were hand-embroidered very precisely, using various methods and colors, several types of cloths, needles, and a special white net. This essay details these tools and the way they embodied ethno-mathematical knowledge that was used to consistently embroider precise geometric shapes.

The importance of cultural values

Following the development of the ethnomathematical trend as an educational-cultural field, several proposals were suggested and numerous attempts were made to develop curricula with multicultural mathematical ideas that include traditional cultural values, traditions, symbols, and mechanisms for the purpose of aiding the instruction of mathematical subjects. This trend testifies to the importance and centrality of ethnomathematics, which appears now not only as a matter of enrichment or the property of a certain society of power, but as a matter that requires an overall effort to develop. Therefore, cultural values must be utilized in mathematical education and instruction, out of solidarity and respect for all cultures as such, while preserving their future existence (Shirley, 2001).

According to D'Ambrosio (2002), educators are responsible for the learning process and therefore they must develop informal curricula that refer to the reality in which the student lives, while integrating traditional values in their cultural-educational context in the mathematical instruction and learning process. Teaching mathematics without cultural context on the pretext that it is abstract and universal is the reason for the failure of students in this subject. On the other hand, when students are exposed to various cultural links and reflect upon them together, they develop a desire to learn and their self-confidence grows. A similar result was found in a research we conducted and that included the development and implementation of an ethnomathematical curriculum among two groups of Bedouin students in Israel (Amit & Abu Qouder, 2017).

Bedouin Embroidery: General Technique

Bedouin women used the following main tools in their embroidery: (1)Choosing the type, color, and amount of desired cloth.(2)Choosing the type, color, and amount of threads. (3) A white checkered net, which is called "a mirka" in Arabic. (4)Scissors. The most commonly used method of embroidery by Bedouin women is the “X method”, in which small dots are created by stitching

X shapes on the cloth. This method can be applied in two ways. In one method, the embroiderer moves from left to right on the “mirka” net: Beginning with one square on the mirka, the embroiderer makes a diagonal stitch from the bottom to the top of the square (/). Then she returns and makes a second diagonal stitch in the same square, this time moving from right to left and from top to bottom (\), thereby forming an X in one square. The second method consists also of stitching from left to right, but diagonal stitches from the bottom up are created in several squares at once, and then the X’ are completed by stitching the diagonal in the reverse direction, from right to left and from the top to the bottom.

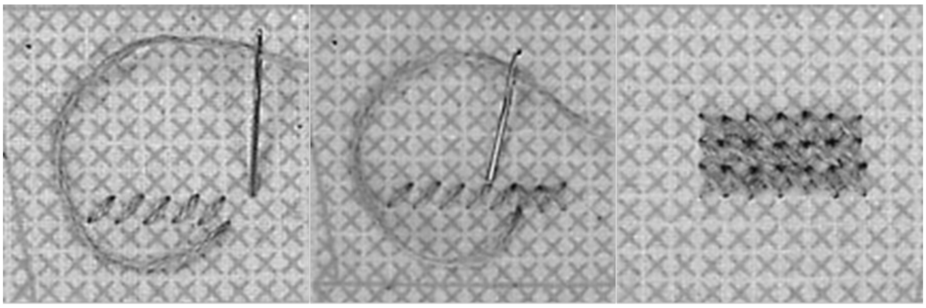

Figure 1. The common “X method” of embroidery

1. Geometry in Bedouin embroidery

After examining the basic methods and tools used in Bedouin embroidery, we will now proceed to examine several geometric shapes that appear in Bedouin embroidery, including the isosceles triangle, the right triangle, the square, rectangle, rhombus, and kite shapes, as well as types of lines and the general concept of symmetry. In each case, the mathematical properties of these shapes will be presented, and then we will describe how they are executed by Bedouin women, using traditional knowledge.

1.1 Embroidering the isosceles triangle

The various properties of the isosceles triangle can be used to prove that the triangle in Bedouin woman embroidery is indeed an isosceles triangle. One such property is that in an isosceles triangle, the altitude bisects both the base and the angle of the apex and is also perpendicular to the base. Further, in an isosceles triangle, the base angles are equal.

How Bedouin women traditionally embroidered the isosceles triangle

In order to embroider the shape of an isosceles triangle, the Bedouin woman uses a counting method in which an odd number of squares are stitched one beneath the other, beginning with the apex. For example, if the embroiderer desires to create a triangle from 16 stitches, she begins with one stitch at the apex of the triangle. Then embroiders three stitches and then five stitches. Finally, she finishes by embroidering a line of seven stitches, which forms the base of the triangle. The result is an isosceles triangle as defined above and as shown graphically in figure 2. Figure 3 shows such an isosceles triangle in an actual Bedouin dress.

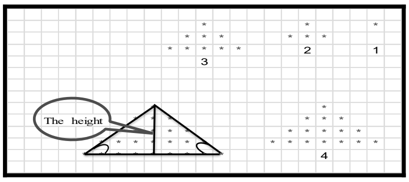

Fig.2. Counting method for embroidering an isosceles triangle

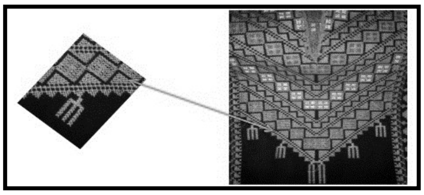

Fig.3. An isosceles triangle as embroidered on a Bedouin dress.

1.2 The Square

The properties of a square: A square is a regular quadrilateral that has four equal sides and four equal angles of 90^0.

How Bedouin women traditionally embroidered a square

In order to embroider a square, Bedouin women begin with any number of stitches, whether odd or even, in a horizontal line. For example, and as shown in figure 10, an embroiderer could begin with a horizontal line made of six stitches, and then embroider a vertical, 6-stitch line on the right side of the horizontal line. The second vertical 6-stitch line on the left would follow, and then, finally, the embroiderer returns to the first vertical line on the right and embroiders a horizontal line of six stitches from one vertical line to the other, thereby completing the square. When Bedouin women embroider a square, they are concerned with using an equal number of stitches on all sides. Therefore, we can argue that the Bedouin cultural method of embroidering a square applies the two basic properties of the square shape: first, that all the sides in the square are equal, and second, that all angles are right angles. Figure 4 shows such a square embroidered on a dress.

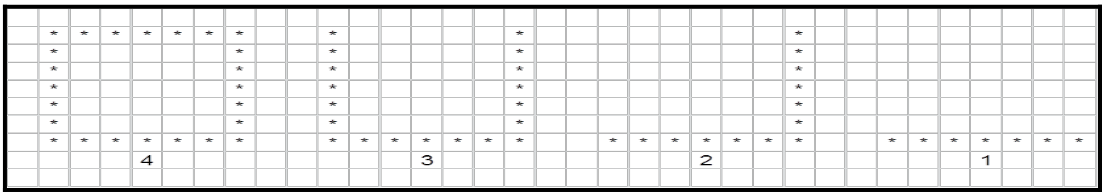

Fig 4. Traditional Bedouin method for embroidering a square.

1.3 The Rhombus

The properties of a rhombus: All sides have equal length; all the opposite sides of the rhombus are parallel and all the opposite angles are equal; the diagonals of a rhombus bisect each other at right angles, creating four right triangles within the rhombus.

How Bedouin women traditionally embroidered a rhombus

In order to embroider the rhombus shape, Bedouin women employ the method of counting the number of squares they embroider on the "mirka" in order to determine the desired lengths of the sides and the overall size of the rhombus.

Bedouin women embroider the rhombus in two steps. In the first step, an isosceles triangle is embroidered as described above. In the second step, after completing the first triangle, the exact same triangle is embroidered on the base of the first triangle. The embroider starts this second triangle by embroidering five stitches on the base of the first triangle. Following this, three stitches are embroidered and the triangle is completed with one stitch that is actually the apex of the new, second triangle. As shown in figure 5, the diagonals of the embroidered rhombus divide it into four equal, right triangles, which is one of properties of the rhombus.

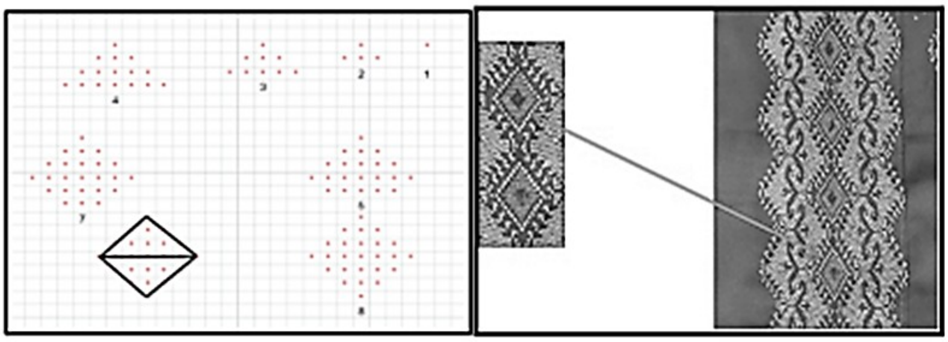

Fig. 5. Traditional method of embroidering a rhombus

Discussion: A different look, Simple math

One of the topics included in my ethnomathematics doctoral research program which was supervised by Prof: Amit Miriam, among the Bedouin population in the south of the Israel was Bedouin embroidery and its mathematical-cultural values, which are expressed in a variety of geometric shapes and mathematical rules. In order to identify and collect the mathematical-cultural knowledge that is embedded and expressed in Bedouin culture, a general search was conducted within this population. The search process was based on personal interviews with older women with experience in the field of embroidery. Interviews were recorded with video, transcribed, and analyzed. The data collected permits several conclusions as follows:

1. The size of the embroidered shapes in the Bedouin fabric, their color the amount of decoration, and the type of fabric used bear a socio-cultural message that depends on the personal social situation of the embroiderer. Thus, the shapes, colors, clothes, and the amount of ornaments used in the dress of a Bedouin girl are different from those used in the dresses of older women. For example, girls' embroidery uses shapes such as squares, triangles, small flowers, stars, and many circles, which are embroidered in bright colors (especially the color red) and adorned with excessive decoration.
2. The general strategy used by Bedouin women in embroidery is the counting method that is based on the counting of embroidered squares in the "mirka", the special white net that is designed especially for embroidery. By using this method, Bedouin women determine the size and circumference of the embroidered shape. This means that the size

and the embroidered shape depend on the number of squares used in the “mirka”.

3. Since Bedouin women have no awareness of the mathematical operations, their work can be described as "spontaneous" work, which is easy to perform and based on (1) planning and implementing the desired shape by counting the number of embroidered squares used to form each shape, which then serves as a means of measuring and determining the extent and area of each shape, and (2) different techniques of repetition, such as symmetry, that are used to embroider various shapes.

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Mathematics Education for the Future: Evidence from Mathematics Education and Research Centre in a Rural University

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Abstract

South African rural mathematics education is confronted by poor learner performance in all phases of schooling. To address this, a Mathematics Education and Research Centre was established in a rural university. Numerous innovative and developmental activities designed for the advancement of teaching and learning of mathematics are conducted through the centre for pre-service and in-service teachers. Research was conducted on the effectiveness of these programs in the centre. The qualitative data from all the stakeholders indicated that the innovative activities created a positive impact and made mathematics more accessible. The successes of the different programs suggest that such innovative and developmental activities are needed to make mathematics more accessible, which in turn can address the poor learner performance.

Introduction

South African rural mathematics education is confronted by poor learner performance in all phases of schooling. For decades, South Africa has been experiencing poor performance in school mathematics. This has been attested by the Trends in International Mathematics and Science Study (TIMSS) (Pournara, Hodgen, Adler, & Pillay, 2015) and by the different documents published by the Department of Basic Education (NSC Examinations; Schools Subject Report; Grade 12 Results- Eastern Cape Results). It has been affirmed that the continued crisis, especially in poor and rural areas are due to many factors such as, disparities in education levels; lack of adequate infrastructure both in the schools and in homes; lack of learning support, lack of sufficient skills, resources, and subject knowledge of teachers (for example, studies by Alex & Juan (2017); Alex (2019); Bansilal, Brijlall, & Mkhwanazi (2014); Kriek & Grayson (2009); Legotlo, Maaga, & Sebego (2006). This created a need for innovative and sustainable practical solutions to improve the quality of teaching and learning of mathematics. The South African government, Non-Profit Organisations (NPOs), and education sector are working to improve mathematics education in the country through various initiatives and programs. This paper reports on such initiatives in a rural university in South Africa. In this paper, drawing on the impact of the initiatives, I portray the answers to the research question "What evidence can be drawn

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Alex, Jogymol Kalariparampil (2023). Mathematics Education for the Future: Evidence from Mathematics Education and Research Centre in a Rural University, in Morska, Janina and Rogerson, Alan (Eds) Innovative Teaching Practices, Proceedings of the First International Symposium of The Mathematics Education for the Future Project, Oxford University, Aug 14-18, 2023 (pp. 7-12). Münster: WTM. https://doi.org/10.37626/GA9783959872508.0.02

from the Mathematics Education and Research Centre in a rural university" which shows how my prior work in the teaching and learning of mathematics laid the foundation for future directions and innovations for pre-service and in-service teachers.

Research on school performance and teaching reveal poor teaching of mathematics in the great majority of rural schools in South Africa (Alex & Juan, 2017). Given that teaching plays such an important part in the learning process, South African Government needs to place a high priority on improving the standard of its mathematics education if it hopes to realise its development objectives. It has become a truism that teachers' subject matter expertise influences what they teach, how they teach it, and the knowledge that their students acquire. To address this issue, several research projects were conducted in the rural South African university.

Mathematics teaching initiatives in the rural university

The research is housed at Walter Sisulu University (WSU), which is both a Historically Disadvantaged Institution (HDI) and a Historically Black University (HBU). As a Comprehensive Institution, WSU is a rural university in the former Transkei region of the Eastern Cape. This university's service area contains hundreds of schools. In addition, it serves a predominantly rural and underdeveloped region. The preponderance of students at WSU hail from the eastern portion of the Eastern Cape. They cannot afford the resources necessary to improve their education and advance their careers as Eastern Cape teachers.

A study was undertaken in WSU to determine if the student teachers who are studying to teach mathematics have a strong understanding of the subject matter they will be teaching in schools. This research aimed to assess the student-Level disciplinary content knowledge (SLDCK) of a sample of 40 Bachelor of Education (Mathematics) students enrolled in the four-year Bachelor of Education (BEd) Further Education Training (FET) programme. The results revealed that the student teachers (prospective teachers) had limited SLDCK in the topics they were expected to teach in schools, which was attributed to the curriculum constraints of the university.

The study provided recommendations for successful teaching measures as well as adjustments in the curriculum for teacher training in order to improve the preparation of future teachers in the country (Alex, 2019). Based on my own reflective practice and from the recommendations, I tried and tested some theory-based strategies such as learner centred approach to teaching, cooperative learning strategies, and flipped learning through ICT integrated teaching. This scholarship was the guiding force for the interventions put in place for effective teaching and learning. The notion of scholarship of teaching and learning served as the foundation for the interventions that were implemented and the reporting of those interventions. As a direct result of the research that was conducted on the SLDCK of the student teachers, the interventions have been put into action.

Mathematics Education and Research Centre

The Mathematics Education and Research Centre was established with the purpose of promoting scholarship of teaching and learning. This involved reflecting on and researching on my own teaching practices. The centre was established with the goal of improving the quality of mathematics instruction and learning in the region. The Eastern Cape Province is typified by an extremely high failure rate in school mathematics, which has persisted despite numerous initiatives, such as vacation classes, weekend classes, and free study materials provided by the Department of Basic Education and NPOs. The poor performance of the student teachers highlighted both the pervasiveness of the problem of learner performance in mathematics as well as the vicious cycle. Therefore, the main goal of the Mathematics Education and Research Centre was to improve the mathematics competencies of the student teachers so that it can act as a centre of mathematics teaching and learning excellence. The facility consists of a computer lab with a smartboard, internet access, and the Computer Aided Mathematics Instruction (CAMI) software installed on each of the computers.

This centre is run from a space that serves multiple purposes, and it has several different components that operate at various times, including the following: Baseline tests with an online programme; intervention courses based on the Baseline tests; student led-unit for the production of lesson plans, which consists of compiling and saving lesson plans onto computers and a Micro-teaching Lab, where student teachers can improve their teaching abilities by using interactive whiteboards.

Interventions for Pe-service teachers

One of the focuses of the centre is to address the various need of the student teachers enrolled in the four-year Bachelor of Education (BEd) Senior and Further Education Training (SP & FET) programme. The student teachers are provided with the specialised pedagogical expertise necessary for teaching and assessing mathematics learners in the SP and FET Phases (Grades 8-12) in the teaching modules.

In these teaching modules, students are given the opportunity to acquire the methodology and content that are pertinent to teach SP and FET Mathematics in schools. In the past three years, our entry level Bachelor of Education (BEd) student teachers were subjected to writing online Computer Aided Mathematics Instruction (CAMI) baseline tests for SP and FET Phase at the onset of the mathematics education programme. Theories on mathematical knowledge for teaching (MKfT) and curriculum knowledge framework provided the analytical lens for the performance in the baseline tests. The lower than anticipated achievements of the student teachers were a cause of concern. The low performance of the student teachers can be directly attributed to the low level of mathematics knowledge that the student teachers bring into the programme.

To address this low level of mathematics of the intake into the university programme, I started collaborating with other higher education institutions which offer professional development courses for in-service teachers. This is in anticipation that if the teachers in the schools (in-service teachers) are trained to improve on their subject content knowledge and pedagogical content knowledge, and if resources are made available, the vicious cycle of poor performance can be addressed. Thus, the Mathematics Education Centre started addressing community and country needs especially the rural communities where Walter Sisulu University serves.

Many collaborative projects were initiated in the centre to promote mathematics competencies of in-service teachers with a strong focus on the improvement of the achievement of mathematics of school learners in the underprivileged rural schools. The generous funding from different international, national, government, and NPOs contributed to the initiatives. For this paper, I am reporting on one of the initiatives, called “Family Maths” initiated by the Science- For- The-Future Centre by University of Free State (UFS) with funds from SANRAL (South African National Roads Agency Limited).

Family Maths: Intervention for in-service teachers

The “Family Maths” Programme as named by the Science-For-The-Future Centre seeks to address the issues in many South African primary schools, especially in rural areas. Some of these issues are teachers’ lack of the content knowledge and teaching skills, insufficient or no teaching resources at school or at home, lack of parental involvement, and the language of learning and teaching which is different from home languages (UFS,2021). These issues are critical to effective mathematics teaching and learning.

The project has adopted a “triangular” project strategy in which, the teachers, students and parents are trained by the trained facilitators at UFS. A total of 25 teachers from 15 underprivileged, under-resourced schools are selected every year with the assistance of the Department of Basic education for the roll-out of the project. In addition to incorporating the Family Maths activities into the school curriculum, the Grade 3 teachers conduct training sessions for the parents of their students at their respective local schools (UFS,2021). Science-for-the-Future manufactures and provides the participating teachers with adequate activity materials, such as manipulatives and other activity materials, for use in the classroom and during parent training sessions. This enables parents to participate in their children's mathematics instruction and learning at school.

Evidence of the impact and effectiveness of the programme

The data on the different initiatives show positive impact of the projects. Due to page constraints, the impact of the projects on student teachers are not included here. They are available from https://scholar.google.co.uk/citations?user=jxSVQHQAAAAJ&hl=en.

The impact of the Family Maths Project on in-service teachers has the following (selected) evidence:

From Facilitators:

1. "I have thoroughly enjoyed working with my teachers. I have seen the "lights" come on and they leave inspired, guided and encouraged to go forward. Without SANRAL'S financial input and Family Maths project monitored extensively, we would not be having this enthusiasm for the Maths drive going forward"

From in-service teachers:

1. "Parents have commented that the "fear" of maths has gone. They never realised that Maths is "Everywhere" especially when dealing with sorting of numbers, linking it to sorting the laundry. They feel we have made it practical and clear for ALL to understand. Parents feel "empowered" after being at the parents' workshop!"

From Parents:

1. "Family maths is a very informative and helpful program. During our school days, maths was a very scary subject, and it was known as a difficult subject, but family maths has made it easy for us parents, to help our kids. It was conducted well to us in an interesting manner, and we have enjoyed it. Keep up the good work.!"
2. "The Family maths idea is a wonderful idea as it has made maths such an interesting and understandable subject contrary to the previous belief system. I think it is very important that our little ones catch the concepts at this level, so they may progress to other grades with confidence in themselves when it comes to maths. Family maths seems to make maths part of our everyday day life and I really like that. Maths is not left for the classroom alone anymore, thanks to Family maths!"
3. "I found the parents workshop such a huge eye opener. This has given me hope that at least this generation of scholars will enjoy maths rather than dread it. Hope they continue with the idea!!"
4. "The opportunity created by family maths evening to experience one's child learning environment is huge. Besides affording the opportunity to meet other parents, it imprinted a mental image of the learning space that will inform parents involvement in the learning process of their children. Furthermore, family maths evening puts parent on the same page with the teacher. By this, parent's involvement in their children's homework will reaffirm teachers' approach".

Conclusion

The responses from the facilitators, in-service teachers, and parents in the Family Maths program indicated that initiatives like "Family Maths" makes "maths part of their everyday day life" and they feel that "the program made it practical and clear for all to understand". This shows the positive impact of such initiatives.

Recommendations

The successes of the different programs suggest that such innovative and practical solutions are needed to make mathematics more accessible, which in turn can address the poor learner performance in similar contexts. Such initiatives are needed for the future.

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Cultivating Inquiry in Advanced Mathematics with the Next Generation of Mathematics Professors

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Abstract

How can we shift the culture of lecture in university mathematics? The purpose of this study was to provide an intervention for mathematics majors on effective practice in teaching advanced mathematics, inspiring the next generation of mathematics professors to engage in inquiry and discourse so their future students can internalize critical concepts. This workshop will reflect those weekly seminars, beginning with puzzles to motivate topics, followed by exploring Introduction to Analysis, Number Theory, and Topology through the 5E Model. Participants completed weekly exit tickets, providing positive feedback regarding the Think-Pair-Share approach, and the Engage, Explore, and Explain aspects of the 5E Model. Survey responses indicated a strong belief in providing students with opportunities in class to collaborate and engage in discourse.

Background

Many students arriving to university mathematics courses lack the background knowledge and tools necessary to internalize mathematical concepts, sometimes due to a focus on procedural fluency in high school in order to perform well on standardized exams. Still, passive lecture is not going to provide students with these necessary tools. Topping et al., (1996) showed that undergraduate students benefit from peer tutoring and collaborative classroom environments. Opportunities for collaboration during advanced mathematics classes are not common, as students are expected to attend passive lectures, then work independently outside of class on exercises suggested by the professor. Meanwhile, the professors that do provide these opportunities are in the minority, and are generally not encouraged to share best practices with their colleagues.

Communities of Practice (Kezar et al., 2017) are less common in university Science, Technology, Engineering, and Mathematics (STEM) departments than in K – 12 departments. In addition to a primary focus on research, mathematicians tend to collaborate less on effective teaching strategies, focusing more on thorough explanations of complex concepts. While mathematics professors care about their students understanding advanced mathematics, and succeeding in coursework and research, most simply believe that the best way for that to happen is by disseminating knowledge through lecture. This belief prevents many professors from implementing any

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cooperative learning strategies because they do not feel as though their students have enough background knowledge to engage in meaningful discourse in the first place. Particularly in the context of teaching Abstract Algebra, Chowdhury et al., (2021) found that “beliefs about content, beliefs about how students learn, and beliefs about student ability are influential in determining instructional practice” (p. 573). Thus, this shift in the norm in university mathematics classrooms requires a change in the foundational, pedagogical beliefs of current and future mathematics professors.

Theoretical Framework

The participants in this study are part of the next generation of mathematics professors, who met weekly to learn how to implement what they know is needed in advanced mathematics courses. These weekly seminars draw on the Multidimensional, Problem-Solving Framework by Carlson and Bloom (2005) by acknowledging advanced mathematics as a non-linear process of persisting in productive struggle, and as some mathematicians informally refer to it, ‘failing in the right direction’. Students need to see their professors modelling this process so that it is accessible, rather than the current perspective of many undergraduate students that mathematicians are able to immediately complete an exercise or construct a proof.

This overarching focus on engaging in productive struggle guided the discussions and activities during the seminars, along with the 5E Instructional Model (Bybee et al., 2006) to promote inquiry and engagement in advanced mathematics classrooms. The 5E Model provided a foundation for what realistic inquiry and discourse can look like in a rigorous, advanced mathematics course. Dawkins (2014) argues that inquiry-oriented practices are “especially important at the advanced undergraduate level where mathematics majors are being inducted into modern mathematical practices of defining, conjecturing, and proving” (p. 103). For most students, this is their first experience with mathematical analysis, and they need support in developing the necessary tools to internalize critical concepts.

An Engaging Intervention

Current undergraduate and graduate mathematics students understand first-hand that lecture alone is ineffective, even if it is sufficient for a few independent learners. During our weekly seminars, these participants continued to address how most of their peers are unable to internalize mathematical concepts, as they lack the tools, background knowledge, and experience. They know something needs to change, and they attended these seminars to explore what that change could look like.

It was clear from the first weekly seminar that the idea of engaging in discourse during class was neither the norm, nor a conceivable reality to most of the students who attended. Like many conversations in professional development sessions with mathematics instructors, the common rhetoric was that there was no time for such inquiry given the extensive curriculum that needs to be ‘covered’ in advanced mathematics classrooms. When the researcher pushed back on what students learn when all of the material is ‘covered’, the response was “not much”. Even though students acknowledge the lack of understanding of most of their peers, it took several weeks of discussions and reflection to get many mathematics students to consider the benefits of establishing socio-mathematical norms (Yackel & Cobb, 1996) in an advanced mathematics classroom in order to promote inquiry and discourse.

How to Realistically Move Away from Lecture

An important distinction between effective practice in high school and university mathematics courses was established from the beginning of the weekly seminars. While some aspects of student engagement and discourse are universally effective, it is important to note that there are distinct differences when considering advanced mathematics courses. For example, there is always space for a warm-up question to establish students’ background knowledge or to check their understanding of the most recent topic, some inquiry and discourse, and often an exit ticket. However, many advanced mathematics curricula do not provide space for project-based learning, or even extensive inquiry-based learning. Direct instruction from mathematicians is critical, which is why the 5E Model is appropriate to guide the next generation of mathematics professors (Bybee et al., 2006). In this manner, the Explain aspect of the 5E Model is maintained, only encouraging student participation as a minor change to the passive lecture approach.

As a mathematics educator with experience teaching grade 7 – 12 mathematics courses and undergraduate mathematics courses, the researcher observed that students demonstrate an algebraic understanding, yet lack a graphical representation of critical concepts. There seems to be a perception in advanced mathematics that we no longer need to provide a graphical depiction, as mathematical analysis tends to more abstract concepts. However, in the context of complex analysis, Piña-Aguirre and Farfán Márquez (2023) “showed that the production of mathematical knowledge… requires the symbolic-formal and the graphical-descriptive conceptualization of complex quantities” (p.11). This suggests that the formal, direct instruction aspect of advanced mathematics can, and should be supplemented with graphical representations, ideally those that students can actively explore during class.

Methods

The private university where these weekly seminars take place does not offer a teaching course for mathematics majors, or for graduate teaching assistants in the mathematics department. The graduate students who attended the seminar lamented that they did not have opportunities to teach, only to grade assessments. This seminar was created in order to establish the need for a credited course at the university to support future mathematics professors, in addition to providing tools for effective practice in teaching advanced mathematics to current undergraduate and graduate mathematics students.

Initially the researcher contacted the head of the mathematics department to request permission to distribute flyers and information sheets about the research study to instructors and professors, to be shared with undergraduate and graduate mathematics students, after receiving ethics committee approval. The department head exceeded expectations, and distributed the information to all students, increasing reliability by establishing the mathematics department as the source of information supporting the study, rather than a researcher from the department of educational sciences.

Students were encouraged to email the researcher with questions, or just show up to the weekly seminars, so it was unclear what to expect with regard to participation. Throughout the semester, 12 students attended the weekly seminars, some consistently, and others sporadically. The research design evolved into a case study with a group of five students who attended consistently, and were invested in the mathematical content of the seminars, and in contributing to a shift in the culture of passive lecture in university mathematics courses.

All students were given a weekly exit ticket, and three surveys were distributed to the group via Qualtrics throughout the semester. This data collected was analysed for descriptive statistics, career plans, perceptions of how students learn mathematics, and for common themes about the current issues in advanced mathematics courses and how they can be improved. The students involved in the case study continue to actively participate in the development of a credited course, along with cultivating inquiry and discourse in additional topics such as Probability Theory, while continuing our work in Number Theory, Mathematical Analysis, and Topology.

Weekly Seminar Structure

Each seminar began with a puzzle to promote critical thinking and productive struggle in mathematics, while cultivating a safe space for discussion. Participants arrived to a new puzzle each week, with varying levels of difficulty in order to be both accessible and challenging, as the researcher was unsure of the backgrounds of the voluntary participants. Coffee, tea, and cookies were provided as well, promoting a safe and informal atmosphere where honest discussions about what needs to change in university mathematics instruction could occur.

Each puzzle was critiqued for its connection and relevance to advanced mathematics. This motivated the discussion on warm-up problems and puzzles for the Engage aspect of the 5E Model. The researcher suggested a puzzle to motivate a new topic, then short problems or exercises for warm-up questions throughout each unit in the curriculum. Most puzzles were well received, especially the Topology puzzles that will be explored in this workshop, which assisted in participants' understanding of the value of exploration in play, even in advanced mathematics courses.

Following the puzzles, we explored sample lesson plans to motivate discussions on how to realistically implement inquiry, using the 5E Model for an Introduction to Mathematical Analysis course, the first course in their undergraduate studies that can be considered advanced mathematics. These sample lesson plans, created by the researcher using the lecture notes from previous courses made available by the current mathematics professor of record (Gheondea, 2021) were analysed for consideration of time, accessibility, and realistic implementation. Weekly seminars concluded with an exit ticket, both to model the Evaluate aspect of the 5E Model, and to gather data on the insights and reflections for the participants.

Workshop

This workshop will mirror the weekly seminars, by beginning with an engaging puzzle to motivative a topic in advanced mathematics. Next, aspects of the 5E Model for inquiry will be explored using topics from Mathematical Analysis, Number Theory, and Topology. Then suggestions for realistic implementation of inquiry and discourse in advanced mathematics will be explained, followed by opportunities for elaboration on cooperative group structures. Finally, exit tickets will be distributed to collect reflections and insight into effective practice in teaching advanced mathematics, in the hope of creating a credited course at a private university for future mathematics professors.

Exercises in this workshop will include the famous “eggs in a basket” problem by Brahmagupta (Strayer, 2002, p. 58) to motivate Elementary Number Theory, where participants provided many avenues to the solution, from modular arithmetic to methodically analysing multiples of each value, building an understanding of how to model scaffolding (Anghileri, 2006). Topology exercises will also be explored (Kreuger, 2011), which brought out a playful side to the generally serious undergraduate and graduate mathematics students, even those who were most skeptical about taking the time to play and explore mathematical concepts.

Discussion and Implications

It is well established that lecture alone is not only insufficient for success and retention in STEM majors, it results in much higher failure rates than active learning in a STEM classroom (Freeman et al., 2014). Unfortunately, passive lecture is still the expectation when a student arrives at an undergraduate mathematics course. This needs to change, in realistic and meaningful ways, starting with the next generation of mathematics professors which will hopefully contribute to an increase in retention in STEM fields.

The 5E Instructional Model (Bybee et al., 2006) provided a valuable, concrete structure for the next generation of mathematics professors to understand how aspects of inquiry and discourse could be realistically implemented into advanced mathematics courses. Both the undergraduate and graduate mathematics students agreed during weekly discussions, and in survey responses, that the Engage aspect of the 5E Model is beneficial to implement consistently. They clearly acknowledged the importance of the Explain aspect, with the slight modification of direct instruction rather than passive lecture. Additionally, participants suggested varying combinations of the Elaborate and Evaluate aspects to account for time restraints while still providing in-class opportunities to support students with the tools they need to internalize mathematical concepts, and to build students’ confidence in working through exercises independently, outside of class. This modified version of the 5E Model will be the foundation for the credited course in effective practice in teaching advanced mathematics for future mathematics professors.