

Dmitry Livanov

The Physics of Planet Earth *and Its Natural Wonders*



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ISBN 978-3-031-33425-2 ISBN 978-3-031-33426-9 (eBook)
<https://doi.org/10.1007/978-3-031-33426-9>

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Preface

The book you are holding in your hands is about the physics of the world. The world around us, which is made up of the planets and stars that we see when looking at the sky, mountains and rivers on the Earth's surface, the seas and oceans with their storms and lulls, our planet's atmosphere with lightning and thunder, wind, snow and rain—all of this is like a huge laboratory in which physical experiments are taking place every minute and every second. Steven Hawking wrote, "The subject of science is often taught in school in a dry and boring way. Children learn to mechanically memorize material in order to pass tests, but do not see any connection between science and the world around them." The aim of this book is to show that this connection really does exist and to explain several physical phenomena, which we encounter every day.

Over the centuries people have asked the question: Why is our world like this? By providing us with the knowledge that we need, physics gives us information about the world, an understanding of what happens in nature and why and also predicts what will occur in the future.

What is the special significance of physics for the development of our civilization and what distinguishes it from other natural sciences?

First, while describing and explaining natural phenomena, physics constructs a scientific picture of the world of modern man. Everyone should have at least a general idea of how the world in which they live works. This is fundamental not only for our general development; a love for nature implies that we also respect everything that happens in it. In order for this to happen,

we need to understand the laws that cause these natural processes to take place so that we leave our children a world in which they can live. Neither all properties of the material world nor all laws of nature have been studied; nature is still fraught with many mysteries. As physics develops, we become more knowledgeable about the world around us.

Second, physics determines mankind's technological development. Everything that distinguishes modern-day civilization from society of past centuries has arisen as a result of practical application thanks to discoveries in physics. Research in the field of electromagnetics, for example, led to the development of household electrical appliances, cell phones and the Internet, which are so essential today, while discoveries in mechanics and thermodynamics resulted in the production of automobiles and trains. Moreover, advancements in the physics of semiconductors gave rise to the unveiling of the computer, while in aerodynamics, airplanes, helicopters and rockets were developed. In return, innovations in engineering and technology make it possible to conduct fundamentally new research.

Third, physics forms the foundation of all the other natural sciences—astronomy, chemistry, geology, biology and geography—because it explores fundamental common factors. Chemistry, for example, studies atoms and molecules, the substances of which they are composed and the transformation of one kind of matter into another. The chemical properties of a substance are determined by the physical properties of atoms and molecules, which are described in such branches of physics as thermodynamics, electromagnetics and quantum mechanics.

Fourth, physics is closely connected with math because math provides a framework with which the laws of physics can be precisely developed. Physical theories are almost always formulated as mathematical equations. Mathematical formulas had to be included in this book, as they make the essence of physical phenomena clearer. Math makes it possible to quantify what occurs around us and to establish common factors and connections between physical quantities, thus making it possible not only to explain, but also to predict, and, in so doing, take control over the future. Without question, only those mathematical relationships that can be verified and measured observationally and with experiments are of value in physics. Furthermore, the level of complexity of mathematical tools should correspond to the approximation of the physical model that is used. Everyone knows the joke made by Albert Einstein, Nobel Laureate in Physics, who, when referring to using overly complex mathematical tools in physics said, "Since the mathematicians have invaded the theory of relativity, I do not understand it myself anymore." Therefore, the level of mathematical description used for each

problem in this book is of the simplest nature and does not go beyond the scope of the material that is presented in a school curriculum. It is also usually limited to qualitative explanations and approximate estimates.

Fifth, observations and experiments form the basis of physical research. By generalizing them, it is possible to highlight those patterns that are overarching and the most substantial, as well as aspects of observed phenomena. In the early stages of experiments, these underlying characteristics are primarily empirical, i.e., they describe only the properties of physical objects and not the internal operations that produce these properties. By analyzing empirical regularities, physicists use appropriate mathematical tools to develop physical theories, which explain the phenomena being researched based on today's ideas of the structure of matter and the interaction between its constituent parts. In so doing, this gives clarity to the way that systems work and reasons for the occurrence of different phenomena. General physical theories help to formulate the laws of physics, which are undisputed until large quantities of new experimental results do not require that they be clarified and reviewed.

I invite you to venture into the fascinating and complicated world of physics. I will end this short preface with a quote from another Nobel Laureate in Physics, Peter Kapitza, who said, "Nothing prevents a person from becoming smarter tomorrow than they were yesterday."

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Acknowledgements Author would like to thank Dr. Jill A. Neaendorf for the excellent work of translating the book from Russian into English. The English translation of the book would not have been possible without the extensive support and help of my friend and colleague Dr. Timothy E. O'Connor.

About This Book

This book is meant for high school students, university students, professors and teachers of physics, as well as everyone who wants to understand what is happening in the world around them and develop a scientific perspective on the vast number of natural phenomena that exist. Every section of this book has essentially a set of physics problems, which enable the reader to strengthen their understanding of physical laws and learn to apply them in interesting situations.

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About the Author

Dmitry Livanov has held a variety of roles in his life. He began his career in science in the field of theoretical physics and will be remembered for several widely publicized articles in the field of superconductivity and the physics of metals for which he was awarded the Golden Medal of the Russian Academy of Sciences for Young Scientists. After developing his own course on solid-state physics and writing his own textbook, Dmitry became heavily involved in teaching. Within the following decade, Dmitry climbed the ranks from Associate Professor to Chancellor of the Moscow State Institute of Steel and Alloys (MISiS) and during that time he did a great deal to make MISiS into a modern-day European scientific and technical university. While holding the position of Minister of Education and Science of the Russian Federation, he worked from 2012 to 2016 on reforming Russia's system of science and education. However, regardless of the capacity in which he works or the position he holds, Dmitry remains, first and foremost, a scientist with creative drive and a rational perspective on the world.



1

The Earth in the Solar System

Abstract We start with a discussion of the two milestones of Nature—the law of universal gravitation and Kepler’s laws, and the latter is the sequence of the first. These laws account for the formation of the Solar system. The Sun is considered with special attention as the main source of energy inside the Solar system. Then we review the main physical features of the planets in the Solar system. The rotation of the Earth around its axis is then discussed, and the associated physical phenomena on the Earth’s surface as well. In concluding the first chapter, we look at the physical background of our calendar.

Planet Earth is the home of all human beings and people have long sought to understand how it works. What is the shape of our planet? Why and how does it move in relation to the Sun and stars? Why do different phenomena on the Earth’s surface, deep inside of it and around it occur exactly as we see them? These are perhaps the primary questions that mankind has always sought to answer.

To our ancient ancestors, the Earth seemed to have a flat surface like that of a disk resting on elephants or turtles (Fig. 1.1). They reasoned that a starry sky, through which heavenly bodies moved, hung above the flat Earth. Today such an idea would make even elementary students laugh, but at that time it was an excellent concept. It explained all natural phenomena: the Earth seemed flat to someone standing on it and earthquakes were thought to be caused by the movement of that very gigantic animal supporting the Earth’s

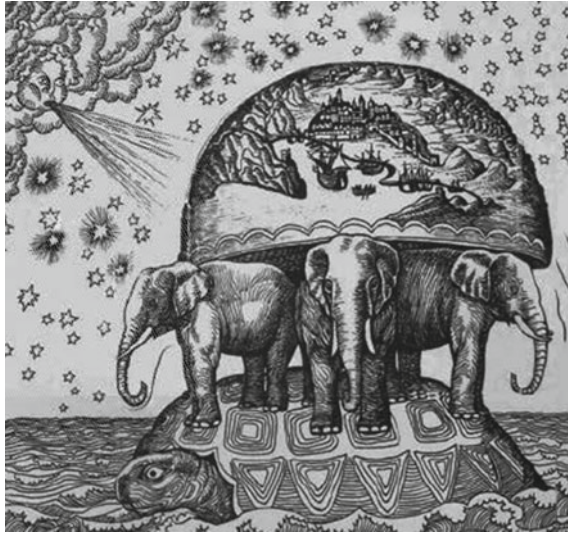


Fig. 1.1 The Earth as imagined by our ancient ancestors

foundations on its back. No one had seen the ends of the Earth because it is so big. Moreover, our ancestors understood the concept of “down” as a direction perpendicular to the Earth as a disk.

However, more than two thousand years ago, the ancient Greeks understood that the Earth is round. Aristotle, the great philosopher of ancient times, was the first to prove that the Earth has a spherical shape. He noticed that during a lunar eclipse, the shadow of the Earth is round and the constellations that are visible from the Earth change places when one travels along its surface. Aristotle surmised that the motionless Earth was located in the center of the world, around which all cosmic bodies rotate in circular orbits (Fig. 1.2). This was called a *geocentric model*. Today we sometimes think in terms of the geocentric system when we say, for example, that “the sun rises” and we imagine that it emerges from a motionless forest instead of a forest that is rotating around the Earth’s axis. However, every child today knows that the Earth revolves around the Sun in its orbit (Fig. 1.3). Moreover, the Earth, just like a top, spins around its axis. But what is the shape of the orbit of the Earth and of other planets? Does the angle between the plane of the Earth’s orbit and the axis of its rotation change? And why don’t the Earth and other planets fly away from the Sun, and the Moon fly away from the Earth? What are their laws of motion? We will examine these questions in the first chapter of this book with the help of physics and math.

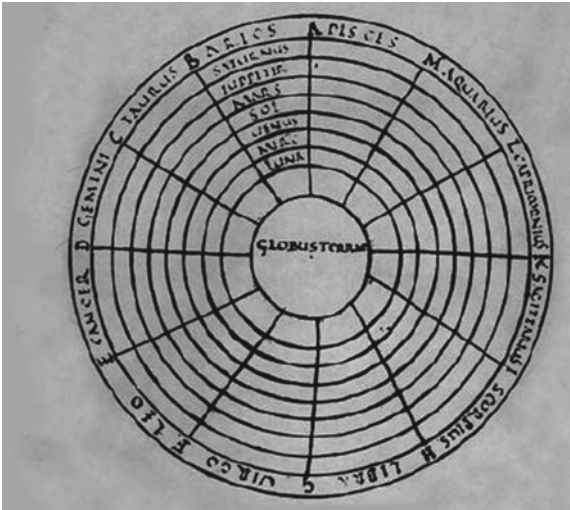


Fig. 1.2 One of the earliest images of the geocentric system that has survived. Macrobius, a manuscript from the ninth century BC



Fig. 1.3 The Solar system. An illustration from Nicolaus Copernicus' book *On the Revolutions of the Celestial Spheres*, 1543

1.1 The Law of Universal Gravitation and Kepler's Laws

Astrologists of the Middle Ages unsuccessfully tried to predict the life events of specific people based on the movement of celestial bodies. Their predictions turned out (and still do) quite badly, but their observations and descriptions of the movement of planets have been extremely useful. The Dane Tycho Brahe, who developed new methods of observation that made it possible to minimize measurement errors and achieved a level of accuracy that was unprecedented for the sixteenth century, made particularly correct predictions. Thanks to data from his observations, Johannes Kepler discovered the laws of planetary motion in the seventeenth century. Based on these laws, Isaac Newton formulated the law of universal gravitation in his book *The Mathematical Principles of Natural Philosophy*, which was published in 1687.

Newton introduced the law of universal gravitation in his book *The Mathematical Principles of Natural Philosophy*, which was published in 1687, and in which he did not mention anything about the gravitational constant. It was only after a little more than 100 years, in 1798, that Henry Cavendish introduced it on an experimental basis and the formula took on its final form.

This is how the historical chain of discoveries progresses, but the logic of physical theories does not always coincide with this progression. Although Kepler's laws were discovered prior to the discovery of the law of universal gravitation, we will first consider this law as the reason for the movement of celestial bodies, and thereafter examine Kepler's laws as a result of the law of universal gravitation.

The law of universal gravitation quite simply states:

Two material points with masses m_1 and m_2 are mutually attracted and the force from their mutual attraction is proportional to the product of their masses and inversely proportional to the square of the distance between them.

$$F = G \frac{m_1 m_2}{r^2} \quad (1.1)$$

If we are not dealing with material points, but with round bodies of finite sizes, then the law of universal gravitation will include the distance between

the centers of these spheres (Fig. 1.4). However, not only round bodies are attracted, but also bodies of any shape. In the latter case, it is necessary to split each of the bodies into very small parts and sum up the interaction of these parts in order to get the force of gravitational pull.

Now it becomes clear that the direction “down” coincides with the direction of the force that acts on a body by the Earth. In this case, “down” is the direction toward the center of the Earth.

The law of universal gravitation is one of the most important laws of physics. It is both simple and universal. From atoms and molecules to stars and galaxies, this law is applicable to all bodies of the universe, the distance between which is much larger than their size. But why don't we notice, for example, a pull between books lying on a table? The reason is because of the G coefficient, which is called the *gravitational constant*. Its value is very small: $G = 6.67 \times 10^{-11} \frac{\text{m}^2}{\text{kg s}^2}$.

Consequently, the force of gravitational pull becomes noticeable only when a body's mass is not just large but very large! Which body close to us has the largest mass? The Earth, of course. This is precisely why we feel the pull of all bodies toward the Earth, which we call gravity, and we absolutely cannot detect any gravitational pull of objects on a table toward each other.

Thus, according to the law of universal gravitation, let us assume that a planet and the Sun are pulled toward one another: force is directed along a straight line that connects the centers of their mass and is inversely proportional to the square of the distance between them. If these conditions are met, the movement of bodies is described by Kepler's three laws.

Kepler's First Law The path of the planets around the Sun is elliptical in shape, with the center of the Sun being located at one focus.

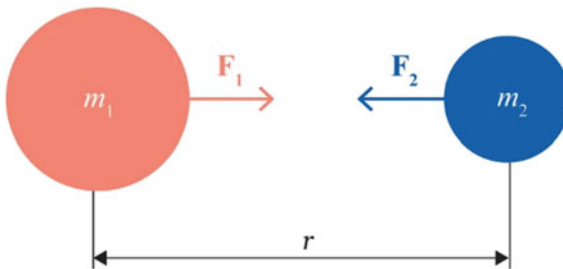


Fig. 1.4 The forces of gravitational pull acting between two bodies

Planets do not move around the Sun in a perfect circle, as ancient astronomers believed, but rather in elliptical orbits (Fig. 1.5). The scientific community was not eager to accept this fact because by default it was thought that “the celestial sphere was the epitome of perfection” and the circle was officially considered the most perfect geometrical figure. Therefore, all celestial bodies were “required” to move only in a circle. However, to this end nature had its own opinion, which had to be reckoned with.

An Ellipse What exactly is an ellipse? Figuratively speaking, an ellipse is an elongated circle (this definition is sometimes seen in crossword puzzles). A circle has a center and the distance between the center and any point of the circle is the radius, which is always the same. Now imagine that there are two centers and they have begun to separate. In order to imagine this, we will conduct a small experiment. We will take a piece of paper (cardboard is better), poke two needles or pins into it that are about 5 cm (1.97 in.) apart, connect them with a ring of thread and then, while pulling on the thread with a pencil, draw a line, making sure that we are always pulling on the thread (Fig. 1.6). Now we have an ellipse!

Half of the “length” of the ellipse is called the *semi-major axis* and is denoted by a , and half of the “width” of the ellipse is called the *semi-minor axis* and is denoted by b . If we move the needles further apart from each other, the ellipse will be more elongated; if we move them closer together, it will be less elongated. In the most extreme case, when the needles are very

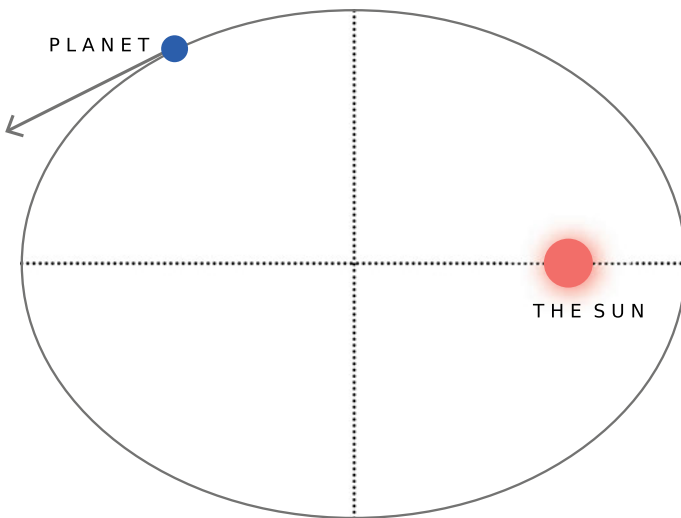


Fig. 1.5 A planet's orbit in the Solar system

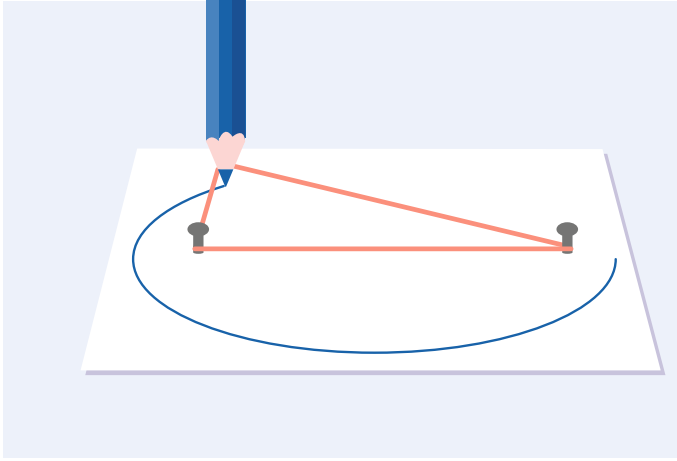


Fig. 1.6 How to draw an ellipse

close together, the “width” is equal to the “length” and a circle forms, i.e., $a = b$. The elongation of the orbit of a celestial body is determined by the eccentricity $e = \sqrt{1 - \frac{b^2}{a^2}}$ ($e = 0$ is a perfect circle and $e = 1$ is when the ellipse degenerates into a line segment).

The equation of an ellipse is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (1.2)$$

If $a = b$, the equation of an ellipse turns into a center-radius form with a radius of a .

An ellipse is usually characterized by the value of the semi-major axis a and of the eccentricity $e = \sqrt{1 - \frac{b^2}{a^2}}$. The foci of the ellipse are two points that are symmetrically located on a large axis and the distance between them is equal to $2ae$ (Fig. 1.7). Those who are interested in geometry can easily prove that for any ellipse point, the sum total of the distances to the foci is constant and equal to $2a$.

We will calculate the area of an ellipse. In order to do this, we imagine a cylinder with a height h , a radius of the base b , and a volume equal to $V = \pi b^2 h$. We cut the cylinder along a plane at an angle α (Fig. 1.8a). An ellipse with semiaxes $a = \frac{b}{\cos \alpha}$ and b is obtained in the cross section.

We attach the truncated top of the cylinder to it from below (Fig. 1.8b), but the volume of the cylinder does not change. Now we cut the cylinder into a large number of n disks that are parallel to the new base. The area of

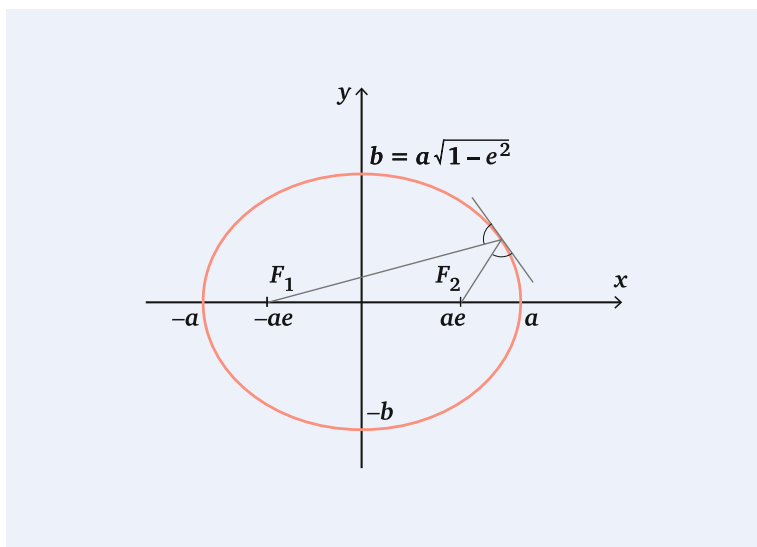


Fig. 1.7 Parameters of an ellipse

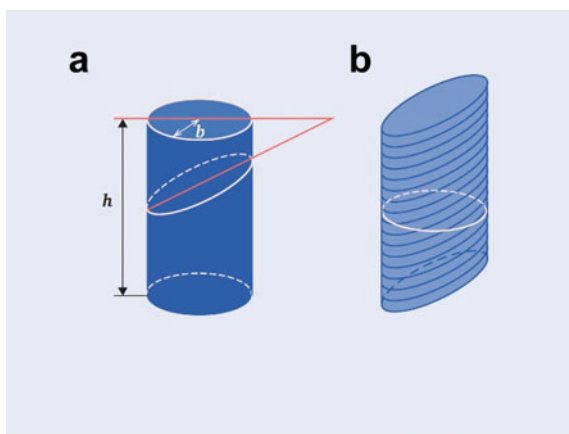


Fig. 1.8 How to calculate the area of an ellipse

each disk is S and the height is $\frac{h}{n} \cos \alpha$. When we make the volume of the cylinders equal, we get $S = \pi ab$.

Now that we know what an ellipse is, we can move on to Kepler's second law.

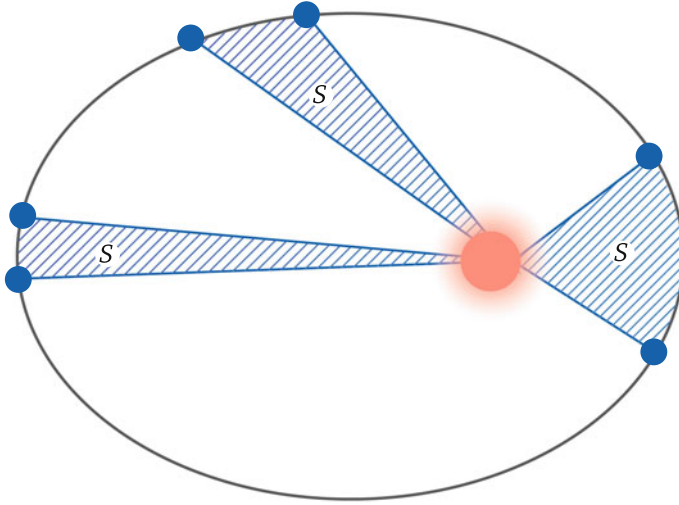


Fig. 1.9 Illustration of Kepler's second law

Kepler's Second Law The radius vector drawn from the Sun to each planet sweeps out equal areas in equal intervals of time (Fig. 1.9).

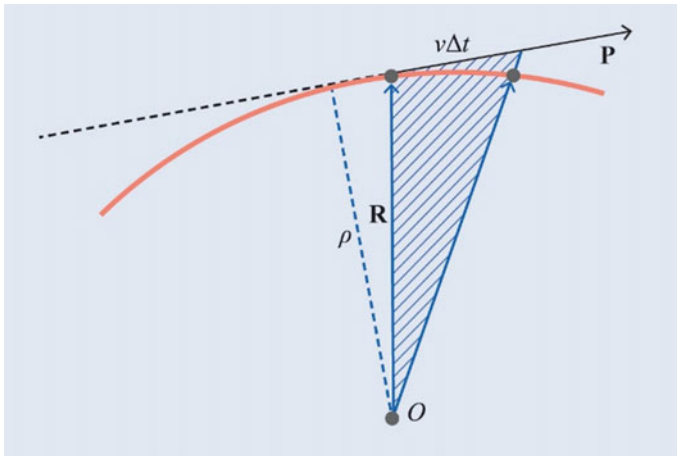


Fig. 1.10 Illustration of Kepler's second law

Support for Kepler's Second Law Let us consider a planet with the mass m moving in the field of gravity of the Sun, which is located at point O . We will disregard the influence of other celestial bodies on the planet's movement.

We will denote the planet's speed as \mathbf{v} . Its momentum is then $\mathbf{P} = m\mathbf{v}$ and is directed along the tangent to the planet's trajectory (Fig. 1.10).

We will fix the origin of coordinates at point O and drop the perpendicular from it onto a line that is defined by the vector \mathbf{P} . We will denote the length of that perpendicular as ρ . The product is called the *angular momentum*:

$$L = \rho P = \rho mv. \quad (1.3)$$

Because the moment of gravity relative to the origin of coordinates is zero, the angular momentum of the planet relative to the Sun does not change when the planet moves. During Δt the planet orbits the distance $v\Delta t$. Let's consider a shaded triangle with the base $v\Delta t$ (see Fig. 1.10). Its area is

$$\Delta S = \frac{1}{2} v \Delta t \rho = \frac{1}{2} \frac{L \Delta t}{m}. \quad (1.4)$$

If Δt time is short, then the base of the triangle practically coincides with the portion of the trajectory through which the planet passes. In this case, the triangle itself is a section of the area that the radius-vector \mathbf{R} of the planet sweeps out during Δt . Since the angular momentum is constant in time, the area swept by the line segment is proportional to the time interval Δt , that is, for equal periods of time the radius-vector of the planet will sweep out equal areas. This is the principle of Kepler's second law.

Let's imagine that an imaginary thread connects the Sun and a planet. The area over which the planet has passed remains constant each time for the same intervals of time. By applying Kepler's second law, we can easily calculate the linear speed of the planet, the velocity value of which can greatly differ depending on the place where the planet is located at that particular moment. In perihelion, which is the point in a planet's orbit that is closest to the Sun, planet speed is at its maximum, while in aphelion, which is the furthest point from the Sun, planet speed is minimal. Therefore, the speed of the Sun has the highest possible velocity value in perihelion $v_{\max} = 30.3 \text{ km (18.83 mi)/s}$. In the furthest point in orbit the formula is $v_{\min} = 29.3 \text{ km (18.21 mi)/s}$. This is why in January when the Earth reaches its perihelion, the Sun's speed in the sky is a little bit faster than in July when it is at aphelion. Admittedly, it is very difficult to observe this with the naked eye due to the fact that the shape of the Earth's orbit is almost circular.

However, this is not the case with other celestial bodies such as comets. Many of them travel on extremely elongated paths. For example, the orbital eccentricity of Halley's Comet is 0.967. Imagine that you are flying on that comet further and further away from the Sun until it is nothing more than a bright star. Your speed in relation to its speed becomes slower and slower... In the darkness and silence, you travel for decades toward aphelion by cosmic standards at a snail's pace of 0.9 km (0.56 mi)/s. Now you have passed through aphelion and the comet starts to pick up speed. The Sun keeps expanding and finally the comet passes through perihelion with incredible speed—54.5 km (33.86 mi)/s! During that very short trip radiation from the Sun causes the comet's surface to quickly become very hot. Owing to this, particles of the comet frantically evaporate and it grows a tail millions of kilometers (hundreds of miles) long. Imagine if the Earth had the same eccentricity as Halley's Comet. Without any sunshine in aphelion the temperature would drop to almost zero; even the air would freeze and precipitation would fall on the cold and lifeless surface. In perihelion the Sun would turn into a brutal fiery ball that would make oceans dry up and rocks melt.

Kepler's Third Law The ratio of the squares of the orbital periods of planets around the Sun (Fig. 1.11) is equal to the ratio of the cube of the length of the semi-major axis of its

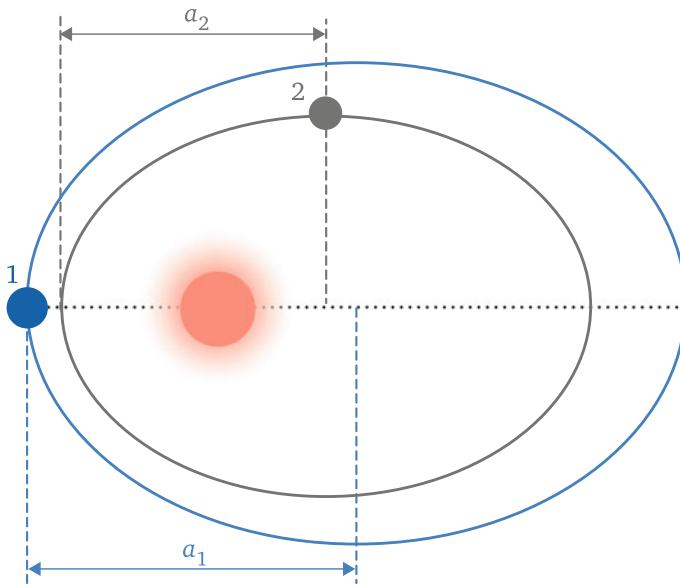


Fig. 1.11 Illustration of Kepler's third law

elliptical orbit:

$$\frac{T_1^2}{a_1^3} = \frac{T_2^2}{a_2^3}. \quad (1.5)$$

Connection Between Kepler's Third Law and the Law of Universal Gravitation Newton deduced the law of universal gravitation from Kepler's third law. Let's try to follow his train of thought.

Let's assume that there are several planets that move around a star and for simplicity's sake, let's say that this movement follows a circular pattern. We will denote the radii of planetary orbits as R_1 , R_2 , etc., and their orbital periods around the star as T_1 , T_2 , etc. Based on Kepler's third law it follows that:

$$\frac{R_1^3}{T_1^2} = \frac{R_2^3}{T_2^2} = \dots = \text{const.} \quad (1.6)$$

After introducing the angular velocity of the planets $\omega = \frac{2\pi}{T}$, Eq. 1.6 can be written as follows: $\omega_1^2 R_1^3 = \omega_2^2 R_2^3 = \dots = \text{const.}$

Newton assumed that the force of interaction of a planet with a star is an exponential function of the distance between them, i.e., it follows that: $F = AR^n$.

Then the accelerated velocity that the planet receives when it comes in contact with a star is proportional to the distance as well: $a = BR^n$.

Newton understood that when movement occurs around the periphery of a circle, centripetal acceleration is proportional to the squared velocity and inversely proportional to the distance, i.e.,

$$a = \frac{v^2}{R} = \omega^2 R \Rightarrow \omega^2 = BR^{n-1}. \quad (1.7)$$

By virtue of Kepler's third law, the product $\omega^2 R^3$ should have a constant value, specifically, it does not depend on distance. On the other hand, $\omega^2 R^3 = BR^{n+2}$, which shows that the required condition is met in the case of $n = -2$. In that event, $\omega^2 R^3 = B$. Newton also surmised that the constant value B is proportional to a star's mass M : $B = GM$. For acceleration we then get: $a = G \frac{M}{R^2}$.

The force that passes such acceleration on to the planet with the mass m will be equal to:

$$F = G \frac{Mm}{R^2}. \quad (1.8)$$

This is the law of universal gravitation. Newton is not to be commended as much for the fact that he discovered a way to express the force of gravitational pull as he is for universalizing this law.

The derivation of Kepler's third law is also quite intriguing.

We will find the time of a planet's complete period of rotation around the Sun, which is the orbital period T . According to Kepler's second law, during Δt the radius-vector of the planet sweeps out the area $S = \Delta t \frac{L}{2m}$. This means that one can calculate the orbital period after having divided the area of the ellipse by the sweep speed: $T = \frac{S}{\frac{L}{2m}}$.

The area of the ellipse is equal to $S = \pi ab$ where b is its semi-minor axis. Then $T = \frac{2\pi abm}{L}$.

From the laws of conservation of energy and momentum, we can obtain the formula $b = \frac{L}{\sqrt{2mE}}$ and then $T = \pi a \sqrt{\frac{2m}{E}}$. After expressing energy in terms of the semi-major axis a ($E = \frac{GmM}{2a}$), we get:

$$T = \frac{2\pi}{\sqrt{GM}} a^{\frac{3}{2}}. \quad (1.9)$$

What is the use of knowing about Kepler's third law? First, we can compare planets' orbits. Second, when we understand the orbital period of a celestial body, we can find the point of the semi-major axis of its orbit. Alternatively, after measuring the point of the semi-major axis of a celestial body's orbit, we can confidently determine what its orbital period is. The further a celestial body is from the Sun, the longer its orbital period.

Kepler's laws have proven just how versatile they are. In particular, they "work" well not only when calculating the orbits of celestial bodies around the Sun, but also when determining the parameters of the motion of man-made satellites and natural satellites of other planets. Information obtained from studying other galaxies has validated that Kepler's laws are carried out in outer space, which makes it possible to receive a great deal of significant and fascinating data.

It was recently reported that astronomers discovered a galaxy in which Kepler's third law does not "work": in this particular galaxy, which has a high velocity of rotation, hydrogen should have been emitted into more distant orbits, but this did not happen. This is because in this galaxy mass is "in short supply." But this is precisely how the natural sciences differ from the liberal arts in that laws that have been discovered and proven cannot be "incorrect" or "out of date." If you find out that a law that has proven its validity millions of times over can suddenly be disproven, that can only mean one thing—there is some new factor at work here that is unknown to you. This was the case in this situation. If we assume that in a galaxy there is a significant mass of a certain type of matter that we have not observed, then a law will once again be applied. In order to find this additional mass, scientists estimated the mass of gas between the stars. But that was not enough. Modern-day physics was faced with a mystery—what is the invisible substance called

“dark matter”? Kepler, who had formulated his laws several centuries ago, still contributes to the development of modern-day science.

1.2 A Star Called the Sun

The most important place in the homes of ancient people was the hearth. It gave off warmth, light and it was where people cooked. In those days when people did not know how to make a fire, a cold hearth could lead to the death of an entire tribe. The Sun, just like the hearth, gives light and warmth to the entire Solar System. Without the Sun no life forms on the Earth could exist. This is exemplified by the fact that in our energy-based biosphere there are planets that store the Sun's energy in the process of photosynthesis. It is not a surprise that in the religions of different countries the Sun God (Ra, Helios, or Jarilo) always existed and was among the most highly revered and powerful gods (Fig. 1.12). Therefore, we will devote some attention to our “cosmic hearth,” under the rays of which life began and exists.

The mass of the Sun is $M_S = 1.99 \times 10^{30}$ kg. Although it is difficult for us to imagine such a weight value within the categorical concepts to which we are accustomed, when speaking about celestial bodies such a quantity is nothing out of the ordinary. The Sun's mass makes up no less than 99.9% of the entire Solar System. In a manner of speaking, the mass of the Sun is the mass of the entire Solar System. Therefore, the Sun's supremacy over all other celestial bodies in the Solar System cannot be doubted. Its mass is large enough to keep planets and other celestial bodies of the Solar System in orbit around it.

But on the scale of the Galaxy, the Sun is of average size and an ordinary star; there are, according to various estimates, between 200 and 400 billion such planets in our Galaxy alone, which is called the *Milky Way*. We are located deep in the Galaxy (Fig. 1.13) at a distance of 26,400 light years from the center of it. It is a quiet place and it is precisely there, according to one hypothesis, that the speed of the stars and the spiral arms of the Galaxy come together. For this reason, it is difficult for us to fall out of the Galaxy, overtake our neighbors or lag behind them. This is called the *corotation circle* and we are very lucky to be located inside of it. After all, if a collision occurs between a celestial body in our Galaxy and another star, the existence of something as insignificant as our planet will not be of great concern to our celestial neighbors. However, thanks to the corotation circle we have little reason to worry about this actually happening.



Fig. 1.12 The Sun God Ra, ancient Egypt, 901–713 BC

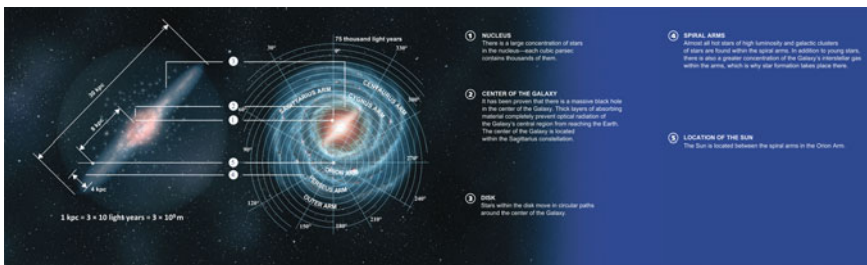


Fig. 1.13 The structure of our Galaxy

The galactic year of our planet, i.e., one complete revolution around the center of the Galaxy, is approximately 200 million years.

Since we have already described the Sun's location, now we need to speak about its age. The Sun was formed approximately 4.5 billion years ago when a molecular cloud composed of hydrogen, helium and other elements rapidly compressed under the influence of gravitational force. A star with a mass

such as that of the Sun has a lifespan of approximately 10 billion years. Thus, according to the standards of a star, the Sun is in its prime.

Ancient astronomers already knew the average distance from the Earth to the Sun: $R_{E-S} = 1.496 \times 10^{11}$ km, which stems from the laws of gravitational astronomy. If one were to fly on an airplane at the speed of 800 km (497 mi)/h, it would take more than five years to cover this distance. However, it takes a beam of light 8 min and 19 s to do this.

Because of the fact that from the Earth the Sun resembles a ball with an average angular diameter $\alpha_S = 9.3 \times 10^{-3}$ rad = $31' 59''$, it is easy to calculate the Sun's radius. It is: $R_S = 6.7 \times 10^8$ m, which is 109 times greater than the Earth's radius. The average solar density is $\rho_S = 1.4 \times 10^3$ kg/m³. We see that solar density is just slightly greater than water density and approximately four times less than the average density of the Earth.

The internal structure of the Sun is well studied today. With the help of various devices, including spectroscopes and different types of telescopes, the Sun's electromagnetic radiation is recorded in a variety of ranges and its surface and activity are observed, which enables us to draw conclusions about its internal structure.

The chemical composition of the Sun mostly consists of hydrogen (about 90%) and helium (about 9%) atoms. The remaining elements (iron, oxygen, nickel, nitrogen, silicon, sulfur, carbon, magnesium, neon, chromium, calcium and sodium) account for less than 2%.

The primary value of the physical characteristics both on the surface and in the inner regions of the Sun, as well as the nature of energy that the Sun (and other stars) constantly emits, is also well known. How did this information become known? After all, it is impossible to fly to the Sun and measure its temperature with a thermometer. Knowing physics and mathematics help make everything clearer.

Constitutive Relation of Solar Matter We will identify some of the physical characteristics of the processes that take place on the Sun.

The intensity of solar radiation is characterized by a value called the *solar constant*. It is the total solar radiation energy per unit of area perpendicular to the Sun's rays and at the Earth's average distance from the Sun. According to data obtained from exo-atmospheric measurements, the solar constant is: $S = 1367$ W/m². Despite its name, the solar constant does not remain constant over time. Its value is determined by two main factors: the distance between the Earth and the Sun, which changes throughout the year (the annual variation is 6.9%, which is from 1412 W/m² at the beginning of January to 1312 W/m² at the beginning of July) and changes in solar activity.

If we multiply the solar constant by the area of the sphere with a radius R_{E-S} , we can find out how much total energy is emitted by the Sun in 1 s, i.e., the solar radiation output or the

solar luminosity. It is equal to $L = S \times 4\pi R_{E-S}^2 = 3.83 \times 10^{26} \text{ W}$. We can also find the solar energy flux density, i.e., the amount of energy that is emitted per second by a square meter (square foot) of the Sun's surface. This is, in fact, its brightness: $R = \frac{L}{4\pi R_S^2} = 6.29 \times 10^7 \text{ W}$. The energy flux density emitted by an object is related to its temperature according to the Stefan-Boltzmann law: $R = \sigma_B T^4$, where $\sigma_B = 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \text{K}^4}$. From here it follows that we can calculate the temperature on the Sun's surface: $T_{S,\text{surf}} = 5780 \text{ K}$.

We can obtain the constitutive relation of solar matter, specifically, the relationship between temperature, pressure and density. If the gas on the Sun's surface consists mainly of electrically neutral atoms (weakly ionized plasma), then when immersed deep into the Sun and when the temperature and pressure are increased, the electrons of the atomic shells detach themselves from their atoms, thus forming plasma, the degree of ionization of which reaches 100%.

Let's assume that solar plasma is made up of hydrogen nuclei (protons), helium nuclei and electrons in a ratio of 91:9:109, respectively. Solar plasma is, in fact, a mixture of three gases: hydrogen nuclei, helium nuclei and electrons of the same temperature. For each of the gas mixtures, which can be considered ideal, the constitutive relation $p = nk_B T$ is true, where the concentration of gas particles is $n = \frac{\rho}{m}$ (ρ is the gas density and m is the mass of gas particles) and $k_B = 1.38 \times 10^{-23} \text{ J/K}$.

In addition, the total plasma pressure is the sum total of the pressure of individual gases: $p = p_H + p_{He} + p_e$, while the overall concentration is $n = n_H + n_{He} + n_e$. We will designate the fraction of hydrogen ions in the total number of particles as N . Then the fraction of helium ions will be proportional to $1 - N$, and the fraction of electrons will be proportional to $2 - N$. The concentration, pressure and gas density of hydrogen ions can be written as

$$n_H = \frac{N}{3 - N} n, \quad (1.10)$$

$$p_H = \frac{N}{3 - N} nk_B T, \quad (1.11)$$

$$\rho_H = m_H \frac{N}{3 - N} n. \quad (1.12)$$

This is similar for the gas of helium ions:

$$n_{He} = \frac{1 - N}{3 - N} n, \quad (1.13)$$

$$p_{He} = \frac{1 - N}{3 - N} nk_B T, \quad (1.14)$$

$$\rho_{He} = m_{He} \frac{1 - N}{3 - N} n. \quad (1.15)$$

For the gas of electrons, it is:

$$n_e = \frac{2 - N}{3 - N} n, \quad (1.16)$$

$$p_e = \frac{2 - N}{3 - N} nk_B T, \quad (1.17)$$

$$\rho_e = m_e \frac{2 - N}{3 - N} n. \quad (1.18)$$

We will take into account that the mass of a helium ion is four times greater than the mass of a hydrogen ion, $m_{\text{He}} = 4m_{\text{H}}$, while the mass of the electron is small to negligible as compared to the mass of the proton, $m_e \ll m_{\text{H}}$. Then we can assume that $\rho_e \approx 0$. We get:

$$\rho = \rho_{\text{H}} + \rho_{\text{He}} + \rho_e = \frac{4 - 3N}{3 - N} m_{\text{H}} n. \quad (1.19)$$

The constitutive relation of solar matter will take on a modern-day appearance of the ideal gas law (also known as the Mendelev-Clapeyron equation):

$$\rho = \frac{\rho}{\mu m_{\text{H}}} k_{\text{B}} T, \quad (1.20)$$

for gas with a molar mass:

$$\mu = \frac{4 - 3N}{3 - N} = 0.61. \quad (1.21)$$

This mass turned out to be very small due to the fact that, although electron gas exerts pressure, it has no bearing on a change in density. This explains the low density of solar plasma.

Since we know the constitutive relation, we can find the temperature and pressure in the central region of the Sun. The pressure, which is created inside the Sun, is due to the gravitational compression of matter. If we consider a column of matter with the density ρ and the height H in a gravitational field with an acceleration of gravity g , the pressure it creates will be equal to: $p = \rho g H$. This formula can be roughly used in this case, although the rate of acceleration of gravity for stars naturally varies with depth. We get:

$$p_{\text{S}} \approx \rho_{\text{S}} g_{\text{S}} R_{\text{S}} \approx \frac{GM_{\text{S}}^2}{R_{\text{S}}^4} \approx 10^{15} \text{ N/m}^2. \quad (1.22)$$

By using an equation of condition, one can even estimate the temperature of the central region of the Sun: $T_{\text{S}} \approx \frac{p_{\text{S}} m_{\text{H}}}{k_{\text{B}} \rho_{\text{S}}} \approx \frac{GM_{\text{S}} m_{\text{H}}}{k_{\text{B}} R_{\text{S}}} \approx 2 \times 10^7 \text{ K}$.

Our estimate of 20 MK roughly corresponds to information about precise calculations. But is gravitational energy enough for the Sun and other stars to exist? We will estimate the potential energy of the Sun after it has been compressed by the force of gravity: $E_{\text{p}} \approx \frac{GM_{\text{S}}^2}{R_{\text{S}}} \approx 4 \times 10^{41} \text{ J}$.

This energy can provide the Sun's brightness that we see: $L = 3.83 \times 10^{26} \text{ W}$ for a period of $t = \frac{E_{\text{p}}}{L} \approx 3 \times 10^7 \text{ years}$.

The lifetime of the Sun can, in fact, last almost as long as five billion years. The solution to our above-calculated equation illustrates that, in addition to gravitational energy, a different and much more powerful energy source is needed to warm the Sun and other stars.

We will first focus on the structure of the Sun (Fig. 1.14) and the primary physical features of its layers (Fig. 1.15). Thereafter, we will analyze those physical mechanisms that cause physics to be at work in the universe.

The central part of the Sun is the *core* having a radius of approximately 151,000 km (93,827 mi). Matter in the core is extremely dense. It is about $1.5 \times 10^5 \text{ kg/m}^3$, which is 150 times higher than water density. The temperature in the center of the solar core exceeds $1.5 \times 10^7 \text{ K}$.

It seems to us that the Sun is a burning ball, but burning is actually an example of a chemical change, while an energetically more powerful process occurs in the solar core: a thermonuclear reaction that makes hydrogen nuclei fuse into helium nuclei. Every second the Sun loses 4.3 T (4.74 sh. tn.) of hydrogen. But there is no need to worry—scientists estimate that because the Sun has a mass of $2 \times 10^{27} \text{ T}$, there is enough solar fuel to last about five billion years.

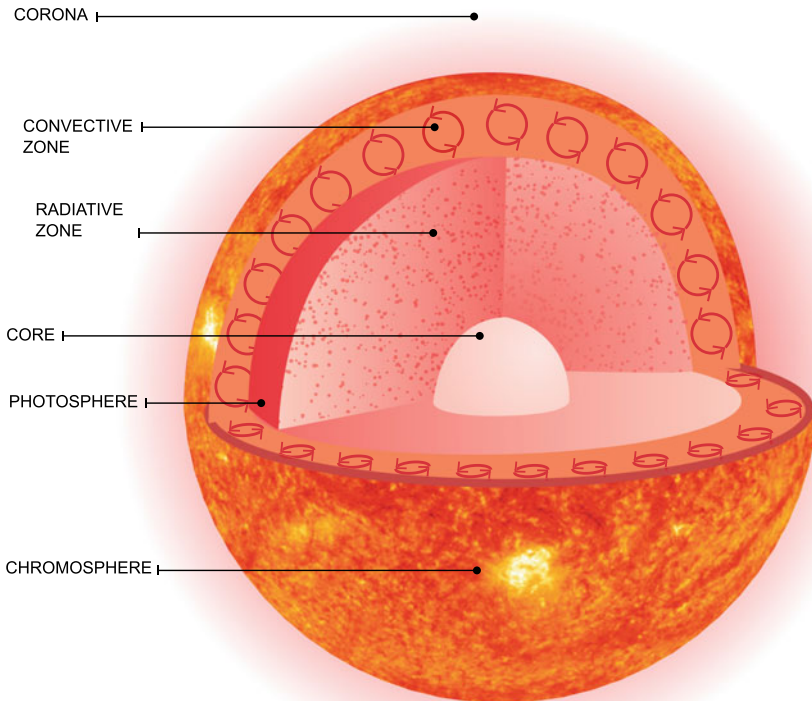


Fig. 1.14 The structure of the Sun