LEARNING MADE EASY

Calculus II

WORKBOOK

for dummies®

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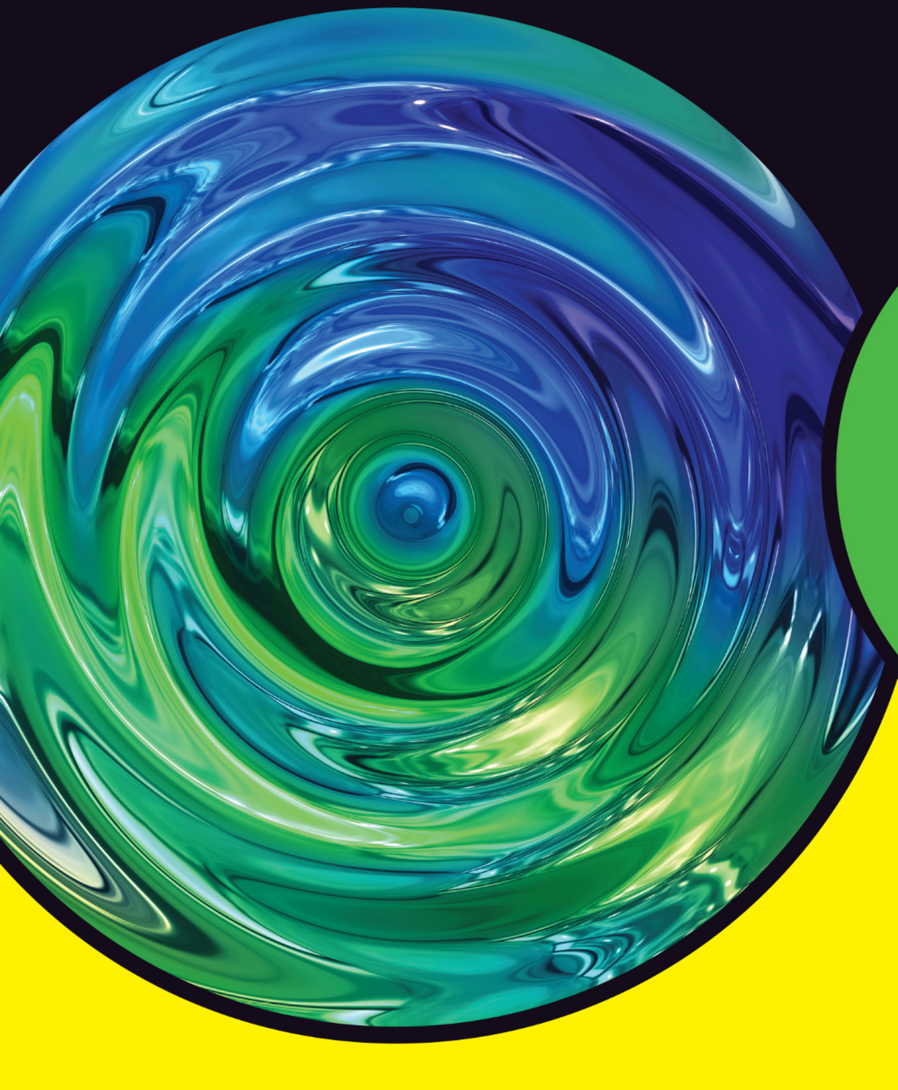

Master tricky integration techniques

Study with hundreds of practice problems

Review key math concepts

Mark Zegarelli

Math Guru

Calculus II Workbook

for dummies

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by Mark Zegarelli

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Contents at a Glance

Table of Contents

Introduction

Calculus II is tough stuff by just about any standard.

In this book, I've done my best to explain the key topics that tend to hang students up as clearly and concisely as possible. I've also provided practice problems that I think will help you to make sense of this information, so you'll be ready to take on similar problems that your teacher or professor might throw at you.

About This Book

Chapter by chapter, this workbook follows the typical order of topics in a standard Calculus II course. But please feel free to jump back and forth throughout the book in whatever order makes sense for you.

Chapter 1 provides an overview of the course, and then Chapters 2 and 3 give you a review of the Pre-Calculus and Calculus I topics you'll need to know to move forward. In Parts 2 through 4, you learn a variety of integration methods, all of which are commonly taught in Calculus II. Part 5 focuses on basic applications of integration. Part 6 covers sequences and series. And in Part 7, I give you a couple of Top Ten lists related to calculus.

Additionally, for convenience, this workbook follows the same chapter-by-chapter format as *Calculus II For Dummies*, 3rd edition. Of course, you don't have to buy that book to make good use of this one. But if you do want to use both books, you'll find that corresponding numbered chapters cover the same topics, so you'll be able to flip back and forth between them easily.

Foolish Assumptions

I assume that you either want or need to learn Calculus II. So, either you're interested in the topic and want to study it on your own, or like many people, you're taking a high-school or college course in the subject.

Icons Used in This Book

In this book, I use a variety of icons to give you a heads-up about what information is important and what you can safely skip over when you're in a rush.

EXAMPLE

The example icon accompanies a sample question followed by a step-by-step solution. In most cases, examples should help you get a handle on difficult material in a way that makes sense.

REMEMBER

This icon alerts you to key information that you may need to pay special attention to, especially if you're currently studying for a test.

TIP

Tips provide you with a quick and easy way to work on a problem. Try them out as you work your way through the book, so you can use them in assignments and on tests.

TECHNICAL STUFF

The Technical Stuff icon marks information of a highly technical nature that you can normally skip if you are in a hurry.

WARNING

This icon warns you of typical errors that students tend to fall into. Keep an eye on these little traps so that they don't get you, too!

Beyond the Book

If you want to explore Calculus II in greater depth, or get an additional perspective on it, look no further than my book *Calculus II For Dummies*, 3rd edition. This workbook has been written in conjunction with that book to provide an even more complete picture of the topics taught here.

This book also includes accompanying online material in the form of a Cheat Sheet. To access it, go to `www.dummies.com` and type **Calculus II Workbook For Dummies** in the Search box.

Where to Go from Here

Success in a Calculus II class is built on a foundation of a whole lot of other math you've been studying since you learned how to count. To help you shore up this foundation, here are a few additional resources:

- If you need more help with some of the foundational math than this book provides, an easy place to start is my book, *Basic Math and Pre-Algebra For Dummies.*
- If you need a refresher on any algebra concepts not covered here, check out *Algebra I For Dummies* and *Algebra II For Dummies*, both by Mary Jane Sterling.
- If you find that your trig skills are in need of a makeover, *Trigonometry For Dummies*, also by Mary Jane Sterling, gives you wider and deeper coverage of the topic than you'll find here.
- Even if you've passed Calculus I, you may want a refresher. Check out *Calculus For Dummies* and its accompanying workbook by Mark Ryan for a closer look at basic calculus topics.

1

Introduction to Integration

IN THIS PART . . .

See Calculus II as an ordered approach to finding the area of unusual shapes on the *xy*-graph

Use the definite integral to clearly define an area problem

Slice an irregularly shaped area into rectangles to approximate area

Review the math you need from Pre-Algebra, Algebra, Pre-Calculus, and Calculus I

IN THIS CHAPTER

- » Measuring the area of shapes with classical geometry
- » Finding the area of shapes on the *xy*-plane
- » Using integration to frame the area problem

Chapter 1

An Aerial View of the Area Problem

In Calculus I, you discovered how to use math to solve a single problem: the *tangent problem* or *slope problem*, which involved finding the slope of a tangent line to a function on the *xy*-graph. Calculus II is also devoted to solving a single problem: the *area problem* — finding the area of a region of the *xy*-graph under a function.

In this chapter, you use a variety of simple methods to frame and solve area problems. First, you use formulas from classical geometry to measure the area of familiar shapes on the *xy*-graph. Next, you discover how to define the area of a region under a function as a definite integral.

To finish up, you put those two skills together, both setting up and solving definite integrals to solve area problems for relatively simple functions.

Ready to get started?

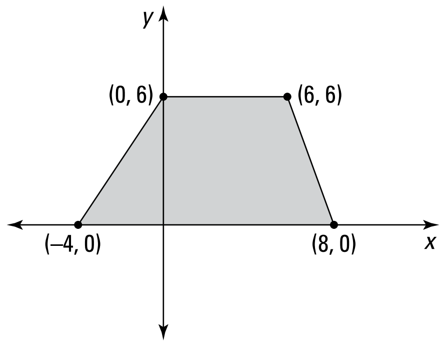
Measuring Area on the *xy*-Graph

People have been calculating the area of shapes for thousands of years. More than 2,000 years ago, Euclid developed a thorough system of geometry that included methods for finding the area of squares, rectangles, triangles, circles, and so on.

Descartes's invention of the *xy*-graph in the early 17th century made the study of geometry possible from an algebraic perspective, laying the foundation for calculus.

But you don't need calculus to measure the area of many basic shapes on the *xy*-graph. Here are a few formulas you know from geometry that are just as useful when measuring area on the *xy*-graph.

Square:	Rectangle:	Triangle:
Area $= s^2$	Area $= bh$	Area $= \frac{1}{2}bh$
Trapezoid:	Circle:	Semi-Circle:
Area $= \frac{b_1 + b_2}{2}h$	Area $= \pi r^2$	Area $= \frac{1}{2}\pi r^2$

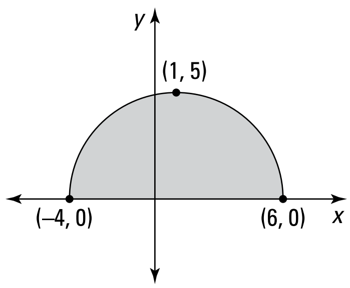


Q. What is the area of the shape in this figure?

EXAMPLE

A. **54.** The figure shows a trapezoid with bases of lengths 12 and 6, and a height of 6. Plug these values into the formula for a trapezoid:

$$\frac{b_1 + b_2}{2}h = \frac{12 + 6}{2}(6) = 54$$



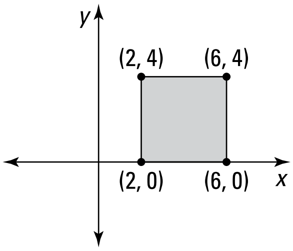
Q. Find the area inside the semi-circle shown in the figure.

EXAMPLE

A. **12.5π.** The semi-circle has a radius of 5, so plug this value into the formula for a semi-circle: $\frac{1}{2}\pi r^2 = \frac{1}{2}\pi(5)^2 = 12.5\pi$

In each of the following *xy*-graphs, find the area of the shape.

1



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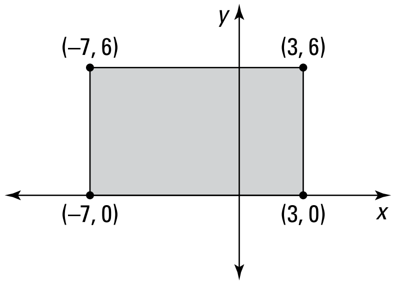

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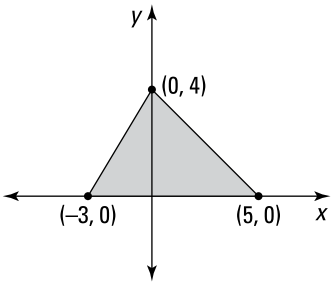

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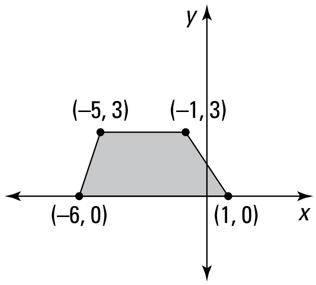

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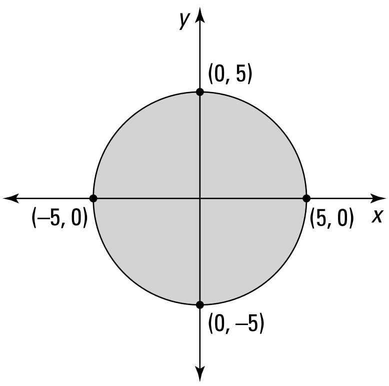

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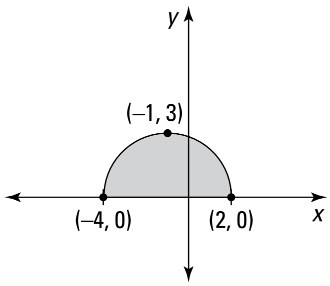
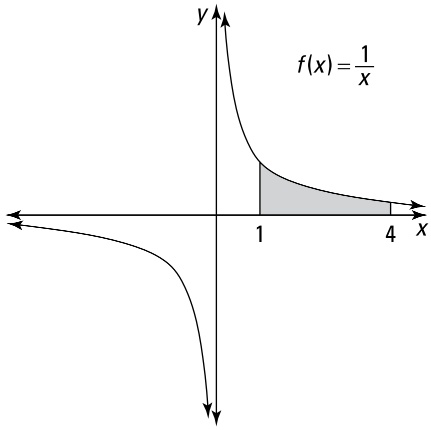


Defining Area Problems with the Definite Integral

The *definite integral* provides a systematic way to define an area on the *xy*-graph. For functions that reside entirely above the *x*-axis, a definite integral $\int_a^b f(x)\,dx$ defines an area that lies:

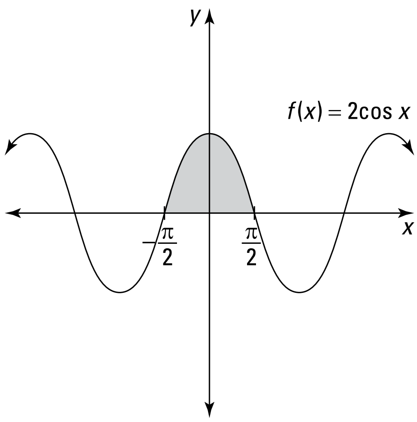
- Vertically between the function *f*(*x*) and the *x*-axis
- Horizontally between two *x*-values, *a* and *b*, called the *limits of integration*



EXAMPLE

Q. What definite integral describes the shaded area in the figure?

A. $\int_1^4 \frac{1}{x}\,dx$. The shaded area resides vertically between the function $\frac{1}{x}$ and the *x*-axis, and horizontally between $x = 1$ and $x = 4$.



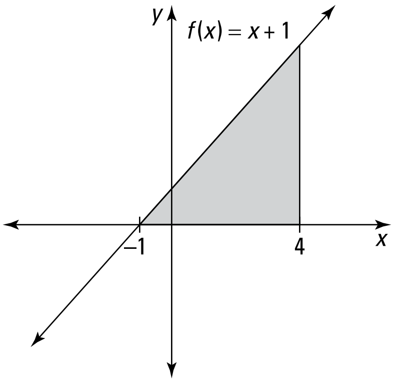
EXAMPLE

Q. Write a definite integral that defines the area of the shaded region in the figure.

A. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\cos x \, dx$. The shaded region is vertically between $f(x) = 2\cos x$ and the x-axis, and horizontally between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$.

In each of the following xy-graphs, write the definite integral that represents the shaded area.

7



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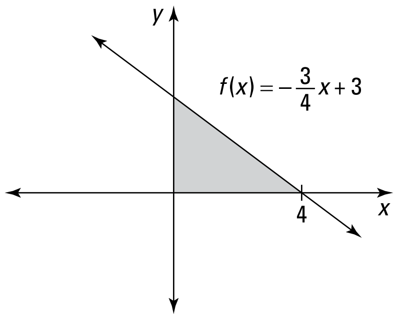

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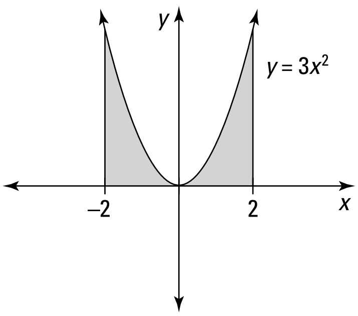

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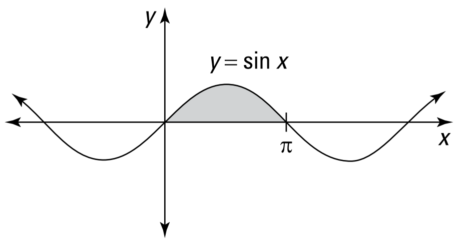

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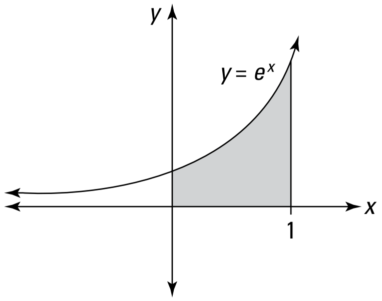

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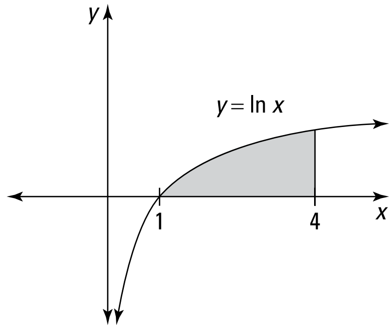
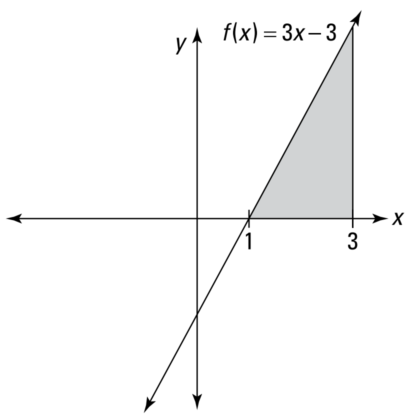


Calculating Area Defined by Functions and Curves on the *xy*-Graph

In the previous section, you discover how to use the definite integral to frame an area problem on the *xy*-graph. In Calculus II, you'll learn a ton of different ways to evaluate definite integrals to solve area problems.

But for many simple functions, you don't need calculus to evaluate a definite integral. For example:

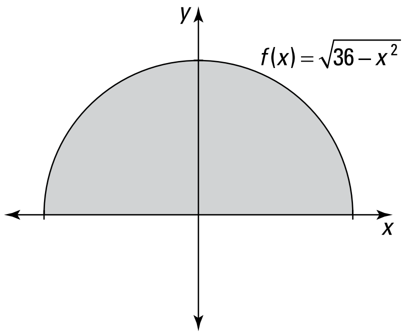


EXAMPLE

Q. Write a definite integral to describe the shaded area in the figure, and then use geometry to evaluate it.

A. $\int_1^3 3x - 3\, dx = 6$. The shaded area is the triangle that resides vertically between the function $3x - 3$ and the x-axis and horizontally between $x = 1$ and $x = 3$. The base of this triangle is $3 - 1 = 2$. To find the height, plug in $f(3) = 3(3) - 3 = 6$. Now, to find the area, plug in 2 for the base and 6 for the height into the formula for a triangle:

$$\text{Area} = \frac{1}{2}bh = \frac{1}{2}(2)(6) = 6$$



EXAMPLE

Q. Use a definite integral to define the shaded region shown, and then find the area of that region without calculus.

A. $\int_{-6}^{6} \sqrt{36 - x^2}\, dx = 18\pi$. The shaded area resides vertically between the function $\sqrt{36 - x^2}$ and the x-axis. To find the limits of integration, set this function to 0 and solve:

$$0 = \sqrt{36 - x^2}$$
$$0 = 36 - x^2$$
$$x^2 = 36$$
$$x = 6 \text{ and } -6$$

Thus, the limits of integration are $x = -6$ and $x = 6$. To find the area, plug in 6 for the radius into the formula for the area of a semi-circle:

$$\frac{1}{2}\pi r^2 = \frac{1}{2}\pi(6)^2 = 18\pi$$

In each of the following *xy*-graphs, write and evaluate the definite integral that represents the shaded area.

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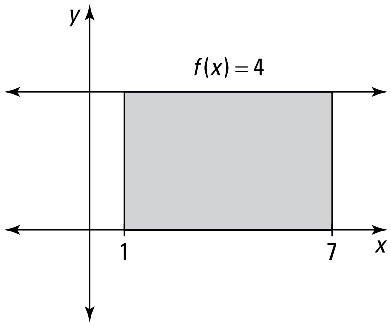

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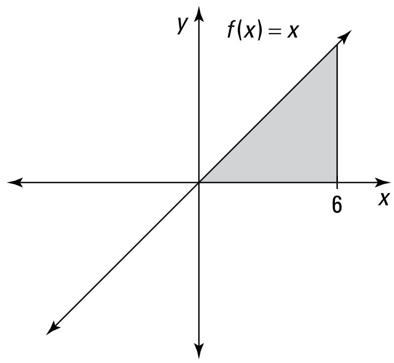

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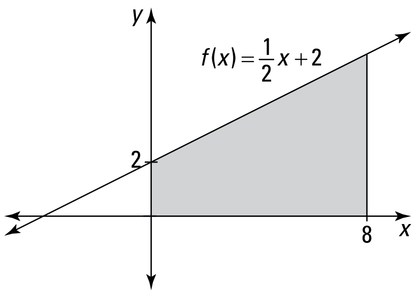

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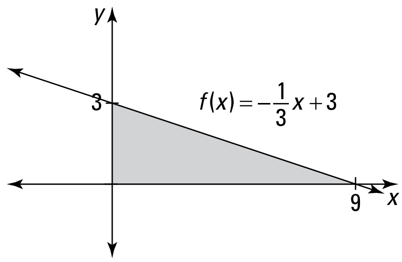

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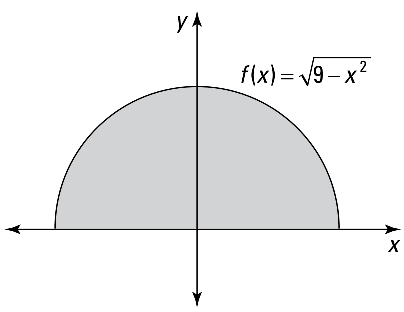

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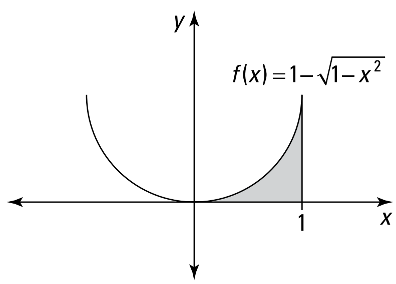

Answers and Explanations

1. **16.** $s^2 = 4^2 = 16$

2. **60.** $bh = (10)(6) = 60$

3. **16.** $\frac{1}{2}bh = \frac{1}{2}(8)(4) = 16$

4. **16.5.** $\frac{b_1 + b_2}{2}h = \frac{7+4}{2}(3) = 16.5$

5. **25π.** $\pi r^2 = \pi(5)^2 = 25\pi$

6. **32π.** $\frac{1}{2}\pi r^2 = \frac{1}{2}\pi(3)^2 = \frac{9}{2}\pi$

7. $\int_{-1}^{4} x + 1\, dx$

8. $\int_{0}^{4} -\frac{3}{4}x + 3\, dx$

9. $\int_{-2}^{2} 3x^2\, dx$

10. $\int_{0}^{\pi} \sin x\, dx$

11. $\int_{0}^{1} e^x dx$

12. $\int_{1}^{4} \ln x\, dx$

13. $\int_{1}^{7} 4\, dx = (6)(4) = 24$

14. $\int_{0}^{6} x\, dx = \frac{1}{2}(6)(6) = 18$

15. $\int_{0}^{8} \frac{1}{2}x + 2\, dx = \frac{2+6}{2}(8) = 32$

16. $\int_{0}^{9} -\frac{1}{3}x + 3\, dx = \frac{1}{2}(9)(3) = 13.5$

17. $\int_{-3}^{3} \sqrt{9 - x^2}\, dx = \frac{1}{2}\pi(3)^2 = 4.5\pi$

18. $\int_{0}^{1} 1 - \sqrt{1 - x^2}\, dx = 1 - \frac{\pi}{4}$

IN THIS CHAPTER

- » Calculating with fractions and factorials
- » Working with exponents and simplifying rational expressions
- » Remembering radian measure
- » Proving trig identities
- » Understanding important parent functions and their transformations
- » Converting an infinite series from sigma notation to expanded notation

Chapter 2

Forgotten but Not Gone: Review of Algebra and Pre-Calculus

Most students have been studying math for at least 10 years before they enter their first calculus classroom. This fact leaves many students overwhelmed by all the math they *should* know, and perhaps *did* know at one time, but can't quite recall.

Fortunately, you don't need another 10 years of review to be ready for Calculus II. In this chapter, I get you back up to speed on the key topics from your Pre-Algebra, Algebra, and Pre-Calculus classes that will help you the most this semester.

To begin, you go all the way back to middle school for a quick review of fractions. I also give you some practice calculating factorials.

After that, I remind you how to work with exponents, and especially how to use negative and fractional exponents to express rational and radical functions. Then I cover a few important ideas from trigonometry that you're sure to need, such as radian measure and trig identities.

Next, I give you an overview of how to sketch the most important parent functions on the *xy*-graph: polynomials, exponentials, radicals, logarithmic functions, and the sine and cosine functions. You use these to work with a variety of function transformations, such as vertical and horizontal transformations, as well as stretch, compress, and reflect transformations.

Fractions

When finding derivatives in Calculus I and integrals in Calculus II, you'll often need to add 1 to (or subtract 1 from) a fraction. Here's a trick for doing both of those operations quickly in your head without getting a common denominator:

EXAMPLE

Q. What is $\frac{7}{5}+1$?

A. $\mathbf{\frac{12}{5}}$. To do this calculation in your head, add the numerator and denominator, and then keep the denominator of 5:

$$\frac{7}{5}+1=\frac{7+5}{5}=\frac{12}{5}$$

EXAMPLE

Q. What is $\frac{5}{6}-1$?

A. $-\mathbf{\frac{1}{6}}$. To calculate this value in your head, subtract the numerator minus the denominator, and then keep the denominator of 6.

$$\frac{5}{6}-1=\frac{5-6}{6}=\frac{-1}{6}=-\frac{1}{6}$$

1. Add 1 to the following fractions and express each answer as a proper or improper fraction (no mixed numbers).

a. $\frac{3}{5}+1=$

b. $\frac{11}{6}+1=$

c. $-\frac{2}{3}+1=$

d. $-\frac{13}{5}+1=$

2. Subtract 1 from the following fractions and express each answer as a proper or improper fraction (no mixed numbers).

a. $\frac{1}{4}-1=$

b. $\frac{7}{3}-1=$

c. $-\frac{1}{5}-1=$

d. $-\frac{21}{2}-1=$

Factorials

In Calculus II, when working with infinite series, you also may need to make use of factorials. Recall that the symbol for factorial is an exclamation point (!). The factorial of any positive integer is that number multiplied by every positive integer less than it. Thus:

REMEMBER

$1! = 1$	$2! = 2 \times 1 = 2$	$3! = 3 \times 2 \times 1 = 6$
$4! = 4 \times 3 \times 2 \times 1 = 24$	$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$	$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

Also, by definition, $0! = 1$.

When you know how to expand factorials in this way, simplifying rational expressions that include them is relatively straightforward. Always look for opportunities to cancel factors in both the numerator and denominator.

EXAMPLE

Q. Simplify $\frac{(6!)(2!)}{(4!)(3!)}$.

A. **10.** Begin by expanding the factorials:

$$\frac{(6!)(2!)}{(4!)(3!)} = \frac{(6 \times 5 \times 4 \times 3 \times 2 \times 1)(2 \times 1)}{(4 \times 3 \times 2 \times 1)(3 \times 2 \times 1)} =$$

Now, cancel factors in both the numerator and denominator, and simplify the result:

$$\frac{(6 \times 5 \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1})(\cancel{2} \times \cancel{1})}{(\cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1})(3 \times \cancel{2} \times \cancel{1})} = \frac{30}{3} = 10$$

EXAMPLE

Q. Simplify $\frac{(n+2!)}{n(n-1!)}$.

A. $\boldsymbol{(n+2)(n+1)}$**.** Expand the factorials as follows:

$$\frac{(n+2!)}{n(n-1!)} = \frac{(n+2)(n+1)(n)\ldots(3)(2)(1)}{n(n-1)(n-2)(n-3)\ldots(3)(2)(1)}$$

Cancel factors in both the numerator and denominator, and simplify the result:

$$\frac{(n+2)(n+1)(\cancel{n})\ldots(\cancel{3})(\cancel{2})(\cancel{1})}{\cancel{n}(\cancel{n-1})(\cancel{n-2})(\cancel{n-3})\ldots(\cancel{3})(\cancel{2})(\cancel{1})} = (n+2)(n+1)$$

3 Simplify each of the following factorial expressions.

a. $\frac{5!}{3!}$

b. $\frac{7!}{9!}$

c. $\frac{100!}{101!}$

d. $\frac{(19!)(8!)}{(11!)(17!)}$

4 Simplify each expression in terms of n.

a. $\frac{n!}{(n+2)!}$

b. $\frac{(n-1)!}{(n+3)!}$

c. $\frac{(n+1)!(n-1)!}{n!}$

d. $\frac{n(n-2)(n+2)!}{(n^3-4n)(n+1)!}$

Negative and Fractional Exponents

REMEMBER

When an expression has a negative exponent, you can rewrite it with a positive exponent and place it in the denominator of a fraction. For example:

$$x^{-1}=\frac{1}{x} \qquad x^{-2}=\frac{1}{x^2} \qquad x^{-3}=\frac{1}{x^3} \qquad x^{-4}=\frac{1}{x^4}$$

When an expression has a fractional exponent, you can rewrite it as a radical. For example:

$$x^{\frac{1}{2}}=\sqrt{x} \qquad x^{\frac{1}{3}}=\sqrt[3]{x} \qquad x^{\frac{1}{4}}=\sqrt[4]{x} \qquad x^{\frac{1}{5}}=\sqrt[5]{x}$$

More complicated fractional exponents can be written in two separate and equally valid ways as a combination of a radical and an exponent. For example:

$$x^{\frac{5}{3}}=\sqrt[3]{x^5} \qquad x^{\frac{3}{4}}=\sqrt[4]{x^3} \qquad x^{\frac{5}{4}}=\sqrt[4]{x^5} \qquad x^{\frac{11}{5}}=\sqrt[5]{x^{11}}$$

$$x^{\frac{5}{3}}=\left(\sqrt[3]{x}\right)^5 \qquad x^{\frac{3}{4}}=\left(\sqrt[4]{x}\right)^3 \qquad x^{\frac{5}{4}}=\left(\sqrt[4]{x}\right)^5 \qquad x^{\frac{11}{5}}=\left(\sqrt[5]{x}\right)^{11}$$

When a fractional exponent is negative, you can rewrite it as a radical in the denominator of a fraction:

$$x^{-\frac{1}{2}}=\frac{1}{\sqrt{x}} \qquad x^{-\frac{3}{2}}=\frac{1}{\sqrt{x^3}} \qquad x^{-\frac{5}{3}}=\frac{1}{\sqrt[3]{x^5}} \qquad x^{-\frac{7}{8}}=\frac{1}{\sqrt[8]{x^7}}$$