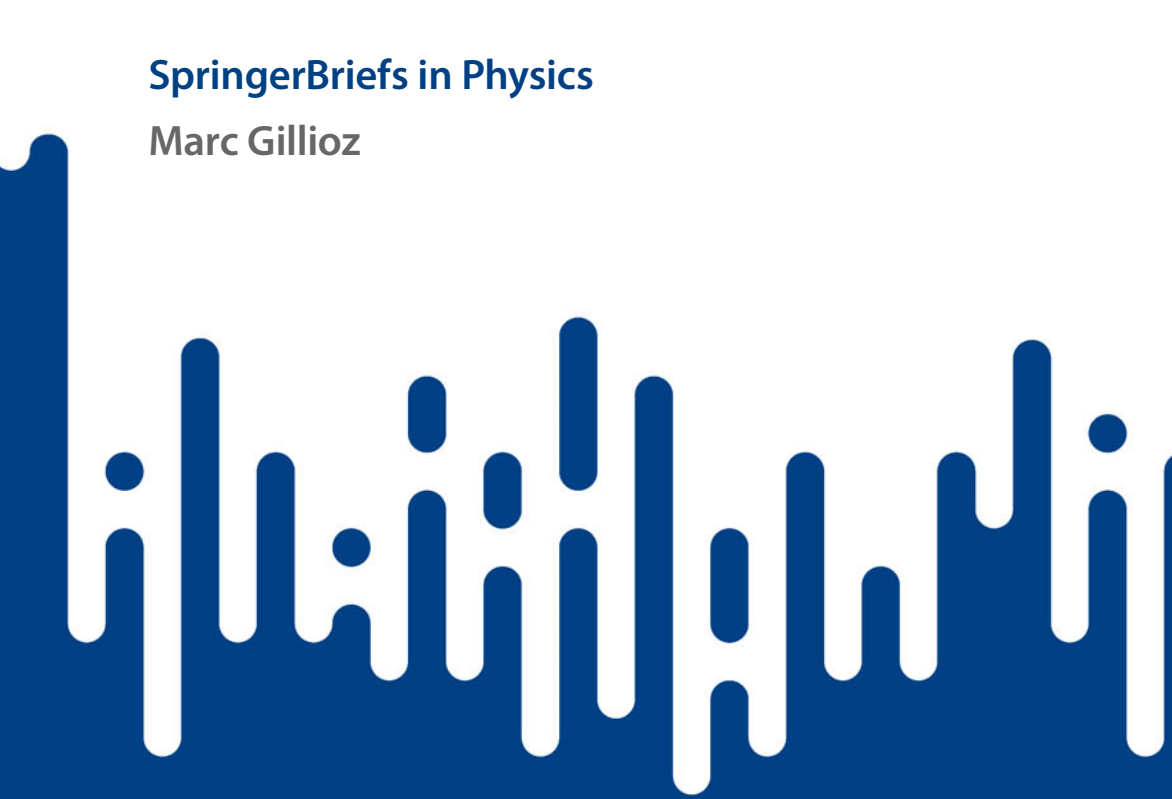


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Marc Gillioz



Conformal Field Theory for Particle Physicists

From QFT Axioms
to the Modern Conformal
Bootstrap

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Springer

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ISSN 2191-5423

ISSN 2191-5431 (electronic)

SpringerBriefs in Physics

ISBN 978-3-031-27085-7

ISBN 978-3-031-27086-4 (eBook)

<https://doi.org/10.1007/978-3-031-27086-4>

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This book is dedicated to all the people who spend their time, their energy, and even their lives in fundamental research and get so little in return. No matter how small, their contribution is a blessing for current and future generations.

Preface

The story behind “Conformal field theory for particle physicists” is before anything else the story behind my own academic path. I started as a particle physicist, with a Ph.D. and a first postdoc mostly focused on high-energy phenomenology and beyond-the-Standard-Model physics. At some point, I realized that my research was not really contributing to the overall advancement of science, as I had imagined it would (or more realistically, I was not able to bring it anywhere). So I decided to start working on conformal field theory, as I was hearing from various sources how great a subject it was. This is not really something you are expected to do in our modern academic world, and I think I got extremely lucky to be able to make such a move.

Even if it did not turn out to be very smart from a career perspective, I never regretted it. I found conformal field theory a fascinating subject. One in which significant advances are made every year. One in which there are great outstanding problems, and sometimes not enough people around to work on them. One in which the applications range from solid-state physics to string theory and cosmology. And one in which I was finally able to make sense of what I learned in quantum field theory beyond perturbation theory.

Learning conformal field theory turned out to be quite difficult for me. Not that the underlying mathematical methods are fundamentally more difficult than in particle physics; for sure, CFT requires quite a bit of mathematics, including complex analysis, group theory, or differential geometry. But these are all standard tools that a particle physicist has (or will) encounter in their own work. No, for me the most difficult part was the change of perspective. Using correlation functions as the central observable instead of scattering amplitudes. Working in position space instead of momentum space, and in Euclidean coordinates instead of Minkowski space-time ones.¹ The big difficulty for me was often more to understand *why* I would have to work on some CFT project, rather than *how*. And even though there are many excellent introductory books and reviews on the topic of conformal field theory, none of

¹ In fact, this is something I must never have completely accepted, as I ended up specializing on the momentum-space representation of conformal correlators in Minkowski space-time.

them was able to answer my questions and help me connect CFT with what I knew from particle physics.

The goal of this book is precisely to fill what I think is a gap in the literature. Conformal field theory is introduced here as just another quantum field theory like the ones we study in particle physics. There are two more properties—scale symmetry and special conformal symmetry—that let us make so much more progress without having to go into the murky details of some specific theory. And from these features, we are able to work our way to the modern conformal bootstrap and gain an unprecedented understanding of the dynamics of a strongly coupled quantum field theory. This approach means that the reader should be familiar with quantum field theory—at least some of it, but the more the better—but no prior experience in conformal field theory is required. The book is typically aimed at graduate students or at more advanced researchers in theoretical physics. The presentation is self-contained, so it might as well be interesting for a more diverse audience than just particle physicists, including condensed matter physicists, cosmologists, or string theorists. Given the unusual approach followed by the book, it might even be of interest to people who are already familiar with some aspects of conformal field theory. The presentation includes topics that are not easily found in the literature (and particularly dear to me), such as subtleties of conformal transformations in Minkowski space-time, the construction of Wightman functions and time-ordered correlators in both position and momentum space, unitarity bounds derived from the spectral representation, and the appearance of UV and IR divergences, even if each of these topics can only be covered superficially in such a short volume.

This book would not have been written if it had not emerged from the lecture notes that I prepared for a graduate class at the University of Bern in the spring semester of 2022. I would like to thank the Institute for Theoretical Physics—Mikko Laine and Thomas Becher in particular—for giving me the opportunity to teach my favorite topic, but also all the students and postdocs who took part in the class for their involvement and their critical questions. All of them contributed to the current state of this book.

Last but not least, I received feedback from various people, both on the particle physics and on the conformal side, including Johan Henriksson, Slava Rychkov, Flip Tanedo, and Luca Vecchi, and benefitted from discussions with various members of the conformal bootstrap community. Thanks a lot to all of them. In spite of this and of the editorial support of Springer Nature, there are certain mistakes and imprecisions left over, and I hope that you will not hesitate to contact me should you notice any. Like any introductory work, this book is also doomed to be incomplete. It will be even more so in the future, as conformal field theory is likely to see fundamental advances in the next few years.

Zürich, Switzerland
December 2022

Marc Gillioz

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