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Martin Kolmar · Magnus Hoffmann

Workbook for *Principles of Microeconomics*

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Acknowledgements

If you want to learn the piano, you have to sit down and start practicing. This is tedious in the beginning and will not sound particularly pretty but things will improve over time. The same applies to economics (and other scientific theories as well). If you want to use them to better understand and analyze certain aspects of economic and social reality, you have to make them your own, understand their internal "mechanics" and work with them. Reading textbooks or listening to lectures is only a poor substitute. Limiting yourself to it would be like wanting to learn how to play piano by just listening to a piano player and studying piano scores.

This is why we have collected a series of problems and exercises that are intended to help you to adopt step by step the theories introduced and discussed in the textbook "Principles of Microeconomics: An Integrative Approach". You will find a chapter with different types of problems and sample solutions that corresponds to a chapter in the main book. We distinguish between three different types of exercises that focus on the development of specific and complementary skills and competencies.

The first type is true or false exercises; statements that can either be true or false. At the end of each section you will find the solutions along with short explanations, as well as links to the textbook.

The second type of problems have the character of short case studies or word problems, to answer which you are will be required to develop a more complex train of thoughts. Problems like these do not have one and only one correct solution but can usually be approached from different directions. Nevertheless, this book offers you sample solutions at the end of this section that illustrate *one* possible approach. Over the years during which we have developed the problems and used them in class we have also been able to identify typical lines of faulty reasoning. We will look into these and explain how they can be avoided.

Finally, you will find multiple-choice questions to answer in which you will have to identify one correct answer from a choice of given answers. Please note that all references to chapters are to those in the textbook "Principles of Microeconomics: An Integrative Approach" by Martin Kolmar, unless otherwise specified.

The teaching material collected in this book has grown over many years and is a result of efforts of a great many people, first of all the students who worked with them. We are most thankful for their innumerable suggestions that helped us improving the set of problems. Further, we thank Dario Fauceglia, Jürg Furrer, Carolin Güssow, Katharina Hofer, Alfonso Sousa-Poza, and Fred Henneberger for pointing out many errors, inconsistencies, and ways to improve the problems included in this book. Last but not least we thank our student assistants Corinne Knöpfel, Jan Riss, and Jan Serwart without whom the book would not be as it is.

Science that aims at both better understanding the reality and practical application of theory is similar to jazz. A good economist is like a good pianist: both have to master their instruments to be able to improvise. You know you're there when it starts to swing. We hope that this book will help you on your way to reaching this goal.

St. Gallen, in July 2017 Martin Kolmar and Magnus Hoffmann

Contents

First Principles

1

1.1 True or False

1.1.1 Statements

1.1.1.1 Block 1

1. The more aspects of reality are taken into consideration, the more useful an economic model is.
2. According to Karl Popper, a basic requisite for the quality of scientific theories is that one can refute them.
3. Concerning logical statements: one can derive a false hypothesis from false assumptions.
4. Modern microeconomics is macro-founded.

1.1.1.2 Block 2

1. Economics, as a positive science, tries to explain why social phenomena work the way they work. Economics, as a negative science, tries to explain why social phenomena do not work the way they work.
2. If an economist tries to determine how a country should increase taxes in the best possible way, she or he is practicing normative science.
3. If an assumption in a scientific theory is incorrect, the theory must be discarded because it cannot contribute to the understanding of reality.
4. In economics, one investigates the interplay of human behavior on the individual level.

1.1.1.3 Block 3

1. With regards to economics, positive science answers the question as to how humans should cope with the phenomena of scarcity.

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2. Quantities of goods that are not on the production-possibility frontier, cannot be produced.
3. Modern macroeconomics is not "micro-founded" because it concentrates on economic aggregates.
4. Opportunity costs are costs of the past that cannot be influenced any longer.

1.1.1.4 Block 4

1. According to the theory of critical rationalism, the monopoly theory is not a scientifically sound theory, because the assumption of profit maximization has been falsified.
2. According to the theory of critical rationalism, scientific theories can be verified, but cannot be falsified.
3. According to the theory of critical rationalism, a good theory can be falsified in principle, but has not been falsified yet.
4. The implication of applying Ockham's razor to scientific theories is that a theory with fewer assumptions is preferable to one with more assumptions if both theories lead to identical hypotheses.

1.1.2 Solutions

1.1.2.1 Sample Solutions for Block 1

1. **False**. An important criterion for good models is simplicity or frugality. The idea is often referred to as "Ockham's razor" which states that, among competing models, the one with the fewest assumptions should be selected. Ockham's razor necessarily implies that the assumptions of a model should not be realistic in the naïve sense that the assumptions shall fit reality. See Chapter 1.2.3.
2. **True**. According to Karl Popper, scientific theories can never be verified but can, in principle, be falsified by bringing in empirical evidence that is in conflict with the hypotheses of the theory. See Chapter 1.2.6.
3. **True**. See examples in Chapter 1.2.2.
4. **False**. Modern macroeconomics is micro-founded, the converse argument does not hold. See Chapter 1.1.

1.1.2.2 Sample Solutions for Block 2

1. **False**. The goal of positive theories is explaining phenomena. Normative theories, on the other hand, try to determine what people should do in which situation. Thus, they are based on a judgement. See Chapter 1.2.7.
2. **True**. See the sample solution to Block 2, Statement 1.
3. **False**. Assumptions are necessarily simplifications. The important thing is finding the correct balance between reasonable simplification of assumptions and the underlying causal mechanisms on the one hand, and the content explained by the derived hypotheses on the other hand. See Chapter 1.2.4.

4. **False**. On the individual level, one investigates individual humans' behavior. Humans' behavioral interplay is investigated on the interaction level. See Chapter 1.1.

1.1.2.3 Sample Solutions for Block 3

1. **False**. With regards to economics, positive science answers the question as to how humans cope (without judgement) with the phenomena of scarcity. See Chapter 1.2.7.
2. **False**. The production-possibility frontier indicates the maximum quantity that can be produced. Every quantity below the frontier can be produced as well. See Chapter 1.2.5.
3. **False**. The micro-foundation of macroeconomics is a research program that tries to explain the regularities on the aggregate level, like relationships between inflation and unemployment, through individuals' behavior and interactions. The current macroeconomics mainstream is, in this sense, largely micro-founded. See Chapter 1.1.
4. **False**. Opportunity costs are costs that result from, for example, forgoing an alternative use of capital or time (such as the salary that a student forgoes, because he is not working). See Chapter 1.1.

1.1.2.4 Sample Solutions for Block 4

1. **False**. Falsification is disproving hypotheses by confronting them with empirical evidence that conflict with the hypotheses. One can only falsify hypotheses, not assumptions. See Chapter 1.2.6.
2. **False**. According to the theory of critical rationalism, scientific theories can never be irrevocably proven, but they can in principle be falsified. See Chapter 1.2.6.
3. **True**. Theories should be formulated such that their hypotheses are falsifiable. Good theories are those that have a large empirical content, but have not been falsified so far. See Chapter 1.2.6.
4. **True**. This is true by definition. See Chapter 1.2.3 and the sample solution to Block 1, Statement 1.

Gains From Trade

2

2.1 True or False

2.1.1 Statements

2.1.1.1 Block 1

There are two individuals, A and B, who can produce two goods, 1 and 2. The production-possibility frontiers of both individuals are $x_1^A = a - b \cdot x_2^A$ and $x_1^B = c - d \cdot x_2^B$, in which a, b, c and d are strictly larger than zero.

1. If $b > d$, then A has a comparative advantage in the production of good 1.
2. If $a > c$, then A has an absolute advantage in the production of both goods.
3. If $a = c$, then no individual has a comparative advantage.
4. If $a = 100$ and $b = 2$, then A can produce 50 units of the second good at maximum.

2.1.1.2 Block 2

1. A situation in which there is no trade between countries is defined as "autarky."
2. The theory of comparative advantage is only valid for linear production-possibility frontiers.
3. If a country has a comparative disadvantage in the production of a good, it should not trade this good with other countries.
4. All countries always benefit from specialization and trade.

2.1.1.3 Block 3

Charlotte and Phil are both bakers. Charlotte can either bake 20 cakes, 15 pizzas or any linear combination of the two in one day. Phil can either bake 10 cakes, 5 pizzas or any linear combination of the two.

1. Charlotte has a comparative advantage in baking pizza.
2. Charlotte has an absolute advantage in baking pizza.

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3. Phil's opportunity costs for a pizza are equivalent to two cakes.
4. Charlotte's opportunity costs for cake are lower than Phil's.

2.1.1.4 Block 4

1. Assume linear production-possibility frontiers. If two individuals have identical opportunity costs, then neither individual has a comparative advantage.
2. When compared with autarky, two individuals are never worse off if they specialize according to their comparative advantage and subsequently trade.
3. The theory of comparative advantage describes and explains the international trade of goods.
4. The sequence of the integration in a trade community (using a sequential procedure) is irrelevant for the trading partners' assessment of the advantageousness of the community.

2.1.1.5 Block 5

1. Individuals A and B can both produce either roses or computers. If they can become better off by trading, then one of the individuals will consign to only producing roses, while the other will consign to solely producing computers.
2. A comparison of opportunity costs allows one to identify potential absolute advantages.
3. An individual can have a comparative advantage concerning one good and an absolute advantage concerning a different good.
4. Two individuals with identical linear production-possibility frontiers can be better off by trading with each other.

2.1.2 Solutions

2.1.2.1 Sample Solutions for Block 1

Opportunity costs of good i in units of good j for individual k are $OC_{ij}^k = \left|\frac{dx_j^k}{dx_i^k}\right|$, where $i, j \in \{1, 2\}$ and $k \in \{A, B\}$. Individual A's opportunity costs are then $OC_{12}^A = \frac{1}{b}$ and $OC_{21}^A = b$. Individual B's opportunity costs are then $OC_{12}^B = \frac{1}{d}$ and $OC_{21}^B = d$. See Chapter 2.2.

1. **True**. If $b > d$, then A's opportunity costs for good 1 are lower than B's and, thus, A has a comparative advantage in the production of good 1.
2. **False**. One cannot determine that A has an absolute advantage in the production of the second good solely based on $a > c$.
3. **False**. One cannot derive that conclusion from $a = c$. In order for that to be the case, $b = d$ must hold as well.
4. **True**. Individual A can produce $x_1^A = 100 - 2 \cdot x_2^A$ units of good 1. If A produces zero units of the first good, this would mean $0 = 100 - 2 \cdot x_2^A$ and thus $x_2^A = 50$.

2.1.2.2 Sample Solutions for Block 2

1. **True**. This is true by definition. See Chapter 2.1.
2. **False**. See the discussion about strictly concave and strictly convex production-possibility frontiers in Chapter 2.3.
3. **False**. That is exactly where an individual is able to be better off through trade. Because a comparative disadvantage in the production of one good always implies a comparative advantage in the production of another good. The individual can then specialize in the production of the good that he or she has a comparative advantage in and become better off due to trade. See Chapter 2.1.
4. **False**. The sequence of integration plays a role as well. Additionally, countries with identical opportunity costs will not have any gains from trade from trading with each other. See Chapter 2.3.

2.1.2.3 Sample Solutions for Block 3

Opportunity costs of good i in units of good j for individual k are $OC_{ij}^{k} = \left| \frac{dx_j^k}{dx_i^k} \right|$, where $i, j \in \{C, P\}$ and $k \in \{Ch, Ph\}$. Charlotte's opportunity costs are then $OC_{CP}^{Ch} = \frac{15}{20} = \frac{3}{4}$ and $OC_{PC}^{Ch} = \frac{20}{15} = \frac{4}{3}$. Phil's opportunity costs are then $OC_{CP}^{Ph} = \frac{5}{10} = \frac{1}{2}$ and $OC_{PC}^{Ph} = \frac{10}{5} = 2$. See Chapter 2.2.

1. **True**. $OC_{PC}^{Ch} = \frac{4}{3} < 2 = OC_{PC}^{Ph}$. Thus, Charlotte has a comparative advantage when baking pizza.
2. **True**. Charlotte can bake 15 pizzas, while Phil can only bake 5. Therefore, Charlotte has an absolute advantage.
3. **True**. $OC_{PC}^{Ph} = 2$.
4. **False**. $OC_{CP}^{Ch} = \frac{3}{4} > \frac{1}{2} = OC_{CP}^{Ph}$.

2.1.2.4 Sample Solutions for Block 4

1. **True**. Given identical opportunity costs, both individuals have to curb the production of one good by the same amount in order to produce one more unit of the other good. Consequently, no individual has a comparative advantage. See Chapter 2.1.
2. **True**. By specializing in one's comparative advantage, one is able to produce a surplus, which makes at least one of the individuals better off. The consumption under autarky can always be guaranteed. See Chapter 2.3.
3. **True**. The concept of comparative advantage cannot only be applied to individuals, but also to countries. See Chapter 2.1.
4. **False**. It is very relevant. See the detailed discussion about sequential integration in Chapter 2.3.

2.1.2.5 Sample Solutions for Block 5

1. **False**. If two individuals can become better off by trading, then there is a comparative advantage. The individuals will specialize according to their comparative advantage; however, whether they will completely specialize or not depends on their consumption preferences. See Chapter 2.2.
2. **False**. See Chapter 2.2.
3. **True**. Consider the example in Chapter 2.2, where Ann has both an absolute advantage in the production of tomatoes as well as a comparative advantage in the production of pears.
4. **False**. Because neither individual has a comparative advantage, neither of them can be better off through specialization and trade. See Chapter 2.1.

2.2 Open Questions

2.2.1 Problems

2.2.1.1 Exercise 1

Explain the theory of comparative advantage. Point out the theory's importance for economics, business administration, and law.

2.2.1.2 Exercise 2

There are two goods, 1 and 2, and two countries, A and B. Both goods are homogeneous and can be produced by both countries using labor as the only input, with each worker supplying 1 unit of labor. Each worker in A can produce 10 units of good 1, 10 units of good 2, or any linear combination of the two. In B, each worker can produce α units of good 1, 9 units of good 2, or any linear combination of the two. There are 100 workers in each country. The gains from trade are distributed among all workers in a country in a manner that makes everybody better off.

1. Determine and draw both countries' production-possibility frontiers for $\alpha = 8$.
2. Determine each country's opportunity costs of producing goods 1 and 2 for any given $\alpha > 0$.
3. Determine each country's comparative advantage depending on α.
4. Individuals in both countries always consume goods 1 and 2 in equal quantities. Determine the optimal production and consumption plans for $\alpha = 9$.
5. Assume that $\alpha = 10$. Show that both countries are better off, in comparison to autarky, when completely specializing in producing the good for which they have a comparative advantage.

2.2.1.3 Exercise 3

There are three countries, A, B and C, and each of them can produce two goods, 1 and 2. The production-possibility frontiers are given as:

$$x_1^A = 1 - x_2^A,$$
$$x_1^B = 1 - \tfrac{1}{2}\, x_2^B,$$
$$x_1^C = 1 - \tfrac{1}{4}\, x_2^C.$$

Each county is inhabited by individuals who always consume both goods in equal amounts. Potential gains from trade are distributed equally among the inhabitants of a country.

1. Determine the countries' production and consumption plans in autarky.
2. Assume countries A and C establish a free-trade agreement, such that a good produced in one of the two countries can be sold in both countries (without additional shipping costs). Determine the optimal production and consumption plans in these two countries if:
   a) gains from trade are split equally between the countries, with consumption under autarky (see Question 1) serving as a reference point.
   b) country C reaps all the benefits alone.
3. Assume countries A, B and C establish a free-trade agreement. Determine the optimal production and consumption plans of the countries if
   a) gains from trade are split equally, with consumption under autarky (see Question 1) serving as a reference point.
   b) gains from trade are split equally, with consumption under the first AC-agreement (see Question 2a)) serving as a reference point.
   c) gains from trade are split equally, with consumption under the second AC-agreement (see Question 2b)) serving as a reference point.
4. What are the implications for trade policy?

2.2.2 Solutions

2.2.2.1 Solutions to Exercise 1

In general, each individual has a comparative advantage in the production of one good, irrespective of whether she has an absolute advantage in the production of a good or not. An individual has a comparative advantage in the production of a given good if she can produce the good at lower opportunity costs, measured in units of the other good, than the other individuals. According to comparative advantages and trade, specialization has the potential to make all individuals better off. Therefore, the concept of comparative advantage is the basis for all further

Table 2.1 Exercise 2.2. Opportunity costs

	Country A	Country B
OC_{12}^k	$OC_{12}^A = 1$	$OC_{12}^B = \frac{9}{\alpha}$
OC_{21}^k	$OC_{21}^A = 1$	$OC_{21}^B = \frac{\alpha}{9}$

contemplation of the organization of economic activities, because one has to ask how economic activities have to be organized to allow specialization and trade.

Significance for Economics
Economics tries to understand how human beings organize (positive) or should organize (normative) economic activities in order to cope with the phenomenon of scarcity. The theory of comparative advantage explains why the organization of economic activities is at the core of economics and helps with the development of hypotheses about the purpose of organizations.

Significance for Business Administration
Business administration analyzes the unit of a firm or a corporation. However, a firm is nothing more than a specific way to organize economic activities. One can, therefore, apply the same logic to the analysis of the organization of such an entity. What is a firm's comparative advantage? How should it organize its internal activities? Where are a firm's boundaries? Should it produce a component itself (insourcing) or buy it in the market (outsourcing)?

Significance for Law
The backbone of the economy is its legal structure. Laws define rights and obligations. Contract law, for example, defines the types of contracts that can be used to organize economic activities, and competition law regulates firm behavior, to name only two fields. The legal structure of an economy, therefore, promotes, or constrains specialization and trade. Hence, one can ask how laws influence economic activities.

2.2.2.2 Solutions to Exercise 2

1. The production-possibility frontier for $\alpha = 8$:
   Country A: $x_1^A = 1{,}000 - x_2^A$
   Country B: $x_1^B = 100\,\alpha - (\frac{\alpha}{9})\,x_2^B \;\Rightarrow\; x_1^B = 800 - \frac{8}{9}\,x_2^B$.
   Both production-possibility frontiers are illustrated in Fig. 2.1.
2. The opportunity costs of good i for a given country, k, in units of good j, are given by $OC_{ij}^k = \left|\frac{dx_j^k}{dx_i^k}\right|$, where $i, j \in \{1, 2\}, i \neq j$, and $k \in \{A, B\}$. The results are given in Table 2.1.
3. Looking for comparative advantages, one has to compare the countries' opportunity costs, which leads to the following question: For what values of α does

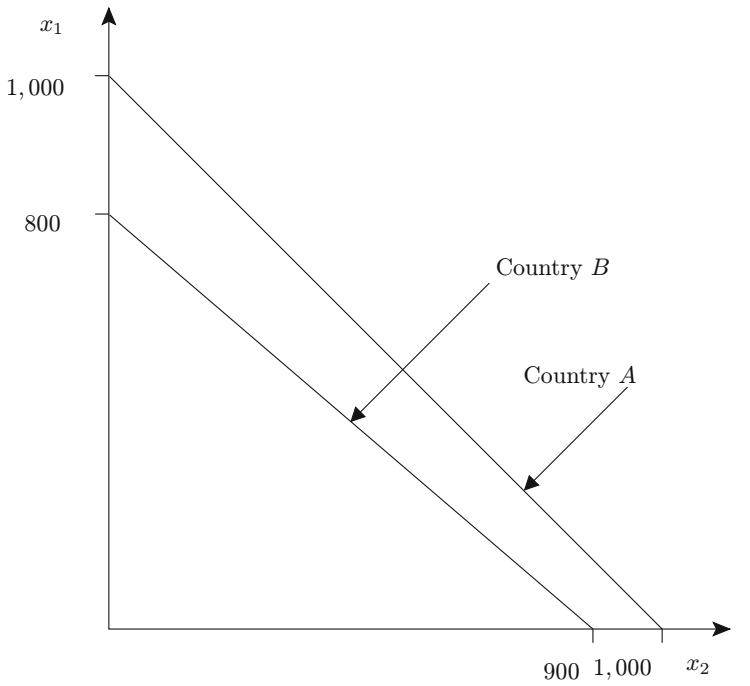

Figure 2.1 Exercise 2.1. The production-possibility frontiers of countries A and B for $\alpha = 8$

one have $OC_{12}^A \left\{ \begin{matrix} \geq \\ \leqq \end{matrix} \right\} OC_{12}^B$? By plugging in the results from Question 2, the conditions simplify to

$$OC_{12}^A \left\{ \begin{matrix} \geq \\ \leqq \end{matrix} \right\} OC_{12}^B \quad \Leftrightarrow \quad 1 \left\{ \begin{matrix} \geq \\ \leqq \end{matrix} \right\} \frac{9}{\alpha} \quad \Leftrightarrow \quad \alpha \left\{ \begin{matrix} \geq \\ \leqq \end{matrix} \right\} 9.$$

- If $\alpha > 9$, then country B has a comparative advantage in the production of good 1 and country A in the production of good 2.
- If $\alpha < 9$, then country A has a comparative advantage in the production of good 1 and country B in the production of good 2.
- If $\alpha = 9$, then opportunity costs are identical in both countries and, hence, neither country has a comparative advantage.

4. Neither country has a comparative advantage and there are no gains from trade to be exploited. Whether both countries remain in autarky or trade does not make a difference for the consumption possibilities. If the countries remain in autarky, each country produces exactly as much as it consumes: $x_1^A = x_2^A = 500$ and $x_1^B = x_2^B = 450$.

Table 2.2 Exercise 2.5. Production plans with complete specialization

	Country A	Country B	Total: $x_i^{AB} = x_i^A + x_i^B$
Good 1	$x_1^A = 0$	$x_1^B = 1{,}000$	$x_1^{AB} = 1{,}000$
Good 2	$x_2^A = 1{,}000$	$x_2^B = 0$	$x_2^{AB} = 1{,}000$

Table 2.3 Exercise 2.5. Production plans under autarky

	Country A	Country B	Total: $x_i^{AB} = x_i^A + x_i^B$
Good 1	$x_1^A = 500$	$x_1^B = 500$	$x_1^{AB} = 1{,}000$
Good 2	$x_2^A = 500$	$x_2^B = 450$	$x_2^{AB} = 950$

5. Since $\alpha = 10$, country B has a comparative advantage in the production of good 1. Complete specialization, in the direction of comparative advantage, implies that $x_1^A = 0$, $x_2^A = 1{,}000$, $x_1^B = 1{,}000$, and $x_2^B = 0$. To be more specific, see Table 2.2.
   Assuming that each country produces 500 units of the first good and uses the remaining resources to produce the second good, one gets the following production plans under autarky (see Table 2.3).
   The total production of good 1 remains unchanged by specialization, but the total production of good 2 increases by 50 units. Through trade, these gains in total production can be split between the countries, making both of them potentially better off. What the new allocation with specialization and trade will look like, however, cannot be determined and depends on each country's negotiation power. In addition, one has to make sure that the gains from trade within each country are distributed in a way that each citizen profits or is, at least, not worse off (methodological individualism forces one to think about economic phenomena from the point of view of individual human beings). See Chapter 1.1.

2.2.2.3 Solutions to Exercise 3

1. Assume that the goods are distributed equally among the citizens within each country to abstract from problems of intra-country distribution. The inhabitants of each country, k ($k \in \{A, B, C\}$), maximize their own consumption, subject to the condition that both goods are consumed in equal quantities, $y_1^k = y_2^k$.
   Under the assumption that, in autarky, production equals consumption (i.e., $y_1^k = x_1^k$ and $y_2^k = x_2^k$), the countries' optimal production plans have the following property: $x_1^k = x_2^k$. The production-possibility frontier (PPF) determines the maximum quantity that can be produced of each good, given the quantity produced of the other good. Applying the constraint $x_1^k = x_2^k$ to the

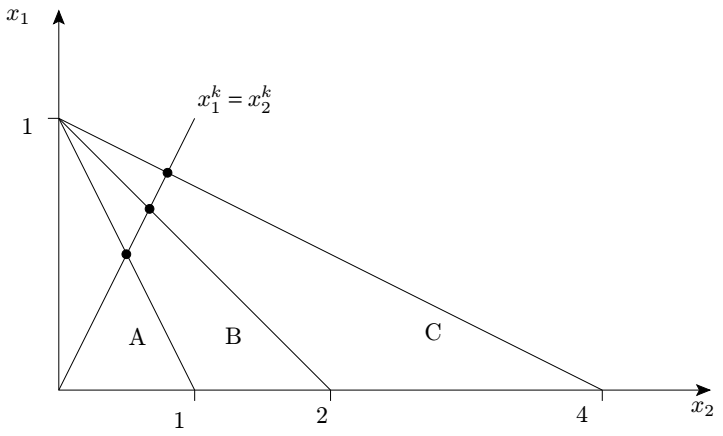

Figure 2.2 Exercise 3.1. The countries' PPFs in autarky and their optimal production plans

PPFs yields the optimal quantities:

$$x_1^A = 1 - x_2^A \wedge x_1^A = x_2^A \quad \Rightarrow x_1^A = \frac{1}{2}, x_2^A = \frac{1}{2}$$
$$x_1^B = 1 - \frac{1}{2}x_2^B \wedge x_1^B = x_2^B \Rightarrow x_1^B = \frac{2}{3}, x_2^B = \frac{2}{3}$$
$$x_1^C = 1 - \frac{1}{4}x_2^C \wedge x_1^C = x_2^C \Rightarrow x_1^C = \frac{4}{5}, x_2^C = \frac{4}{5}.$$

This result is illustrated in Fig. 2.2.

2. If countries A and C establish a free-trade agreement, the countries' joint PPF, as illustrated in Fig. 2.3, includes the points at which each country produces only good 1 or 2, respectively. Starting from point $(x_2, x_1) = (0, 2)$, that is, from the point at which only good 1 is produced, the PPF has a slope of $-\frac{1}{4}$ (the slope of country C's autarky PPF) until it reaches the point $(4, 1)$.
At this point, the slope changes to -1 (the slope of country A's autarky PPF) until the PPF reaches the point at which both countries produce only good 2 (Point $(5, 0)$). The joint PPF is, hence, given as:

$$x_1^{AC} = \begin{cases} 2 - \frac{1}{4}x_2^{AC} & \text{for} \quad 0 \le x_2^{AC} < 4, \\ 5 - x_2^{AC} & \text{for} \quad 4 \le x_2^{AC} < 5, \\ 0 & \text{else}, \end{cases} \tag{2.1}$$

where x_i^{AC} is the quantity of good i jointly produced by countries A and C.

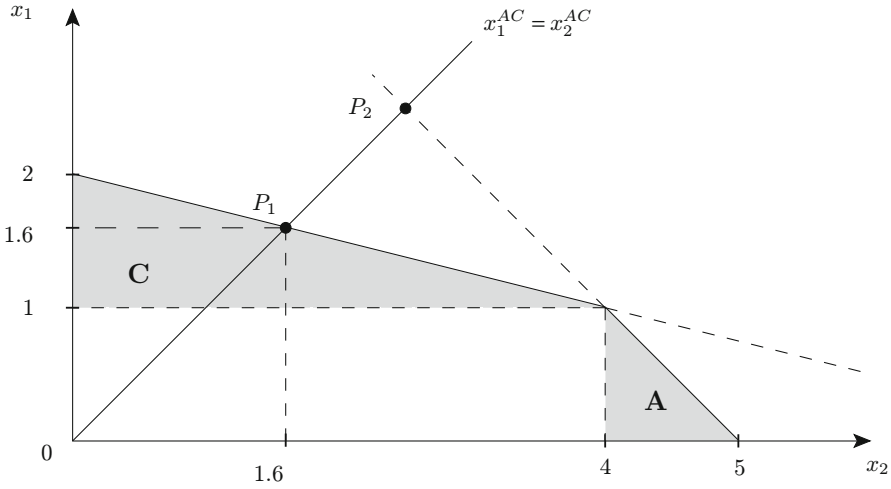

Figure 2.3 Exercise 3.2. Joint PPF of countries A and C

Optimal production maximizes consumption, subject to the condition that both goods are consumed in equal quantities, $y_1^{AC} = y_2^{AC}$ (which, again, can be transformed into $x_1^{AC} = x_2^{AC}$, since we can still assume that all of the goods that are produced are also consumed). Applying the constraint $x_1^{AC} = x_2^{AC}$ to the joint PPF yields the optimal quantities (see point P_1 in Fig. 2.3):

$$x_1^{AC} = 2 - \frac{1}{4} x_2^{AC} \wedge x_1^{AC} = x_2^{AC} \Rightarrow x_1^{AC} = 1.6, x_2^{AC} = 1.6.$$

Country A completely specializes in the production of good 1 ($x_1^A = 1$), whereas country C produces 0.6 units of good 1 and 1.6 units of good 2:

$$\begin{aligned} x_1^A &= 1, & x_2^A &= 0, \\ x_1^C &= 0.6, & x_2^C &= 1.6. \end{aligned}$$

Any piecewise-defined function (like the joint PPF of countries A and C) consists of multiple subfunctions, each of which is paired with an interval (see Eq. 2.1). A common mistake when calculating the optimal production plan of the joint PPF is to not consider the interval to which a specific subfunction applies. In Question 2, this would lead to the following (wrong!) solution (see point P_2 of Fig. 2.3):

$$x_1^{AC} = 5 - x_2^{AC} \wedge x_1^{AC} = x_2^{AC} \Rightarrow x_1^{AC} = 2.5, x_2^{AC} = 2.5. \quad \lightning$$

a) In autarky, countries A and C produce 1.3 units of each good in total (see Question 1). Due to the specialization under the free-trade agreement, this

quantity increases to 1.6. Hence, there are gains from trade of 0.3 units of each good. If these gains from trade are split equally, each country gains 0.15 units of each good in comparison to what they would have made in autarky. For country B, nothing changes in comparison to Question 1, since it is not part of the trade agreement. Consumption then becomes:

- Country A: $y_1^A = y_2^A = \frac{1}{2} + \frac{3}{20} = \frac{13}{20} = 0.65$,
- Country B: $y_1^B = y_2^B = \frac{2}{3} \approx 0.67$,
- Country C: $y_1^C = y_2^C = \frac{4}{5} + \frac{3}{20} = \frac{19}{20} = 0.95$.

b) Now, the surplus is not split equally. Country C reaps all of the gains from trade alone and country A consumes as much as it would under autarky.

- Country A: $y_1^A = y_2^A = \frac{1}{2} + 0 = 0.5$,
- Country B: $y_1^B = y_2^B = \frac{2}{3} \approx 0.67$,
- Country C: $y_1^C = y_2^C = \frac{4}{5} + \frac{3}{10} = \frac{11}{10} = 1.1$.

3. The new joint PPF is

$$x_1^{ABC} = \begin{cases} 3 - \frac{1}{4}x_2^{ABC} & \text{for} \quad 0 \leq x_2^{ABC} < 4, \\ 4 - \frac{1}{2}x_2^{ABC} & \text{for} \quad 4 \leq x_2^{ABC} < 6, \\ 7 - x_2^{ABC} & \text{for} \quad 6 \leq x_2^{ABC} < 7, \\ 0 & \text{else}, \end{cases} \tag{2.2}$$

where x_i^{ABC} is the quantity of good i jointly produced by countries A, B, and C.

The optimal production plan maximizes the total quantity of both goods, under the constraint that both goods are produced (and consumed) in equal amounts. Applying the constraint $x_1^{ABC} = x_2^{ABC}$ to the joint PPF (see Eq. 2.2) yields the optimal quantities (see point P_1 in Fig. 2.4):

$$x_1^{ABC} = 3 - \frac{1}{4}x_2^{ABC} \wedge x_1^{ABC} = x_2^{ABC} \Rightarrow x_1^{ABC} = 2.4,\ x_2^{ABC} = 2.4.$$

Hence, the new optimal total production plan is $x_1^{ABC} = x_2^{ABC} = 2.4$. Countries A and B completely specialize in the production of good 1 and produce 1 unit each, country C produces 0.4 units of good 1 and 2.4 units of good 2.

$$\begin{aligned} x_1^A &= 1, & x_2^A &= 0, \\ x_1^B &= 1, & x_2^B &= 0, \\ x_1^C &= 0.4, & x_2^C &= 2.4. \end{aligned}$$

Please note once more, not considering the interval to which each subfunction applies leads to the following (wrong!) solutions:

- *Point P_2 in Fig. 2.4*:

$$x_1^{ABC} = 4 - \frac{1}{2}x_2^{ABC} \wedge x_1^{ABC} = x_2^{ABC} \Rightarrow x_1^{AC} \approx 2.67,\ x_2^{AC} \approx 2.67. \qquad \lightning$$