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Lydia Patton · Erik Curiel *Editors*

Working Toward Solutions in Fluid Dynamics and Astrophysics

What the Equations Don't Say

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Preface

The editors and authors of this book have been working on aspects of this topic for some time, both separately and in conversation with each other. Erik Curiel's 2010 preprint "On the Formal Consistency of Theory and Experiment, with Applications to Problems in the Initial-Value Formulation of the Partial-Differential Equations of Mathematical Physics" establishes his approach to the partial differential equations of fluid dynamics, including theories of relativistic dissipative fluids. Curiel acknowledges the work of Howard Stein, who did path-breaking research into the question of the relationship between theory and experiment.

Over the past two decades, a significant strain of Susan Sterrett's work has focused on the study of physically similar systems. The themes Sterrett explores in this connection include analogue systems and dimensional analysis.

Colin McLarty is a category theorist who has done substantial research into the history of algebraic geometry, especially the work of Emmy Noether and Alexander Grothendieck.

Jamee Elder's research concerns the methodology and epistemology of large astrophysical experiments, especially those—including LIGO-Virgo-KAGRA and the Event Horizon Telescope—that involve "observing" black holes. This work has encouraged research into the role of equations and models in the interpretation and analysis of observation, including in model validation.

In 2015, Lydia Patton gave a talk called "Fishbones, Wheels, Eyes, and Butterflies" at the MidWest Philosophy of Mathematics seminar at the University of Notre Dame, a longstanding meeting organized by Michael Detlefsen until his passing in 2019, Patricia Blanchette, and Curtis Franks. The 2015 talk was the occasion for Patton, McLarty, and Sterrett to begin a conversation about the mutual interest in the topic of mathematical modeling, the role of equations in physical explanation, and the study of partial differential equations in the philosophy of mathematics and the philosophy of science. That conversation continued in the summer of 2020, in which Susan Sterrett organized a workshop, "What Equations Don't Say", that brought together talks by Patton, McLarty, Sterrett, and Erik Curiel. They were later joined by Jamee Elder, whose work delves into related questions in the philosophy of general relativity and astrophysics. Since the workshop, the authors have continued to discuss and work on their projects, which have culminated in the papers for this volume.

The editors gratefully acknowledge Susan Sterrett's organization of an online workshop in July 2020. The 2020 workshop benefited from the attendance of researchers including Daniele Oriti, George Smith, Bas van Fraassen, Patricia Palacios, Erich Reck, Robert Batterman, Alastair Wilson, Silvia de Toffoli, Germain Rousseaux, Susan Castro, and Brian Hepburn. The authors and editors are very grateful for the workshop participants' contributions to the development of the papers and for the development of conversations on these research topics. Lydia Patton is grateful to Michael Detlefsen, Patricia Blanchette, and Curtis Franks for the invitation to present a paper at the MidWest Philosophy of Mathematics Workshop. Colin McLarty and Susan Sterrett have been galvanizing forces behind the project.

We are grateful to Christopher Wilby, Werner Hermens, and Arumugam Deivasigamani for shepherding the manuscript through the review and publication process.

Blacksburg, USA Lydia Patton

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Chapter 1
Introduction

Lydia Patton

Abstract Systems of differential equations are used to describe, model, explain, and predict states of physical systems. Experimental and theoretical branches of physics including general relativity, climate science, and particle physics have differential equations at their center. Direct solutions to differential equations are not available in many domains, which spurs on the use of creative mathematics and simulated solutions.

Keywords Differential equations · Fluid dynamics · Astronomy · Philosophy of science

Philosophical questions arise from the use of differential equations in physical science and in mathematics. In particular, there is a need for sustained attention to the questions in the philosophy of science that arise from the use of differential equations in physical science.[1]

The papers in this volume analyze the use of differential equations in fluid dynamics and astronomy. The central problem at stake is the fact that direct solutions to differential equations are not available in many domains for which the systems of equations are constructed. Lack of a direct or immediate solution means that an equation or system of equations does not have a solution in a domain without employing a method that either restricts the domain, or extends the equations, or both. Mathematicians may refer to the lack of an 'analytic' solution. Or they may refer to the lack of an 'exact' solution. An equation's being without an exact solution can mean a number of things: that no closed form solution can be written, for instance, or that

[1] Parker (2017), Morrison (1999), Fillion (2012), Curiel (2010), Batterman (2013), and Mattingly and Warwick (2009) are among those who have contributed philosophical studies of the use of differential equations in the analysis of physical systems.

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the available solution does not provide unique exact values for variables of interest, but provide approximate or perturbative solutions instead.[2]

The papers that follow focus on the Navier-Stokes equations in fluid dynamics and Einstein's field equations of general relativity as they are employed in astrophysics. The Navier-Stokes equations do not have immediate exact solutions for many physical fluid systems. Similarly, the Einstein field equations of general relativity do not have direct solutions in many domains of interest, including the merger of astronomical bodies, which emanates gravitational waves.

And yet, scientists and engineers work with these equations every day. Ingenious methods have been devised, which may involve finding simulated solutions for artificial or idealized situations and then extending those solutions to actual cases; or finding 'weak' solutions which may not have derivatives (solutions) everywhere, but which nonetheless satisfy the equations in some restricted sense; or finding another strategy to extend the equations to the domain of interest. In many cases, solutions that initially apply only in restricted or idealized contexts are extended to a wider set of physically realistic situations.

Equations may be used in scientific reasoning to yield predictions and explanations: to predict states of a system and uniquely specify the values of variables. Equations can play these roles directly when immediate solutions are available. This collection goes beyond that familiar case to explore what happens when direct solutions are absent, or when the task at hand is precisely to find a way to connect the equations to a specific physical situation.

One of the most striking results learned early on in real analysis is the fact that there are many more irrationals than rational numbers. Early education often presents irrational numbers as exceptions, so it comes as a surprise to find that they are more numerous than the rational numbers. The application of differential equations to physical contexts reveals a similar situation. Physical reasoning using differential equations usually is presented as follows: scientists select an appropriate equation or system of equations, find a solution, and thereby determine the evolution and properties of a physical system. But cases in which a direct solution is not available and scientists must work out a solution, or work in the absence of a solution, are far from the exception. Working with equations can involve reasoning toward, from, and around equations as well, in the absence of a solution in the domain of interest.

The papers focus on how scientists reason with, around, and toward differential equations in fluid dynamics and astrophysics in the absence of immediate solutions. The process of reasoning may involve extending available solutions to differential equations to initially inaccessible domains.[3] Or, it may involve finding novel ways of simulating or modeling the domain so that it can make contact with the equations, or new methods for calculating, or new ways of measuring. McLarty 2023 focuses

[2] These are not the only definitions in common use. See Fillion and Bangu 2015 and McLarty 2023 for discussion of types of solutions.

[3] A conversation with Hasok Chang put a clear focus on 'stretching' or 'extending' equations, which is a helpful way of describing one aspect of these situations.