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Calculus

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FEATURES

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lessons and problems
End-of-chapter quizzes
Even more questions
online!

Mark Ryan

The Calculus-in-Plain-English Guy



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Introduction

The mere thought of having to take a required calculus course is enough to make legions of students break out in a cold sweat. Others who have no intention of ever studying the subject have this notion that calculus is impossibly difficult unless you happen to be a direct descendant of Einstein.

Well, I'm here to tell you that you *can* master calculus. It's not nearly as tough as its mystique would lead you to think. Much of calculus is really just very advanced algebra, geometry, and trig. It builds upon and is a logical extension of those subjects. If you can do algebra, geometry, and trig, you can do calculus.

But why should you bother — apart from being required to take a course? Why climb Mt. Everest? Why listen to Beethoven's Ninth Symphony? Why visit the Louvre to see the *Mona Lisa*? Why watch *South Park*? Like these endeavors, doing calculus can be its own reward. There are many who say that calculus is one of the crowning achievements in all of intellectual history. As such, it's worth the effort. Read this jargon-free book, get a handle on calculus, and join the happy few who can proudly say, "Calculus? Oh, sure, I know calculus. It's no big deal."

About This Book

Calculus All-in-One For Dummies is intended for three groups of readers: students taking their first calculus course, students who need to brush up on their calculus to prepare for other studies, and adults of all ages who'd like a good introduction to the subject either to satisfy their own curiosity or perhaps to help someone else with calculus.

If you're enrolled in a calculus course and you find your textbook less than crystal clear, this is the book for you. It covers the most important topics in the first year of calculus: differentiation, integration, and infinite series.

If you've had elementary calculus, but it's been a couple of years and you want to review the concepts to prepare for, say, some graduate program, *Calculus All-in-One For Dummies* will give you a thorough, no-nonsense refresher course.

Non-student readers will find the book's exposition clear and accessible. *Calculus All-in-One For Dummies* takes calculus out of the ivory tower and brings it down to earth.

This is a user-friendly math book. Whenever possible, I explain the calculus concepts by showing you connections between the calculus ideas and easier ideas from algebra and geometry. I then show you how the calculus concepts work using concrete examples. Only later do I give you the fancy calculus formulas. All explanations are in plain English, not math-speak.

The following conventions keep the text consistent and oh-so-easy to follow:

- » Variables are in *italics*.
- » Calculus terms are italicized and defined when they first appear in the text.
- » In the step-by-step problem-solving methods, the general action you need to take is in bold, followed by the specifics of the particular problem.

It can be a great aid to true understanding of calculus — or any math topic for that matter — to focus on the *why* in addition to the *how-to*. With this in mind, I've put a lot of effort into explaining the underlying logic of many of the ideas in this book. If you want to give your study of calculus a solid foundation, you should read these explanations. But if you're really in a hurry, you can cut to the chase and read only the important introductory stuff, the example problems, the step-by-step solutions, and all the rules and definitions next to the icons. You can read the remaining exposition later only if you feel the need.

I find the sidebars interesting and entertaining. (What do you expect? I wrote them!) But you can skip them without missing any essential calculus. No, you won't be tested on that stuff.

The most important thing is for you to work out the “Your Turn” example problems and the problems in the end-of-chapter quizzes (solutions are provided). You can't learn calculus (or any type of math) without working out dozens or hundreds of problems. If you want even more practice problems, you can find more chapter quizzes online.

Foolish Assumptions

Call me crazy, but I assume

- » You know at least the basics of algebra, geometry, and trig.

If you're rusty, Unit 2 (and the online Cheat Sheet) contains a good review of these pre-calculus topics. Actually, if you're not currently taking a calculus course, and you're reading this book just to satisfy a general curiosity about calculus, you can get a good conceptual picture of the subject without the nitty-gritty details of algebra, geometry, and trig. But you won't, in that case, be able to follow all the problem solutions. In short, without the pre-calculus stuff, you can see the calculus *forest*, but not the *trees*. If you're enrolled in a calculus course, you've got no choice — you've got to know the trees as well as the forest.

- » You're willing to do some w_ _ _.

No, not the dreaded *w*-word! Yes, that's w-o-r-k, *work*. I've tried to make this material as accessible as possible, but it is calculus after all. You can't learn calculus by just listening to a tape in your car or taking a pill — not yet anyway.

Is that too much to ask?

Icons Used in This Book

Keep your eyes on the icons:



MATH
RULES

Next to this icon are calculus rules, definitions, and formulas.



REMEMBER

These are things you need to know from algebra, geometry, or trig, or things you should recall from earlier in the book.



TIP

The lightbulb icon appears next to things that will make your life easier. Take note.



WARNING

This icon highlights common calculus mistakes. Take heed.



EXAMPLE

Each example is a calculus problem that illustrates the topic just discussed, followed by a step-by-step solution. Studying these example problems and their solutions will help you solve the “Your Turn” practice problems and the problems in the end-of-chapter quizzes.



YOUR
TURN

This icon means it's time to put on your thinking cap. It appears next to practice problems for you to work out. Some of these problems will be quite similar to the example problems from the same section. Others will challenge you by going a bit beyond the garden-variety examples. Solutions are provided.

Beyond the Book

There's some great supplementary calculus material online that you might want to check out.

To view this book's online Cheat Sheet, simply go to www.dummies.com and type **Calculus All in One For Dummies Cheat Sheet** in the Search box. You'll find a nice list of important formulas, theorems, definitions, and so on from algebra, geometry, trigonometry, and calculus. This is a great place to go if you forget a formula.

You'll also have access to additional online quizzes for each chapter of the book, starting with Unit 2. To access the quizzes, follow these steps:

1. **Register your book or ebook at Dummies.com to get your PIN.** Go to www.dummies.com/go/getaccess.
2. **Select your product from the drop-down list on that page.**
3. **Follow the prompts to validate your product, and then check your email for a confirmation message that includes your PIN and instructions for logging in.**

If you do not receive this email within two hours, please check your spam folder before contacting us through our Technical Support website at <http://support.wiley.com> or by phone at 877-762-2974.

Where to Go from Here

Why, Chapter 1, of course, if you want to start at the beginning. If you already have some background in calculus or just need a refresher course in one area or another, then feel free to skip around. Use the table of contents and index to find what you're looking for. If all goes well, in a half a year or so, you'll be able to check calculus off your list:

- ☐ Run a marathon
- ☐ Go skydiving
- ☐ Write a book
- ☒ Learn calculus
- ☐ Swim the English Channel
- ☐ Cure cancer
- ☐ Write a symphony
- ☐ Pull an unnatural double cork 1260° at the X Games

For the rest of your list, you're on your own.

1

An Overview of Calculus

In This Unit . . .

CHAPTER 1: What Is Calculus?

What Calculus Is Not
So What Is Calculus, Already?
Real-World Examples of Calculus

CHAPTER 2: The Two Big Ideas of Calculus: Differentiation and Integration — Plus Infinite Series

Defining Differentiation
Investigating Integration
Sorting Out Infinite Series

CHAPTER 3: Why Calculus Works

The Limit Concept: A Mathematical Microscope
What Happens When You Zoom In
Two Caveats; or, Precision, Preschmidgen

IN THIS CHAPTER

- » You're only in Chapter 1 and you're already going to get your first calc test
- » Calculus — it's just souped-up regular math
- » Zooming in is the key
- » The world before and after calculus

Chapter 1

What Is Calculus?

“My best day in Calc 101 at Southern Cal was the day I had to cut class to get a root canal.”

— MARY JOHNSON

“I keep having this recurring dream where my calculus professor is coming after me with an axe.”

— TOM FRANKLIN, COLORADO COLLEGE SOPHOMORE

“Calculus is fun, and it's so easy. I don't get what all the fuss is about.”

— SAM EINSTEIN, ALBERT'S GREAT-GRANDSON

In this chapter, I answer the question, “What is calculus?” in plain English, and I give you real-world examples of how calculus is used. After reading this and the following two short chapters, you *will* understand what calculus is all about. But here's a twist: Why don't you start out on the *wrong* foot by briefly checking out what calculus is *not*?

What Calculus Is Not

No sense delaying the inevitable. Ready for your first calculus test? Circle True or False.

True or False: Unless you actually enjoy wearing a pocket protector, you've got no business taking calculus.

True or False: Studying calculus is hazardous to your health.

True or False: Calculus is totally irrelevant.

False, false, false! There's this mystique about calculus that it's this ridiculously difficult, incredibly arcane subject that no one in their right mind would sign up for unless it was a required course.

Don't buy into this misconception. Sure, calculus is difficult — I'm not going to lie to you — but it's manageable, doable. You made it through algebra, geometry, and trigonometry. Well, calculus just picks up where they leave off — it's simply the next step in a logical progression.



REMEMBER

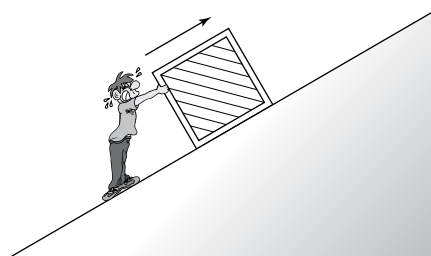
Calculus is relevant. Calculus is not a dead language like Latin, spoken only by academics. It's the language of engineers, scientists, and economists. Okay, so it's a couple steps removed from your everyday life and unlikely to come up at a cocktail party. But the work of those engineers, scientists, and economists has a huge impact on your day-to-day life — from your microwave oven, cellphone, TV, and car to the medicines you take, the workings of the economy, and our national defense. At this very moment, something within your reach or within your view has been impacted by calculus.

So What Is Calculus, Already?

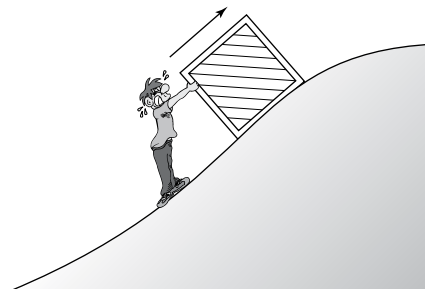
Calculus is basically just very advanced algebra and geometry. In one sense, it's not even a new subject — it takes the ordinary rules of algebra and geometry and tweaks them so that they can be used on more complicated problems. (The rub, of course, is that darn *other* sense in which it is a new and more difficult subject.)

Look at Figure 1-1. On the left is a man pushing a crate up a straight incline. On the right, the man is pushing the same crate up a curving incline. The problem, in both cases, is to determine the amount of energy required to push the crate to the top. You can do the problem on the left with regular math. For the one on the right, you need calculus (assuming you don't know the physics shortcuts).

FIGURE 1-1:
The difference
between
regular math
and calculus: In
a word, it's the
curve.



Regular math problem



Calculus problem

For the straight incline, the man pushes with an *unchanging* force, and the crate goes up the incline at an *unchanging* speed. With some simple physics formulas and regular math (including algebra and trig), you can compute how many calories of energy are required to push the crate up the incline. Note that the amount of energy expended each second remains the same.

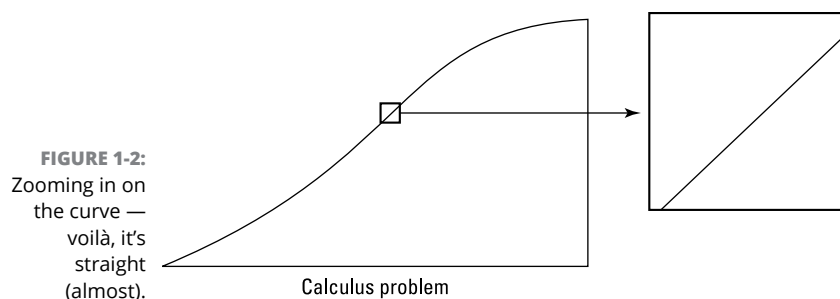
For the curving incline, on the other hand, things are constantly changing. The steepness of the incline is *changing* — and not just in increments like it's one steepness for the first 3 feet then a different steepness for the next 3 feet. It's *constantly changing*. And the man pushes with a *constantly changing* force — the steeper the incline, the harder the push. As a result, the amount of energy expended is also changing, not every second or every thousandth of a second, but *constantly changing* from one moment to the next. That's what makes it a calculus problem.



REMEMBER

Calculus is the mathematics of change. By this time, it should come as no surprise to you that calculus is described as “the mathematics of change.” Calculus takes the regular rules of math and applies them to fluid, evolving problems.

For the curving incline problem, the physics formulas remain the same, and the algebra and trig you use stay the same. The difference is that — in contrast to the straight incline problem, which you can sort of do in a single shot — you've got to break up the curving incline problem into small chunks and do each chunk separately. Figure 1-2 shows a small portion of the curving incline blown up to several times its size.



When you zoom in far enough, the small length of the curving incline becomes practically straight. Then, because it's straight, you can solve that small chunk just like the straight incline problem. Each small chunk can be solved the same way, and then you just add up all the chunks.

That's calculus in a nutshell. It takes a problem that can't be done with regular math because things are constantly changing — the changing quantities show up on a graph as curves — it zooms in on the curve till it becomes straight, and then it finishes off the problem with regular math.

What makes the invention of calculus such a fantastic achievement is that it does what seems impossible: It zooms in *infinitely*. As a matter of fact, everything in calculus involves infinity in one way or another, because if something is constantly changing, it's changing infinitely often from each infinitesimal moment to the next.

Real-World Examples of Calculus

So, with regular math you can do the straight incline problem; with calculus you can do the curving incline problem. Here are some more examples.

With regular math you can determine the length of a buried cable that runs diagonally from one corner of a park to the other (remember the Pythagorean Theorem?). With calculus you can determine the length of a cable hung between two towers that has the shape of a *catenary* (which is different, by the way, from a simple circular arc or a parabola). Knowing the exact length is of obvious importance to a power company planning hundreds of miles of new electric cable. See Figure 1-3.

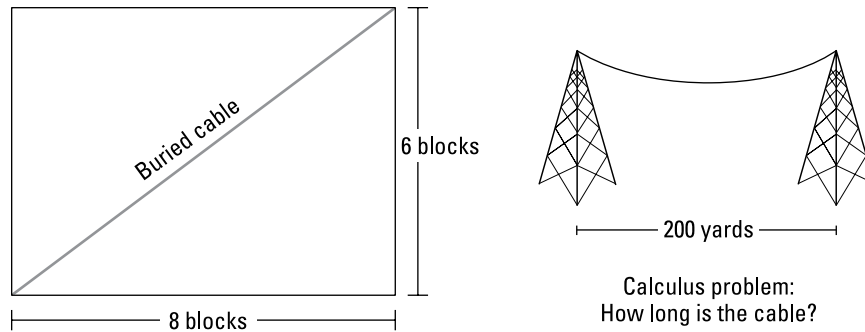


FIGURE 1-3:
Without and
with calculus.

Regular math problem:
How long is the cable?

You can calculate the area of the flat roof of a home with ordinary geometry. With calculus you can compute the area of complicated, nonspherical shapes like the shapes of some sports arena domes. Architects designing such a building need to know the dome's area to determine the cost of materials and to figure the weight of the dome (with and without snow on it). The weight, of course, is needed for planning the strength of the supporting structure. Check out Figure 1-4.

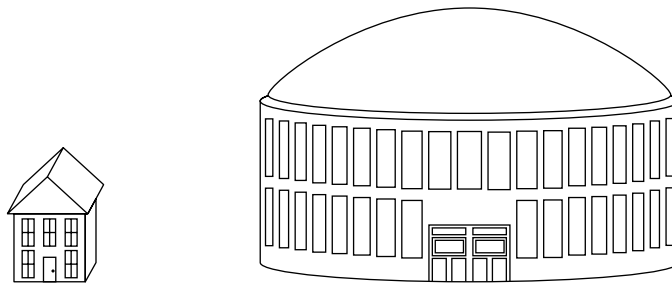


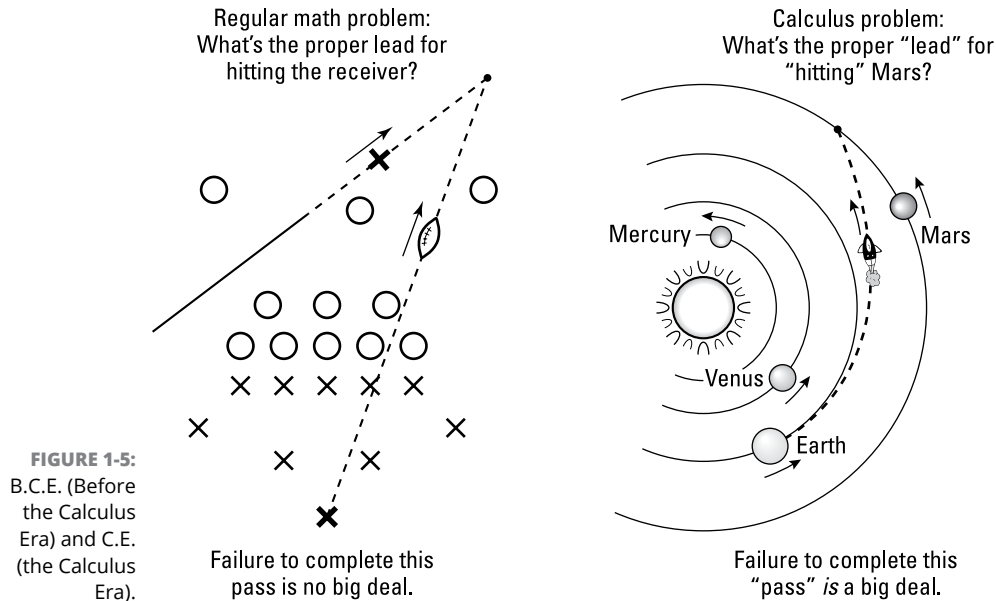
FIGURE 1-4:
Sans and avec
calculus.

Regular math problem:
What's the roof's area?

Calculus problem:
What's the dome's area?

With regular math and some simple physics, you can calculate how much a quarterback must lead his receiver to complete a pass. (I'm assuming here that the receiver runs in a *straight* line and at a *constant* speed.) But when NASA, in 1975, calculated the necessary "lead" for aiming the Viking I at Mars, it needed calculus because both the Earth and Mars travel on *elliptical* orbits (of different shapes) and the speeds of both are *constantly changing* — not to mention the fact that on its way to Mars, the spacecraft is affected by the different and *constantly changing* gravitational pulls of the Earth, the Moon, Mars, and the Sun. See Figure 1-5.

You see many real-world applications of calculus throughout this book. The differentiation problems in Unit 4 all involve the steepness of a curve — like the steepness of the curving incline in Figure 1-1. In Unit 5, you do integration problems like the cable-length problem shown back in Figure 1-3. These problems involve breaking up something into little sections, calculating each section, and then adding up the sections to get the total. More about that in Chapter 2.



IN THIS CHAPTER

- » Delving into the derivative: It's a rate and a slope
- » Investigating the integral — addition for experts
- » Infinite series: Achilles versus the tortoise — place your bets

Chapter 2

The Two Big Ideas of Calculus: Differentiation and Integration — Plus Infinite Series

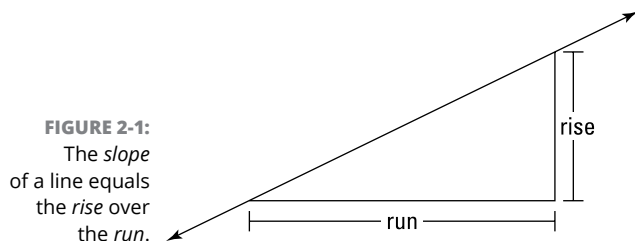
This book covers the two main topics in calculus — differentiation and integration — as well as a third topic, infinite series. All three topics touch the earth and the heavens because all are built upon the rules of ordinary algebra and geometry, and all involve the idea of infinity.

Defining Differentiation

Differentiation is the process of finding the *derivative* of a curve. And the word “derivative” is just the fancy calculus term for the curve’s slope or steepness. And because the slope of a curve is equivalent to a simple rate (like *miles per hour* or *profit per item*), the derivative is a rate as well as a slope.

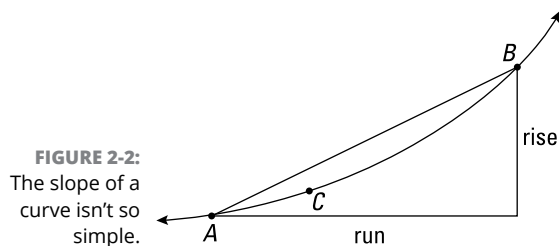
The derivative is a slope

In algebra, you learned about the slope of a line — it's equal to the ratio of the *rise* to the *run*. In other words, $Slope = \frac{rise}{run}$. See Figure 2-1. Let me guess: A sudden rush of algebra nostalgia is flooding over you.

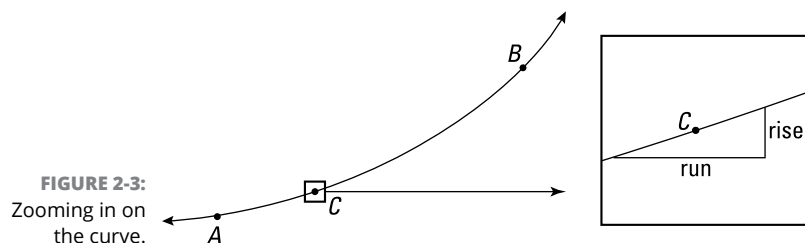


In Figure 2-1, the *rise* is half as long as the *run*, so the line has a slope of $\frac{1}{2}$.

On a curve, the slope is constantly *changing*, so you need calculus to determine its slope. See Figure 2-2.



Just like the line in Figure 2-1, the straight line between A and B in Figure 2-2 has a slope of $\frac{1}{2}$. And the slope of this line is the same at every point between A and B. But you can see that, unlike the line, the steepness of the curve is changing between A and B. At A, the curve is less steep than the line, and at B, the curve is steeper than the line. What do you do if you want the exact slope at, say, point C? Can you guess? Time's up. Answer: You zoom in. See Figure 2-3.



When you zoom in far enough — really far, actually *infinitely* far — the little piece of the curve becomes straight, and you can figure the slope the old-fashioned way. That's how differentiation works.