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Percival McCormack

Vortex, Molecular Spin and Nanovorticity

An Introduction

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Percival McCormack
Department of BioEngineering
University of Illinois at Chicago
218 SEO, 851 S. Morgan St. (M/C 063)
Chicago, IL 60607–7052
USA
pmccorm@uic.edu

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Chapter 1
The Vortex

1.1 Brief History

The first notable works on fluid motions appeared in the early seventeenth century. Descartes in his “Principia Philosophiae” (1644) considered that space was filled with frictional vortices, so that the planets are carried along by the vortex motions. Vortices had fascinated mankind for many centuries before Descartes. It was considered that life had started in the water of the primeval vortex – in whirlwinds and whirlpools.

The oldest description of a tidal vortex was given by Homer (eighth century BC). The returning heroes of the Odyssey had to face the danger of the giant whirlpool Charybde [1]. The Celts and Teutons also believed that life was created in this whirlpool. The connection between cosmic whirl and the Maelstrom is described by de Santillana [2].

Empedocles (492–432 BC) lived in Sicily and believed in four primary substances – earth, water, air, and fire. He also distinguished attractive and repelling “‘forces’ as ‘love’ and ‘strife’”, respectively. He demonstrated the centrifugal effect by the retention of fluid within a rotating, liquid-filled ladle. He used this effect to explain the position of the celestial bodies in the firmament.

Anaxagoras (500–428 BC) did not believe in the basic four elements but thought of matter as a continuum. He extended the vortex concept to the total world process [3]. With hindsight, today it would appear that this vortex concept of Anaxagoras may be close to a universal truth. It is interesting to note that Plato and his disciple Aristotle (384–322 BC) rejected the vortex theory of the micro- and macro-cosmos. However, in his book “Meteorologica” and in the pseudo-Aristotelian scripts “Problemata Mechanica” and “De Mundo”, vortices *are* described for their own sake. The cause, occurrence, and motion of whirlwinds and tidal vortices are considered.

In Roman times, meteorological vortices were described by Seneca (0–65 AD), Pliny (23–79 AD), and Lucan (39–65 AD). In the Middle Ages, the early development of a scientific terminology appeared [4, 5].

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In the thirteenth century, Gerard of Brussels studied the kinematics of rotating bodies – he conceptualized solid-body rotation (see his text – "Liber de motu"). Nicole Oresme (circa 1325–1382) studied the commensurability of rotating celestial bodies. He identified the return of a single body along a circular orbit from an arbitrary point to the same point, as a circulation, whereas the return of several bodies from an arbitrary initial state to exactly the same state (commensurable), or to a similar state (incommensurable), is a revolution.

The Renaissance period saw the climax in the use of art to describe vortex motions, and leading this effort was Leonardo da Vinci (1452–1515). He perceived the vortex and the wave as the manifestation of power and motion, and used spiral and wavy elements in his paintings. He pioneered the concept of turbulent motion; identified the difference between the potential vortex and solid-body rotation; and studied channel vortex flow and vortex formation in the wake of obstacles [6].

1.1.1 Cartesian Vortex Theory (Sixteenth and Seventeenth Centuries)

In this period, Kepler found that the planetary orbits are elliptic not circular. He attributed this distortion to magnetic attraction and repulsion. He conceived of a magnetic vortex caused by the rotation of the sun – the essence of Cartesian vortex theory [7, 8]. The theory is based on the assumption that matter has extension and is identified with space (it is a distributed system). The implication was that there is no vacuum and bodies interact by direct contact. When a body moves the surrounding fluid, particles are induced into motion around it. This is valid for a continuum, but not for celestial bodies. The problem was resolved in 1687 by Newton in his "Principia" – he advocated that material bodies interact over a distance by gravitational attraction. The success of Newtonian mechanics in describing planetary motion overwhelmed all opposition. The eighteenth and nineteenth centuries saw the birth and development of classical mechanics – based upon Newton's theory.

Hadley in 1735 published a theory of the general circulation of the atmosphere. Although since rejected, his theory was the start of a long effort to develop a satisfactory model of atmospheric motions [9]. The effort still goes on today with the aid of high-speed computers. In 1749, Boscovitch published a comprehensive analysis of tornadoes [10]. Kant in 1755 proposed that the sun and planets developed out of a rotating gaseous cloud.

The foundation of fluid mechanics is attributed to Euler [11]. He derived the equations of motion for an inviscid fluid. He used a mathematical term that later was identified as the "vorticity vector." This led to the development of the equations of motion for viscous fluids – known as the Navier–Stokes equations [12]. The analytical treatment of vortex motion started with Herman von Helmholtz's classic paper “On Integrals of the Hydrodynamic Equations Corresponding to Vortex Motion” in 1858. Kelvin in 1869 demonstrated the necessity of Helmholtz’s theorems for the existence of vortex motions and developed a circulation theorem named after him. These works formed the basis for the modern vortex theory. They also led to the extension of the vortex concept into other areas of physics.

Based on the argument that magnetic fields are rotatory, Maxwell in 1861 used the vortex model in his electromagnetic theory. The equivalence of electric and magnetic fields was inherent in this theory and was pointed out by Boltzmann in 1891. During the latter half of the nineteenth century, Lord Kelvin developed a theory of the properties of atoms and molecules based on vortex rings [13–15]. However, vortices in nature are unstable and decay and these properties were incompatible with the atomic model. Although the attempt to interpret the structure of the physical world in terms of a mechanical vortex model failed, the vortex concept and its applications in classical mechanics, fluid, and aerodynamics, and now in the fields of superfluid physics and superconductivity, have been very successful.

1.1.2 The Twentieth Century

The vortex theorems of Helmholtz and Lord Kelvin were further developed by Crocco (1937), Ertel (1942), and Vazsonyi (1945) – see [16]. The Taylor–Proudman theorem has made an important contribution to the analysis of rotating fluid systems [17]. Benard, von Karman, Taylor, and Gortler have identified various vortex configurations, all connected with the development of flow instability. For example, Taylor–Gortler vortices develop in curved flow (over a concave plate) and involve a conversion from two-dimensional disturbances into a three-dimensional configuration [18]. The onset of flow instability is the prelude to transition to turbulence and early research on turbulence is identified with the work of Boussinesq, Taylor, Prandtl, Heisenberg, and Reynolds [19, 20].

The *dynamics* of turbulent flow is based on the hypothesis that the Navier–Stokes equations are valid for turbulent flow. But, the *mechanics*, or structure, of turbulence is a *statistical* mechanics. Turbulent boundary layers are complicated by the fact that their structure is neither homogeneous nor isotropic. Little progress has been made to date on the *theory* of nonisotropic turbulence. Considerable progress has been made, however, in the study of at least locally isotropic turbulence. This has been facilitated by the use of computational fluid dynamics (CFD).

In 1941, a general theory of locally isotropic turbulence was formulated by Kolmogorov which predicted a number of laws governing turbulent flow for large Reynolds numbers. The fundamental physical concepts which form the basis of Kolmogorov’s theory can be summarized as follows:

1. A turbulent flow at large Reynolds numbers is considered to be the result of the superposing of disturbances (vortices or eddies) of all possible sizes. Only the very largest of these vortices are due directly to the instability of the mean flow.

2. The motion of the large vortices is unstable and this produces smaller (secondary) vortices: the latter produce third-order vortices and so on.
3. The motion of the smallest vortices is “laminar” and depends basically on the molecular viscosity.
4. The motion of all but the largest vortices can be assumed to be homogeneous and isotropic.
5. The motion of all vortices whose scales are much smaller than the *local* structure of the flow must be subject to general statistical laws which do not depend on the geometry of the flow or on the properties of the mean flow.

The establishment of these general statistical laws constitutes the theory of local isotropic turbulence.

In aero- and hydro-dynamics, Lanchester made a unique contribution by suggesting that a vortex acting as an aerofoil is the cause of lift.

The bound vortex theory of lift has been developed by many scientists, but of special note are the names of Prandtl et al. [21]. Analytical design of airplanes, propellers, and turbines rapidly followed.

In meteorology, theoretical weather prediction is based on Bjerknes circulation theorem and Rossby’s solution of the vorticity transport equation. The fact that to understand global atmospheric circulation it is essential to know the occurrence and migration of cyclonic and anticyclonic air masses in mid-latitudes, which was discovered by Jeffreys in 1926 [9]. The great contribution of Rotunno in the 1980s to tornado dynamics must also be noted [22]. In 1908, Benard related the vortex concept to sound and this led to the theory of aerodynamic sound generation based on vortical motion in an unsteady flow. Lighthill [23] and Powell [24] have been pioneers in this area, and Sarpkaya [25] in the closely related area of vortex-induced vibration of bodies.

In astronomy and astrophysical dynamics, the vortex phenomenon is common. Hubble identified spiral nebulae as galaxies in the mid-1920s, and in 1943 Weizsacher developed a vortex theory for the generation of the planetary system. In recent decades, studies of exotic stars have been made. Neutron stars with enormously high angular velocities have been postulated and discovered, as well as rotating black holes whose existence and characteristics require the general theory of relativity and quantum mechanics [26]. Magnificent images of the atmospheric vortex arrays from Jupiter have been transmitted back to Earth by Voyagers 1 and 2 [27, 28]. Many unsolved vortex problems remain, such as the motion of planetary atmospheres and convection flows inside stars [29, 30].

With the development of ultra-fast computers with great storage capacity, the ease of programming, and associated graphics capability, it has become possible to solve numerically (even in real time) the basic equations of fluid motion for nontrivial problems. The discipline of computational fluid dynamics (CFD) is now recognized in its own right [31, 32]. For example, numerical solution of the vorticity transport equation was the key to understanding unsteady flows [33]. Finite element, finite difference methods, and extensive meshes with moving frames of reference have facilitated the design of aerofoils, engine intakes, and nacelles in turbulent hypersonic flow.

When the underlying discretization is of the vorticity field, rather than the velocity field, the numerical method for approximating the solution of the incompressible Euler form of the Navier–Stokes equations is known as the vortex method [34, 35]. Leonard [36, 37] has studied a 3D version of the vortex-in-cell method and computed 3D flows.

Advances in the fields of turbulence and unionized and ionized compressible and incompressible flow – all necessitating a statistical mechanical approach – will probably be "rate controlled" by advances in CFD techniques.

In the 1940s, it was found that the peculiar, or superfluid, properties of helium 2 (fountain effect, second sound, and frictionless flow) could be largely accounted for on the basis of the phenomenological two-fluid theory of Tisza [38] and Landau [39]. In 1954, London [40] proposed that below the so-called "λ-point" (a discontinuity in the specific heat versus temperature curve), helium is a quantum fluid whose essential feature is the macroscopic occupation of a single quantum state. Excitations in liquid helium were identified as phonons (similar to those in solids) and rotons, which correspond to possible rotational modes of motion in the liquid. It was Onsager in 1950 [41] who showed that there was a possible wave function for the liquid, which would produce a motion analogous to classical vortex motion, and suggested the possible existence of quantized vortices in helium 2. Feynmann [42] in 1955 developed this conjecture into a theory which has been successful in explaining many of the peculiar phenomena observed in the superfluid. Experimental and theoretical studies of the superfluid state continue today, with the fascinating extension of the quantized vortex to the superconducting state and the interaction with, and generation of, magnetic fields [43, 44]. Moreover, as vortices are described by solutions of field equations, the topology of vortices is now relevant to elementary particle physics [45].

Vortex physics is now well developed, from the microscopic, or atomic, world to the astronomical world. The vortex concept spans an enormous size range from the truly microscopic to the truly macroscopic (10^{-8}–10^{18} cm – a scale factor of 10^{26}!). The significance of the fact that vortex physics spans this enormous range, unchanged apart from the quantum restriction at the microscopic level, remains to be elucidated.

1.2 Kinematics

1.2.1 Definition of a Vortex

A vortex is defined as the motion of fluid particles around a central spin axis. The individual particle paths may be circular, or not – see Fig. 1.1. If the paths are the same in every plane normal to the axis of rotation, one has a cylindrical

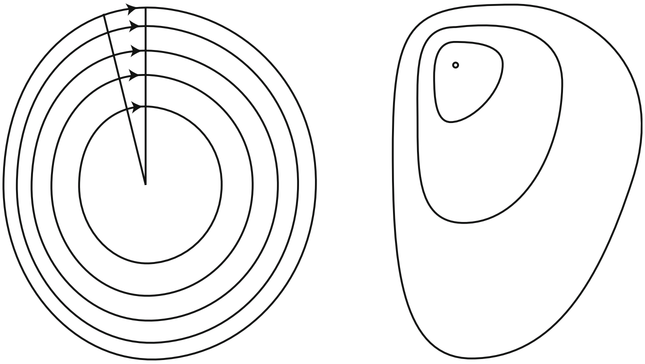

Fig. 1.1

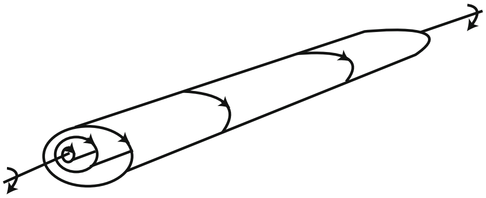

Fig. 1.2

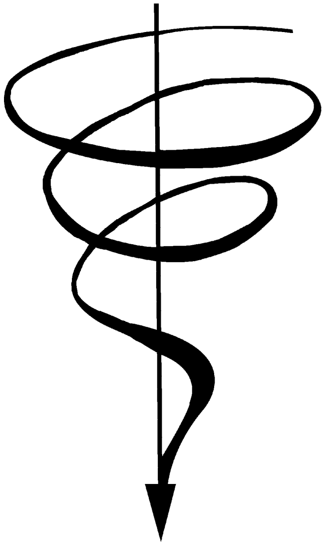

Fig. 1.3

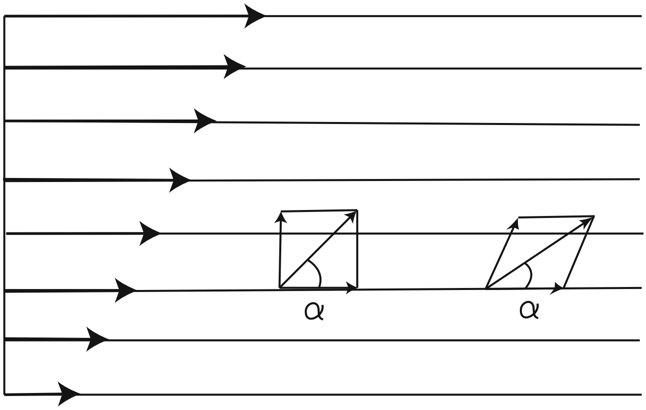

Fig. 1.4

vortex – see Fig. 1.2. Many natural vortices have pathlines that are not perpendicular to the axis of rotation, but are oblique to it. A particular case is the spiral vortex, illustrated in Fig. 1.3. Rather being based on pathlines, the *flow pattern* at a point in space may be used to define the vortex. This is the basis of the Cauchy and Stokes definition [46]. They called the angular velocity of a fluid at a point in space "vorticity" or spin. Thus, vortex motion involves a basic mode of rotation, along with translational and deformational motion.

Other terms for vortex, such as eddy, circulation, spiral, whirl, and cyclone, are used and differ only in their physical, geophysical, or engineering meaning.

All motion of a fluid within a finite space surrounded by a solid boundary must result in rotation. This follows from the conservation of matter. Such rotational motion is also possible in an infinite space, given that the fluid far from the source of the motion is at rest. As mentioned earlier, the angular velocity about a point is spin. Galaxies and virtually zero mass particles all spin. Spin is governed by the conservation of angular momentum – a fact that is fundamental to many analyses in physics. In the atmosphere, in space, and in the ocean, if a specific volume of fluid that is rotating is compacted, the rate of rotation must increase to conserve angular momentum.

It must be noted that the presence of vorticity, or spin, is basic to any vortex, but one can have vorticity and *no vortex*, as in the shear flow illustrated in Fig. 1.4.

1.2.2 Vorticity and Circulation

Stokes (1845) showed for viscous fluids and vonHelmholtz for ideal fluids that the motion of deformable bodies, such as fluids, can be broken down into a sum of a translation, a rotation, and a strain. The strain may be further divided into a pure linear strain and a pure shearing (angular) strain.