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Péter Hraskó

Basic Relativity

An Introductory Essay

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Preface

Traditional presentations of relativity theory start with the introduction of Lorentz-transformations from which the peculiar phenomena of the theory (time dilation, Lorentz contraction, the velocity addition formula, etc.) follow. Though this is certainly the most logical approach, it seems rather unfortunate from a pedagogical point of view, since a convincing and conceptually transparent explanation of the Lorentz-transformation itself presents a task of considerable difficulty. Lorentz-transformation is based on both the constancy of the light speed and Einstein's synchronization prescription, and the interrelation between these two constituents is open to the frequent misunderstanding that constancy of the light speed is enforced by the special synchronization of clocks rather than being the law of nature. In order to avoid this pitfall an ad hoc though rigorous presentation of the theory's perplexing properties in Part 1 precedes the introduction of the Lorentz-transformation (and any synchronization procedure). After the introduction of these transformations in Part 2 those same relativistic effects are reconsidered this time in a systematic manner. Part 3 is devoted to the fundamentals of general relativity.

The book is based on the lectures given at the post graduate course in physics education at the Eötvös Loránd University (Budapest).

Budapest, December 2010 Péter Hraskó

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Chapter 1
From Time Dilation to $E_0 = mc^2$

Abstract Time dilation and the relativity of simultaneity are deduced from the Doppler-effect. Lorentz contraction and the equation of motion are derived from time dilation. Mass-energy relation is proved and its popular interpretation is critically examined.

Keywords Reference frames · Time · Simultaneity · Contraction · Mass · Energy

1.1 Reference Frames and Inertial Frames

Physical phenomena are always described relative to some *object* (laboratory, the surface of the Earth, moving traincar, spacecraft, etc.). Objects of reference of this kind are called *reference frames*.

Though reference frames and coordinate systems are two very different notions they are not always clearly distinguished from each other. When, in order to study a certain phenomenon, a measurement is performed the instruments (including clocks and measuring rods among them) are always at rest in the reference frame used but nothing like 'coordinate system' is found there. Coordinate systems serve to assign a triple of numbers to the points of space in order to make calculations possible, while the purpose of the reference frames is to accommodate measuring apparatuses and their personnel. Phenomena which we try to observe and predict are coincidences, i.e. encounters of bodies, whose coordinates are important but unobservable auxiliary quantities.

A coordinate system requires more than just three mutually perpendicular axes through the origin: the set of coordinate lines must cover a whole domain of space. Such an infinitely dense set of coordinate lines exists only in our minds and a great many misunderstandings could be avoided if the really existing (or imagined as such) reference frames were never called coordinate systems (and vice versa).

Reference frames with respect to which the laws of Nature take their simplest possible form are called *inertial frames*. This rather informal definition

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presupposes that when the basic laws of a new field of physical phenomena have been successfully developed the concept of the inertial frame must be suitably adapted. In the first period of the modern history of physics, before the advent of electrodynamics, it was mechanics that reached a sufficiently high level of sophistication to formulate a precise law, the Newtonian law of *mass*$\times$ *acceleration* = *force*, on which the definition of the inertial frames could be based. It is this formula which in Newtonian physics permits us to select inertial frames from the multitude of reference frames by the absence of *inertial forces*, i.e. by the criterion that in these frames one needs to take into consideration only forces, originating from well identifiable physical objects (*true forces*). In the special case when sources of this kind are absent (or are very far away) an isolated body retains its rectilinear uniform motion or remains at rest (the *law of inertia*). This is a practically applicable criterion to decide whether the reference frame a body is referred to is an inertial frame or not.

A laboratory on the surface of the Earth is not an inertial frame since the plane in which the Foucault pendulum swings rotates with respect to it. This rotation is caused by the Coriolis force which is an inertial force. When the effect of the Coriolis force is negligible such laboratories can be considered as approximately inertial frames. But no laboratory on the Earth can be assumed an *isolated* inertial frame since all bodies in it are subjected to the action of the gravitation which from the Newtonian point of view is a true force.[1] Therefore, in the laboratories on the Earth the law of inertia must be formulated in a counterfactual form: were gravitation switched off (or compensated) the velocities of isolated bodies would remain constant.

Given an inertial frame all the other reference frames which move uniformly or remain at rest with respect to it are, according to Newtonian physics, also inertial frames.

Since in all of the inertial frames the basic laws of mechanics are of the same form these frames are, within the range of Newtonian mechanics, equivalent to each other. On the other hand, owing to the great variety of the inertial forces, generic reference frames are endowed with individual properties which make all of them intrinsically distinguishable from the others.

The fundamental laws of electrodynamics are expressed by the Maxwell equations, according to which light propagates with the same velocity in all directions (isotropy). Vacuum light velocity is denoted by c. Einstein assumed that in their original form Maxwell equations are valid in the inertial frames which means that their observable consequences can be proved true with respect to these frames. In particular, *it is only in the (isolated) inertial frames that speed of light is*

[1] It is a remarkable fact that because of weightlessness in them satellites, orbiting freely around the Earth, have the properties of a truly isolated inertial frame. Nevertheless, in the Newtonian framework they cannot be qualified as such since their center of mass is accelerating and bodies within them are subjected to the action of the corresponding inertial force. However, this force is precisely compensated by the gravitational attraction of the Earth. This question will be taken up again in Sect. 3.2 in connection with general relativity.

equal to the same c in any direction. In this respect propagation of light is fundamentally different from that of sound which is isotropic only with respect to its medium at rest. More generally, inertial frames free from outside influences are, from the point of view of both mechanics and electrodynamics, equivalent to each other. Though the inclusion of electrodynamics does not invalidate the mechanical equivalence of the inertial frames and in particular the validity of the law of inertia in them it leads to a slight modification of the form of the Newtonian equation of motion which retains its original form $mass \times acceleration = force$ only for velocities much smaller than c (see Sects. 1.7 and 2.18).

As far as it is known today the equivalence of the inertial frames extends actually far beyond mechanics and electrodynamics into the realms of weak and strong interactions too. This assumption which is a far reaching generalization of the constancy of the light velocity constitutes the first of the two postulates of *the special relativity theory*. This theory preserves the important property of the inertial frames that their relative motion is uniform and rectilinear. These properties are, however, lost in *general relativity*. As it can be guessed from its name this theory is the generalization of special relativity which emerged from Einstein's attempts to extend this latter theory to the gravitation. In pursuing this aim Einstein realized that gravitation cannot be forced into the Procrustean bed of special relativity but special relativity can be extended so as to provide a surprizingly natural place to gravitation. This more general approach does not, of course, invalidate special relativity but, as can be expected, it recognizes the limits of its applicability. In what follows we will confine ourselves mostly to the special theory which in itself covers important areas of physics. The basic principles of general relativity will be outlined in Chap. 3.

Returning to the electrodynamics let us notice that as far as the considerations are restricted to some given inertial frame the constancy of the light speed presents no problem. It can be experimentally verified by any method which has been accepted as legitimate procedure to measure light velocity as e.g. the rotating disc experiment of Fizeau or Foucault's rotating mirror method.[2] Either procedure is based on the *path/time* notion of velocity and they were performed as two-way experiments rather than unidirectional one's with the only aim to improve accuracy (see Sect. 2.2). But, as a matter of fact, it would be an extremely difficult task to measure light velocity in a number of inertial frames in relative motion with an accuracy sufficient to convince ourselves of its constancy. Instead, we may resort to an indirect reasoning. Should light speed not the same in the different inertial frames to a high degree of accuracy, this fact had already been come to light, owing to its numerous consequences. It is in fact the whole body of the twentieth century physics which testifies in favour of the relativistic postulate of light

[2] Strictly speaking, it would be unreasonable to expect that speed of light should be constant in reference frames, resting on the Earth, since Coriolis force and gravitation do certainly influence the propagation of light. The influence of the rotation of the Earth manifests itself in the Sagnac effect (see Sect. 2.23), but the effect of the gravitation is extremely small (see Sect. 3.7).

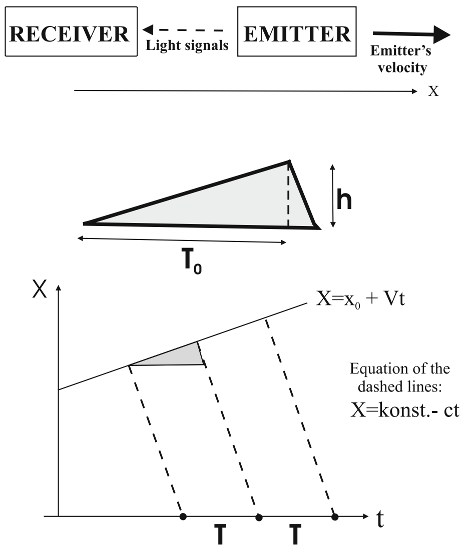

Fig. 1.1 Calculation in the rest frame of the receiver (RFR)

velocity. In what follows we will, therefore, consider the independence of the light velocity of the motion of the inertial frames a well established *empirical fact*.

When, on the other hand, a given phenomenon is analysed simultaneously from the point of view of several inertial frames in different states of motion one, as a rule, runs into conflict with intuition. The essence of special relativity theory is to explicate these paradoxes and explain how to resolve them in a consistent manner. This chapter will be devoted to this theme.

1.2 The Optical Doppler-Effect and Time Dilation

Imagine a light source which is continuously emitting sharp signals with a period of T_0 (i.e. at a rate equal to $\nu_0 = 1/T_0$) and a receiver which detects them. When the latter is at rest with respect to the emitter it will detect the signals with the same frequency. But when it is moving the observed frequency ν (and the period $T = 1/\nu$) will be different from ν_0 (and T_0). This phenomenon is known as the *Doppler-effect*.

Assume that the emitter and the receiver recede from each other with the constant velocity V (and both are inertial frames). Then the ratio ν/ν_0 is smaller than 1 and, according to the equivalence of the inertial frames, its value is the same regardless of whether the emitter or the receiver is taken to be at rest.[3]

[3] Note that in acoustics the propagation of sound is influenced, beside the motion of the emitter and the receiver, by the state of motion of the medium too.