

Ioannis K. Chatjigeorgiou

# Dynamic Behavior of Pipelines for Marine Applications

---

# **Synthesis Lectures on Ocean Systems Engineering**

## **Series Editor**

Nikolas Xiros, University of New Orleans, New Orleans, LA, USA

The series publishes short books on state-of-the-art research and applications in related and interdependent areas of design, construction, maintenance and operation of marine vessels and structures as well as ocean and oceanic engineering.

---

Ioannis K. Chatjigeorgiou

# Dynamic Behavior of Pipelines for Marine Applications

 Springer

Ioannis K. Chatjigeorgiou  
School of Naval Architecture and Marine  
Engineering  
National Technical University of Athens  
Athens, Greece

ISSN 2692-4420                      ISSN 2692-4471 (electronic)  
Synthesis Lectures on Ocean Systems Engineering  
ISBN 978-3-031-24826-9              ISBN 978-3-031-24827-6 (eBook)  
<https://doi.org/10.1007/978-3-031-24827-6>

© The Editor(s) (if applicable) and The Author(s), under exclusive license to Springer Nature Switzerland AG  
2023

This work is subject to copyright. All rights are solely and exclusively licensed by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors, and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, expressed or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

This Springer imprint is published by the registered company Springer Nature Switzerland AG  
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

*To my Varvaroula*  
*To my Maraki*

---

## Preface

The subject of this book concerns the pipelines conveying fluids giving special attention to marine applications. The main interest is focused on pipelines that are free hanging or suspended between two fixed points. It is evident that relevant configurations execute motions that originate from environmental actions in the wet environment. The material contained in this book emphasizes in the very interesting topic of the dynamic behavior of those structures. This is the reason why the bottom seated pipelines are excluded from the material given that in most of the cases their dynamic behavior is not the primary concern. Marine pipelines should not be confused with the simple pipes or tubes which we use in a daily basis, as they are lengthy flow lines. Their length can reach several kilometers, while the order of magnitude of their diameter could be one meter or more. The most important application of marine pipelines is the production and the convey of hydrocarbons in a liquid or an aerial phase. As far as their operation is concerned, it is greatly influenced by the continuous action of the prevailing environmental conditions. In essence, marine pipelines should be considered, and studied, as continuously moving structures operating in an extremely harsh environment.

The *dynamic behavior* considers the time-dependent response of the pipelines under external effects, which are basically the environmental actions, water waves, sea currents as well as the excitations imposed by the floating structure where the pipeline is attached. As a consequence, the motions executed by the pipeline will affect its internal loading condition, i.e., the axial tension, the shear forces, the bending moments and, finally, the torsion. Accordingly, the internal loads will cause the development of stresses which could compromise the pipeline's structural integrity and hopefully lead to failure. Nevertheless, it should not be assumed that a pipeline that operates in sea and is subjected, even, to extreme loading conditions, will fail immediately. The continuous displacements which imply a variable, mainly oscillatory, kinematical behavior lead unavoidably to fatigue and strength reduction. We must also remember that those structures are designed to be installed with extended anticipated operational life. Their design, manufacturing, assemblance and installation are laborious and time-consuming processes, which are connected with enormous costs and expected benefits. Also, the issues associated with the protection of the environment are extremely important. Hence, it is easily understood that the failure

of a marine pipeline, even in a long term, due to fatigue, may have dramatic consequences which are not limited only to economics.

The increased requirements of offshore industry in the hydrocarbons production sector, have made the study of the dynamic behavior of marine pipelines an indispensable part of the process required from conceiving the project, up to the initiation of the operation of a pipeline. This is due to the fact that the production of hydrocarbons is moving to deeper waters making the dynamics of pipelines the prevailing factor of their operation. In addition, the conventional static or quasi-static design and analysis methods, that are based on a plethora of simplifying assumptions, lead to coarse estimations of the actual conditions. Finally, the rules and the regulatory codes that rely on safety factors, are, a fortiori, overconservative approximations, often orientating to erroneous assessments.

The primary task writing this book was to include all “familiar actions” that influence the dynamic behavior of marine pipelines. “Familiar actions” are the phenomena that the experience and the practice highlighted as essential, although their effects are not equivalent in all configurations. Marine pipelines are typically unique structures and are designed to be adapted in one of a kind floating structure. Their design and the utilization are connected, one way or another, with the application and the site of installation. Besides, this is the reason why they are very expensive structures. Perhaps, the advancements of offshore industry may reveal the occurrence of additional phenomena that could be judged as important for the design and the operation of pipelines leading to the enhancement of the “familiar actions”.

The study of the individual subjects discussed in this book is not exhaustive. That would be unfeasible in the frame of a single volume. In fact, each subject considered herein constitutes a specialization domain of engineering discipline. The main goal by writing this book was the incorporation of the most important subjects associated with the dynamic behavior of marine pipelines conveying fluids in the marine environment.

The topics analyzed in the chapters of the book are based on advanced mathematical formulations. The primary task was not to deliver a simple descriptive text. In contrast, effort has been made to correlate the contained mathematics with the reliable estimation of the behavior of the physical subject and to understand its details through the extensive analysis and the use of advanced tools for the solution of the underlying mathematical systems.

I aspire and wish that the topics examined in the book, to be used in more areas of engineering and applied mathematics as well. Besides, pipelines are factually beams which have numerous applications in science and technology. The material contained in this book is based, a fortiori, on my 30 years avocation with the concerned subject, as a Ph.D. candidate, Dr.-Eng. and, finally, a Professor in the School of Naval Architecture and Marine Engineering of the National Technical University of Athens. During this journey however, I was not alone. I was blessed to have distinguished scientists as my teachers and mentors. I am grateful to them for teaching me, helping me and trusting me as the



continuator of their work. This book is dedicated to them as well. Finally, special thanks goes to the Architect Mrs. Evita Fanou who edited several drawings of the book in a very professional manner.

Athens, Greece  
October 2022

Ioannis K. Chatjigeorgiou

---

# Contents

<b>1</b>	<b>Introduction</b>	1
1.1	Land Based and Marine Pipelines	1
1.2	Configurations and Operation of Marine Pipelines-Risers	2
1.2.1	Operation	2
1.2.2	Riser Configurations	3
1.3	Pipeline Design Through Analysis (DTA)	4
1.4	Dynamic Analysis as a Tool for Efficient Pipeline Design	6
<b>2</b>	<b>Nonlinear Dynamics of a Curve in Three Dimensions</b>	9
2.1	General Remarks	9
2.2	Fundamental Formulation	10
2.2.1	Material Derivatives in Time and Space	13
2.2.2	Euler Angles	14
2.2.3	The Kinematics of an Element Along the Distorted Space Curve	17
2.2.4	Balance of Moments	18
2.2.5	Equations of Continuity	20
2.2.6	Distributed Forces	21
2.2.7	The Governing Dynamic System	21
2.3	Static Equilibrium	23
2.4	Two-Dimensional Dynamics	26
2.5	Three-Dimensional Dynamic Behavior of an Originally Plane Beam with no Torsional Effects	28
2.6	The Linearized System	31
2.7	Decoupled In-Plane and Out-of-Plane Vibrations of an Initially Plane Beam	33
2.8	The Last Simplification: Free Bending Vibrations of Horizontal or Vertical Beams	35
2.8.1	Free Elastic Vibrations of Rods	35
2.8.2	Free Elastic Vibrations of Beams	36

2.8.3	Free Vibrations of Vertical Clamped Beams Under Tension, Using Perturbation Analysis .....	37
	Literature .....	43
<b>3</b>	<b>Distributed Forces—Hydrodynamic Loads</b> .....	45
3.1	Description of the Distributed Forces .....	45
3.2	The Morison Equation .....	48
3.2.1	Introductory Information .....	48
3.2.2	The Drag Forces .....	50
3.2.3	A Further Significant Use of Morison’s Equation .....	51
3.3	Variation of the Inertia and the Drag Coefficients .....	54
3.4	Height Frequency Oscillations due to Vortex Shedding—The Vortex-Induced-Vibrations Phenomenon .....	58
3.4.1	Brief Introduction .....	58
3.4.2	Description of the VIV Phenomenon .....	58
3.4.3	The Lock-In Condition .....	60
3.4.4	Motions at Lock-In—The Van der Pol Oscillator .....	62
	Literature .....	67
<b>4</b>	<b>Inner Flow Models</b> .....	71
4.1	Introduction .....	71
4.2	Mathematical Formulation of Pipeline Dynamics with Internal Flow .....	73
4.2.1	Dynamic Equilibrium of the Pipeline’s Element .....	75
4.2.2	Dynamic Equilibrium of the Fluid’s Element .....	76
4.3	Governing Differential Equations .....	77
4.3.1	Equations of Motion .....	77
4.3.2	Distributed Forces .....	78
4.3.3	The Final System .....	79
4.4	The Potential Flow Model .....	81
4.5	Steady-State “Slug Flow” Model .....	84
4.5.1	Two-Phase Steady-State Model .....	84
4.5.2	The Cross Section .....	88
4.5.3	The Friction Forces .....	88
4.5.4	Hold-Up in the Liquid Slug Zone .....	89
4.5.5	The Steady Translational Velocity of the Slug Unit .....	90
4.5.6	The Bubble Velocity .....	90
4.5.7	Liquid’s Velocity in the Slug .....	91
4.6	Combining Pipeline Dynamics with the Inner Steady-State Two-Phase Flow .....	92
4.6.1	Slug Flow Calculations .....	92
4.6.2	Pipeline Dynamics with Internal Slug Flow .....	95

4.7	Unsteady Two-Phase Flow .....	98
	Literature .....	100
<b>5</b>	<b>Linear and Nonlinear Dynamics of Pipelines .....</b>	<b>103</b>
5.1	General Information .....	103
5.2	Free Vibrations of Slender Structures .....	104
5.2.1	Free Vibrations of a Straight, Hollow, Weightless Beam Assuming an Internal Flow .....	104
5.2.2	Free Vibrations of Catenary, Cylindrical Pipelines .....	107
5.2.3	Free Vibrations of Catenary Pipelines with Intermediate Buoyancy Sections .....	114
5.3	Solution Method that Relies on Expansions of Eigenfunctions .....	116
5.4	Solution of Nonlinear Systems in the Frequency Domain .....	122
5.4.1	Introduction—The Fast Fourier Transformation (FFT) .....	122
5.4.2	Nonlinear Dynamics of Catenary Pipelines—Solution in the Frequency Domain .....	124
5.5	Dynamics of Pipes with Nonconventional End Conditions .....	135
5.5.1	A Robin-Type Condition .....	135
5.5.2	Statics .....	137
5.5.3	Dynamics .....	137
	Literature .....	142
<b>6</b>	<b>Local Discontinuities—Buoyancy Devices .....</b>	<b>145</b>
6.1	Introduction .....	145
6.2	Buoys Attached Along the Pipeline .....	146
6.2.1	General Information .....	146
6.2.2	Point Connections of Buoys .....	149
6.2.3	Drag Forces .....	150
6.2.4	Added Masses .....	151
6.2.5	Modification of the Dynamic Behavior .....	153
6.3	Damping Due to the Motions of the Pipeline in the Wet Environment ...	156
6.3.1	Introduction .....	156
6.3.2	Calculation of Damping via the Linearized Problem .....	157
6.3.3	Calculation of Damping in the Time Domain .....	160
	Literature .....	165
<b>7</b>	<b>Extreme Loading—Dynamic Buckling .....</b>	<b>167</b>
7.1	Extreme Loading .....	167
7.2	Mathematical Formulation .....	169
7.3	Series Expansion .....	171
7.4	Mathieu Equations .....	173

---

7.5	Hill's Equation .....	176
7.5.1	The General Form .....	176
7.5.2	Floquet Theory .....	177
7.6	Identifying Instabilities via Perturbation Analysis .....	179
7.6.1	Perturbation Expansions .....	179
7.6.2	Identifying Nonlinear Resonances .....	183
7.7	Dynamic Buckling in Catenary Pipelines .....	185
7.8	Loss of Restoring Due to Tension Cancellation .....	186
	Literature .....	192



## 1.1 Land Based and Marine Pipelines

The physical subject becomes explicitly obvious by the lexical definition. Nonetheless, a distinction must be made between the pipelines which are manufactured to operate in the marine environment and the land-based pipelines. The land pipelines are structures of great length that cross the soils of several countries. They can be braced onto the soil by means of proper footholds, or they can simply lay on it, or even they can be buried. Dynamic effects on land pipelines are unusual given that relevant impacts could be induced by scarce physical phenomena, such as earthquakes. Externally imposed dynamic loads could also be induced by the wind although the time variation of the wind velocity could be considered relatively insignificant. Finally, possible catastrophic phenomena in tubular members in the air, such as galloping in bridge cables, could be neglected because of the large diameter of the pipelines and the relatively small length of the segments between successive braces onto the soil.

In contrast, marine pipelines are used to convey fluids through the sea, seeking for routes alternative to those passing through the land. Compared to the latter, marine pipelines have both advantages and disadvantages. The most important disadvantage refers to the dynamic loads due to the harsh and unpredictable operating environment.

The benefits associated with the use of marine pipelines are connected with geopolitical and environmental issues. With regard to the former, marine pipelines, contrary to the land lines, do not pass through different national soils, avoiding unwanted international political conflicts. As far as the environmental issues are concerned, it is noted that marine pipelines are soundless, environmentally friendly solutions, which do not cross sensitive land sites. In contrast, land pipelines should run enormous distances together with the manufacturing personnel. They should pass through protected areas such as zones of natural importance, rivers, forests, farms. In addition, their manufacturing could generate

issues of legal interest given they could pass through proprietary surfaces. The installation of land pipelines requires strict and comprehensive studies for environmental impacts. Those studies should be approved by local and governmental authorities to allow installation. Finally, from the economical point of view, although marine pipelines require increased initial investment, they can reduce the long-term cost. The additional cost associated with land pipelines is mainly due to the personnel and the effort required for the maintenance of the intermediate compression stations that are necessary to achieve the required fluid pressure. Land pipelines rely on compression stations installed nearly every 200 km. Finally, an additional significant advantage offered by the marine pipelines is the beneficial effect of the hydrostatic pressure which allows the transportation of increased volume of fluid.

Marine pipelines can be used either for production (production, transportation and storage of hydrocarbons) or explicitly for transportation, while in many cases constitute parts of integrated channels assembled by both land and marine sections.

The pipelines which are used solely for transportation are laid on the seafloor connecting two land points. They offer an alternative method for fluid transportation against both land pipelines and surface ships such as tankers and liquefied natural gas/LNG carriers. The most important parameter that influences the laying process and the consequent operation of the pipeline is the installation depth. Also, an essential factor is the bottom topography, i.e., whether is rocky or sandy, if there are sudden changes in the depth, due to the underwater trenches, or if the bottom is characterized by abnormal topography allowing the formation of free span segments between elevated regions. In the latter case, the corresponding segments will be subjected to dynamic loads, for instance due to underwater currents or the action of waves in shallow waters.

The production pipelines, which are usually referred to as “risers”, connect the source of the material (oil or natural gas) with the floating production and storage station. In deep waters, which are also connected with the contemporary technological developments, the floating stations on the free surface are moored by means of multi-line mooring systems. Taking into account that the present technological developments allow production at depths around 10,000 ft, it is easily understood that the length of the suspended part of a production pipeline will be even bigger. This is due to the fact that in these depths, it is preferable to install catenary structures instead of fully perpendicular.

---

## **1.2 Configurations and Operation of Marine Pipelines-Risers**

### **1.2.1 Operation**

Contrary to land pipelines, as well as to bottom seated marine pipelines which are focused explicitly on fluid transportation, the production risers have a great part that is fully suspended, while the remaining part is laying on the seabed. It is evident that the section

of the pipeline that is subjected to dynamic loads, which accordingly induce relevant responses, is the suspended part.

In fact, they constitute fluid transportation channels from the seabed to the floating production infrastructure. Also, they can be used for the reverse process, i.e., for fluid transportation from the floating structure to the seabed. Riser pipelines are usually insulated with special wraps to withstand to corrosion and to the temperatures close to the seabed. Finally, in terms of the distortional behavior, they can be either rigid or deformable.

### 1.2.2 Riser Configurations

There is relatively a large variety of riser configurations that can be encountered as *attached risers*, *pull-tube risers*, *steel catenary risers*, *top-tensioned risers*, *riser towers*, *elastic risers* and *drilling risers*.

The configuration of *attached risers* was the first kind of riser that was developed, and they are explicitly used in bottom seated platforms (gravity or jacket). They are firmly attached to the side of the platform connecting the seabed with the deck above the free surface. They are usually manufactured in sections, while the part of the pipeline that is closer to the bottom is connected with a flow line or an extraction pipe. The rest of the parts are firmly attached to the side of the infrastructure, and the upper section is connected with the processing equipment of the transported material.

*Pull-tube risers* are also installed in bottom fixed structures. They are used as pipeline channels or channels of flowlines, and they are firmly connected to the center of the structure. In those configurations, a large diameter pipeline is pre-installed on the platform that is situated above the free surface. Subsequently, a wire cable is attached to the pipeline or to the flowline on the bottom. The wire cable is attached to the platform bringing together the pipeline or the flowline.

*Steel catenary risers* (SCR) are named after the catenary curve that characterizes their geometry. The configuration on the two-dimensional plane of reference is determined by the “catenary equations” in the static position. SCRs are used to connect the seabed with the production platforms, or to connect two floating platforms together. They are used often in tension leg platforms/TLPs, in floating production, storage and offloading/FPSO systems and in spar platforms. They are also used in bottom fixed, gravity or jacket structures, and they are very efficient solutions for hydrocarbon production because of their low cost, while they are very interesting, in terms of their dynamic behavior.

*Top-tensioned risers* are used in tension leg and spar platforms. They are completely perpendicular, and they connect the seabed with the bottom of the floating structure. TLPs and spars are able to perform oblique displacements, due to the action of wind, waves and sea currents. The pipelines are connected with the sea bottom, while their internal loading condition is influenced by the heaving motions of the point of connection



with the platform. Two possible solutions exist for the specific problem, which engages the structural behavior of particular pipelines. The first involves the utilization of motion compensators, which maintain a constant tension force by controlling the elongation or the shrinkage, relative to the motions of the floating structure. Also, buoys could be attached to the external area of the pipeline in order to maintain buoyancy. Accordingly, the upper part of the pipeline is connected to the floater by means of a flexible pipe that has the ability to better distribute the motions of the platform.

*Riser towers* were used for the first time in Girassol infrastructure of Total in Angola, and they prove to be ideal solutions in deep waters. They were built to elevate pipelines in a large height in order to reach FPSOs at the sea water level. The specific design involves a steel tower which bears at its top a large tank to offer buoyancy. The pipelines are located into the tower and extend from the seabed to the top of the tower and the tank. The buoyancy of the tank maintains the tension at the top of the pipelines. The connection between the vertical pipelines with the floating structure is achieved using additional elastic pipes.

*Elastic risers* can be used in several hybrid designs, and they are able to resist in both vertical and horizontal motions, which makes them ideal for floating installations. They were first used as connections between floating installations and metallic production pipelines, and contemporary are used as primary pipelines as well. A variety of elastic pipelines configurations has been proposed that includes both Steep-S and Lazy-S systems (Fig. 6.4). These designs employ moored buoyancy devices that assist to configure the geometry of the pipeline in its plane of reference.

Finally, contrary to the production pipelines conveying hydrocarbons during the production phase, the *drilling risers* convey surface material (mad) during drilling. They are used as temporary connections between the well and the free surface and they prevent the escape of the drilling material to the environment.

---

### 1.3 Pipeline Design Through Analysis (DTA)

The latest developments in marine technology have enabled the use of state-of-the-art design and analysis tools that make use of advanced material and updated design codes. As a result, it is feasible to design pipelines that are able to perform at operational limits and to assess their operational reliability. This design approach is referred to as design through analysis (DTA). According to the DTA technique, numerical methods are employed (e.g., finite elements methods/FEM or finite differences methods/FDM), to simulate the general behavior of the pipeline (static and dynamic), as well as to calculate its local strength. DTA is a two-stage process and is used in a complementary manner to determine the limiting conditions and to optimize a particular design.

The advantage of the use of advanced methods of mechanical analysis is to reduce the possibility of conservative designs, by means of the more accurate determination of

the environmental effects and the loading conditions of the structure. The general design context should be covered by the relevant rules and design codes, which however involve several uncertainties regarding the employed data and the analysis methods. When the structure itself as well as the loading conditions can be accurately described, then the realistic simulations can possibly reveal that some issues associated with the design codes are extremely conservative for a particular design condition. The contemporary employed numerical methods (e.g., FEM) are able to simulate with sufficient accuracy of the actual structural behavior of the pipeline and to allow the adoption of appropriate measures to revise and optimize the design.

Better quality control during the production of the pipeline enables accurate modeling of the material, while the numerical tools for the prediction of its structural condition provide the capability to simulate the lifecycle behavior of the entire pipeline system. In this manner, it is possible to determine properly the most loaded sections, which can be accordingly entered as predictions into the numerical solution codes. Thus, these codes can be enhanced with the capability to determine the prevailing types of failure and the limiting criteria that correspond to a specific design. The failure types and the set criteria can be compared against existing predictions of the available design codes, allowing the review, revision and update of the regulations. Possible uncertainties in the employed data (e.g., random loading from the sea) can be modeled using statistical evidence to calculate the probability distributions as regards the imposed loads and their effects.

The mechanical analysis of marine pipelines relied for a large amount of time on analytical methods, which considered only specific elements of structural systems. Analytical methods herein refer to approximate simplified relations or even empirical correlations, which were used to calculate the particulars of the loading condition and the behavior of the pipeline. Nonetheless, to anticipate the modes of interaction between the several sections and, most importantly, to anticipate how the examined structural system responds to situations that involve imposed loading, it is necessary to employ nonlinear dynamic models which are able to specify with sufficient accuracy the loads, the characteristics of the material and, finally, the structural configuration of the pipeline. For the calculation of the nonlinear dynamic behavior of pipelines, the current technological demands dictate the use of advanced prediction models, which should be based on robust and detailed mathematical formulations.

In addition, the simulation of the general dynamic behavior and the internal loading condition of the pipeline is required, because, in most of the cases, the design parameters are explicitly related to a specific installation, namely a detailed configuration and specific environmental conditions. For instance, the dynamic simulation for a pipeline that is subjected to cycling loading and displacements could show that the general response is satisfactorily stable or even unstable. In the latter case, the adoption of relevant constraints could be required. The simulation of the pipeline's behavior in a realistic environment, obtained through proper measurements, could assist to the identification of the strong points or even the disabilities of a particular design that would allow the adoption and the

employment of safe and economically advantageous solutions. The traditional, and familiar, method that relies on the formulation in two dimensions neglects the all-important three-dimensional phenomena. This simplified approach may lead to either overconservative, or unsafe designs. DTA has demonstrated the importance of the three-dimensional analyses, especially for heavily loaded pipelines that also experience thermal expansion.

Succinctly speaking, DTA involves the following activities:

- preliminary design according to guidelines and applicable codes,
- calculation of the general dynamic behavior through modeling of the entire system,
- simulations of the dynamic behavior applying the maximum anticipated loading and taking into account the expected operational lifecycle of the pipeline,
- identification of areas which potentially could be sources of problems,
- investigation of possible modes of failure and the involved dynamics through detailed numerical analysis,
- development of strategies for minimizing the cost maintaining the required safety level,
- employment of iterative optimization cycles for the design,
- provision of support for the design and the operation of the pipeline.

---

## **1.4 Dynamic Analysis as a Tool for Efficient Pipeline Design**

Dynamic analysis of pipelines, with a special focus on marine applications, refers to the simulation of the behavior of the structure in realistic conditions. The environment within which those structures are operating is, by nature, variable, depending on time varying laws. The simulation should accordingly account for all effects encountered in the operating environment, namely waves and sea currents as well as the consequences induced by their existence. In this context, a proper dynamic analysis should consider, e.g., the influence of phenomena, such as the interaction with the bottom, vortex-induced-vibrations, excitations originating from the motions of the host facility, while responding to the wave action, etc.

The prerequisite of the dynamic analysis is the static equilibrium of the pipeline. In mathematical terms, this is obtained through the formulation (and the solution) of the static problem, i.e., the system of ordinary differential equations that describe the geometrical configuration of the structure and the variation of the internal loading components along it. Evidently, it is assumed that at static equilibrium, no external loading component, which varies dynamically, is applied, with the only exception of possible steady current and when applicable, second-order steady wave loading. The static equilibrium is accordingly used as the initial condition for the dynamic problem, and it roughly describes the average position of the structure. The pipeline returns to this position when the dynamic loading is canceled.

A complete and comprehensive dynamic analysis of a pipeline typically starts with the consideration of the free-vibrations problem that provides the governing eigensolutions. The latter deliver the eigenfrequencies (or the natural frequencies) and the eigenfunctions (the modes of vibration) that correspond to the natural frequencies. The natural frequencies and the mode shapes are infinite, given that a pipeline is formulated as a continuous dynamic system.

The formulation of the free-vibrations problem (or the eigenvalue problem) relies on the homogeneous system of equations of dynamic equilibrium, namely without considering external effects (e.g., loading) and the nonconservative components (e.g., in the case of pipelines in the marine environment, the drag forces). The homogeneous system is assumed to obey to harmonic motions of a given frequency that is originally unknown and is accordingly calculated to specify each of the infinite natural frequencies. The mode shapes associated with the natural frequencies show the modes of vibration along the slender pipeline.

The solution of the eigenvalue problem dictates knowing the statics of the pipeline which is, in fact, the input to the dynamic problem, while all static components appear in the mathematical system of dynamic equilibrium as functions of the spatial coordinate that takes values along the length of the pipeline. This creates some problems as the resulting complications prohibit the derivation of analytical, closed-form, solutions, and consequently, the sought calculations should rely on efficient numerical analysis.

The treatment of the eigenvalue problem is of paramount importance for determining the details of the pipeline dynamics. Knowing the natural frequencies allows detecting the regimes of possible resonances, both linear and nonlinear. The former are significantly important, as linear resonances are more possible to occur. Nevertheless, nonlinear resonances, which occur at algebraic combinations of the natural frequencies, may induce unwanted impacts associated with magnifications of the response components. Further, the mode shapes, aside from the fact that they deliver an indication of the response, provide the means for the complete solution of the dynamic problem. In particular, proper solutions can be formulated in the form of, infinite, linear superpositions of the mode shapes. This method is quite efficient as the mode shapes satisfy, by default, the boundary conditions. Note that relevant analyses are typically considered as two-point boundary value problems. The mode shapes, given that they satisfy the boundary conditions, are orthogonal scalar functions of the spatial coordinate of the pipeline. The validity of the property of orthogonality can be shown either analytically or numerically depending on the complications involved in the static configurations. At any rate, the employment of this property allows the simplification of the original system of partial differential equations to a system of ordinary differential equations, which can be treated effectively using popular methods of numerical analysis.

Dynamic simulations should account for all possible effects originating from the marine environment, e.g., wave exciting forces, nonlinear drag forces, cycling impact