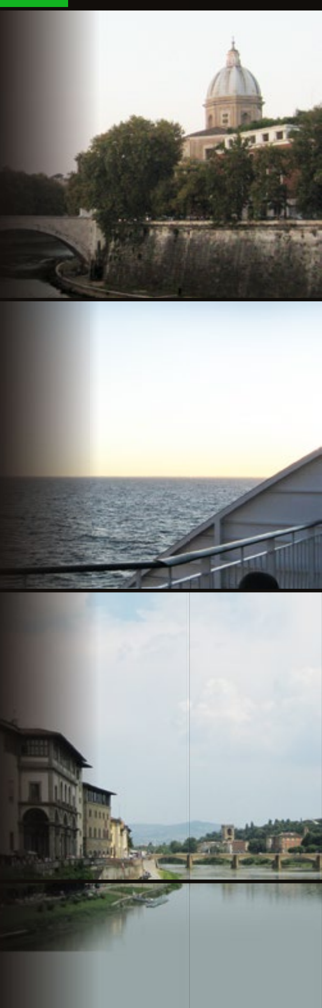
Mathematics, Culture, and the Arts

Emily Rolfe Grosholz

Great Circles

The Transits of Mathematics and Poetry



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The Transits of Mathematics and Poetry

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This book is dedicated to my friends Paula Deitz and Marjorie Lee Senechal, with admiration for their work as writers and editors.

Preface

Philosophy is inherently interdisciplinary, because it can reflect on the conditions of intelligibility or meaningfulness of almost anything. A philosopher is thus especially well suited to explore connections among disciplines and explain the import of those linkages. As a philosopher of mathematics, I have urged the use of historical case studies as a complement to logical investigations, in my co-edited volume *The Growth of Mathematical Knowledge*, and then in two monographs: *Representation and Productive Ambiguity in Mathematics and the Sciences* as well as *Starry Reckoning: Reference and Analysis in Mathematics and Cosmology*. While I often draw my historical examples from classical antiquity or the seventeenth century, I also write about algebra, topology, number theory, and logic in the twentieth century, as well as mathematical models in biology, chemistry, and modern cosmology. At the same time, I am a poet and literary critic who tends to approach poetry philosophically. Many of my essays and poems have appeared in *The Hudson Review*, where I have served as advisory editor since 1984, as well as in the *Sewanee Review*, *PN Review*, *Poetry Magazine, Prairie Schooner, Plume, Journal of Mathematics and the Arts*, *Think* Journal, *Able Muse*, *Literary Matters*, *San Diego Reader*, *Journal of Humanistic Mathematics,* and others. Because mathematics and poetry seem inherently and problematically disjoint to many people, I have spent the last 40 years searching for ways to think the two disciplines together, and this book organizes my reflections. In general, I argue that poetry stands in the same relation to the humanities as mathematics stands to the sciences. Both disciplines generate insight by highly concentrated modes of expression in which formal structure is just as important as content in the creation of meaning. Thus, in these disciplines, close attention to form, and to forms in combination, is essential to the interpretation of texts; the philosopher must also balance an appreciation of timeless form with an historian's sense of the temporality of proof and discovery as human actions, and the changing cultural context of poems.

Human understanding hovers between the timeless realm of concepts, propositions, and arguments that stand in inferential relations tracked by logic and rhetoric, and the historical realm in which discoveries are made and projects framed on the basis of earlier results or events, and in light of as-yet-unsolved problems. A name or concept pulls something that exists out of the flux of time, and by imposing the universal on the particular gives it a kind of local immortality. Arguments organize thoughts so that they can be rehearsed and examined; narratives organize human actions so that they can be relived and their meaning reconsidered. A scientific experiment on the one hand and a theatrical drama on the other deliberately represent situations both as having happened once—physically or dramatically real—and as meant to be repeated—universally true. In mathematics and poetry, the tension on this duality is especially strong. The study of mathematical knowledge and poetic knowledge is therefore central to philosophical epistemology; the dialogues of Plato testify to this. My work in the philosophy of mathematics takes its inspiration from the twentieth century European tradition that begins with Poincaré, Hilbert, and early Husserl; I explain mathematical rationality as an interplay between logical necessity and historical contingency. My literary work locates human action and utterance at the crossroads between the constraints of moral law and the fatal accomplishments of history, and the free play of artistic form and anarchic will, always imagining "what if...?"

And as Keats reminds us, truth is not the whole story: we must also acknowledge the importance of beauty. Art (including poetic art) and mathematics characteristically generate beautiful forms that express human action on the one hand, and on the other hand the stable systems and dynamic processes of nature. As a philosopher of mathematics, I have developed a theory of "hybrids," which examines the growth of mathematical knowledge at the intersection of heterogeneous domains. When one domain is brought in to augment the resources of another, each with its own tradition of representation, the result is the combination, superposition, and metamorphosis of a variety of modes of representation that often produces new mathematical entities. This situation in itself calls into question standard accounts of theory reduction and moreover presents striking examples of constructive ambiguity. What William Empson says so brilliantly in his *Seven Types of Ambiguity* about the poets' exploitation of structured ambiguity offered by the semantic field of dictionary definitions also holds true for the mathematician. The ellipse in Proposition XI of Newton's *Principia* must be read as a trajectory, as a figure derived from Euclid and Apollonius, as a dynamic nexus determined by a central force, and (after Leibniz, the Bernoullis, and Euler) as the solution to a differential equation; its internal articulation must also be read as both finite and infinitesimal. The proof of the proposition hinges on Newton's exploitation of the controlled ambiguity of the ellipse.

In poetry, the line embedded in stanzas and organized by rules of meter and rhyme (or studied violations of them) creates a formal counterpoint of superimposed periodicities that deepens and complicates what it means. Thus in a poem a thought is suspended at the end of a line by the white margin even if it is also continued by enjambment and grammar to the next line, or by the logical structure of an argument to the next stanza. This formal constitution of ambiguity, which exploits aural patterns of repeated phoneme and accentual beat, grammatical structure, poetic lineation, metrical conventions, and logical forms, shows that a poem is not just a string of words but a two-dimensional array that composes a rich plurality of modes of representation. When I interpret action, character, and image in (for example) poems of Keats and Housman, I discover that this constructed ambiguity often mirrors the ambiguity of human intention: whenever we act, we are aware of what we might have chosen but in fact did not choose, and those unrealized possibilities remain with us as part of the meaning of what we did. We act at the crossroads of necessity and freedom, and of the visible and the invisible.

This book explores the many ways in which mathematics and poetry may enrich and inform each other directly, but also how they exhibit important analogies as they shape human existence. I did my best to explain the mathematical contexts "all the way down" for a general audience, as I have learned to do in the classroom, and also to evoke the complexity and allure of poetry for students of mathematics and the sciences. Poetry is to the other genres of literature as mathematics is to the sciences: each provides a home ground from which everything begins, the distillate, the seed. The proportion also suggests relations that take us beyond discourse: mathematics is the middle term between natural systems and scientific theory, and poetry is the middle term between human life as we live it and the historical, theological, philosophical, and, yes, even scientific theories we use to understand its meaning, and to confer meaning upon it.

University Park, PA Emily Rolfe Grosholz

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Part I
A Life in Mathematics and Poetry

Chapter 1
The House of Childhood

In his book *The Poetics of Space*, Gaston Bachelard talks about the house of childhood, the house we never leave because at first we live in it, and afterwards it lives on in us. The house of childhood organizes our experience, first of all determining inside and outside, and then offering middle terms: the front porch and its steps are a middle term between the house and the town, while the back yard and garden are a middle term between the house and the wild. It organizes what is far away, both because we measure "away" by how far it is from home, how many hours or days of travel. Moreover, the windows of the house let in the distances, the dwindling train tracks, river or road, the fields and forest, even the cloudy-blue or starry heavens: they are set squarely on the walls within the window-frames, as light comes through and we see what is outside. The infinite or limitless is framed or mapped, a kind of compactification. (We'll look into middle terms and compactification later.) The house also organizes time, for what lives in the basement or the attic? We ourselves do not eat or sleep or socialize there, although those rooms are part of the house: it is where we put the past, the discarded and the treasured. It is also where sometimes we put the might-have-been, the unrealized possibles. So, finally, the house invites playing. The play room (the nursery, as one used to call it) with its gate, and the fenced-in part of the back yard that displays and bars the wild, are enclosures where the toys are kept and where children go about their business of imitating the adult activities of building and furnishing houses, admonishing and encouraging their dolls, rushing about on small basketball courts and soccer pitches, setting forth amidst the ceremonies of departure and return, celebrating holidays, those middle terms between time and eternity that punctuate and organize the human year (Bachelard 1969).

This brings us back to a song. It is, to my mind, the most beautiful of the poems in Robert Louis Stevenson's *A Child's Garden of Verses*, "Where Go the Boats?" (Stevenson 1941). As a child, I owned a golden vinyl record with this poem on it, recorded as a song: thus I learned it by heart and of course I can still sing it, and very often do. Many of the poems I know by heart I learned as songs, including and especially poems in other languages.

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Dark brown is the river.
Golden is the sand.
It flows along forever,
With trees on either hand.

Green leaves a-floating,
Castles of the foam,
Boats of mine a-boating,
Where will all come home?

On goes the river
And out past the mill,
Away down the valley,
Away down the hill.

Away down the river,
A hundred miles or more,
Other little children
Shall bring my boats ashore.

The house where I grew up was not located by a river, though there was a stream I loved in the woods, about a twenty-minute walk away, flanked by a magic green door built into a hillside. I seemed to revisit that place decades later at the Paleolithic caves of Altamira, Spain, which one entered through a green door, directly into a hillside; then, about a decade after that, it appeared again whenever I took my second son to painting lessons in the nearby town of Lemont, behind another green door perched on stairs overlooking the banks of our local small river, Spring Creek (the finest wild trout stream in Pennsylvania!).

My old house at 2 Forrest Lane, Strafford, Pennsylvania (Photograph by Frances Skerrett Grosholz)

However, earthbound as it was, my childhood house was close to two other kinds of stream. Just one block down the Old Eagle School Road (the Old Eagle School was founded in 1788, and lies next to a Revolutionary War graveyard), ran the Main Line railroad tracks. All day long we heard the train whistles, and the Doppler Effect lowering the pitch, my first, aural introduction to the sorrow of the Red Shift: why are all those galaxies leaving us? Physics tells us that sound is first of all waves, propagating in the medium of the air, with a certain frequency (how quickly the crests of the waves move past a fixed point) and wavelength (the distance between those peaks). Light too is propagated as waves. The Doppler Effect is produced by a moving source of waves relative to a fixed observer: there is an apparent upward shift in frequency for the observer towards whom the source is approaching (blue shift) because the crests of the waves seem to go past more quickly, and an apparent downward shift in frequency for the observer from whom the source is receding (red shift), when the crests of the waves seem to go past more and more slowly. This effect has been discerned in the light of galaxies.

In the early twentieth century, Henrietta Swan Leavitt, an astronomer working at the Harvard College Observatory, discovered the relation between the absolute luminosity and the period of ‘Cepheid variable stars,’ which regularly wax and wane in brightness; they then became the ‘standard candles’ that first allowed astronomers to measure correctly the distance between Earth and very distant galaxies. In 1912, Vesto Slipher, an astronomer at the Lowell observatory in Flagstaff, Arizona (since deserts are the best places for telescopes), used spectroscopy to detect the red shift of the light emitted from distant galaxies, and Edwin Hubble in 1929 put these results together to formulate what is now called Hubble’s Law, demonstrating that most galaxies are moving away from us: the universe is expanding. (We’ll meet these people again, along with Einstein and the Belgian priest Georges Lemaître, in the cosmological poems discussed at the end of this book.)

In the other direction from my house, a block down the Old Eagle School Road, lay the Lincoln Highway, one of the first transcontinental highways in the United States. Route 30, as it was designated under the auspices of the United States Numbered Highway system established in 1926, ran all the way to San Francisco, and when my parents returned from California where my father had served in the Navy during the Korean War, that was the route they came home on. We had a painting of the cypresses “the sailor wind/ties into deep sea knots,” (as Robinson Jeffers wrote) at Point Lobos over our fireplace, and I retained a few fugitive memories of California and the long trip back home, so for me that road always led to California, as well as Exton (where the best ice cream place was), Downingtown (summer camp lay on its outskirts), and Lancaster (where the Amish people at the Farmers’ Market came from), points west that seemed far, far away. On my first road trip in high school, I drove my friend Jackie Dee past Lancaster, north to the Ephrata Cloister—which was like going to eighteenth century southwest Germany, as I later discovered—and felt that I had achieved adulthood, navigating past the Pillars of Hercules into unknown waters. But in this chapter we must return to early childhood.

As Tolkien wrote, in one of my beloved books, *The Hobbit*, “The road goes ever, ever on” (Tolkien 1937). I discovered *The Hobbit* in the back of my fifth grade classroom, long before anyone else I knew had heard of Tolkien, so for a while it was my private world, far over the Misty Mountains cold. That was the same year Jackie and my cousin Trish Grosholz and I and some other friends spent every recess writing and rehearsing a play of Louisa May Alcott’s *Little Women*, which we put on at the end of the year. (You might recall that Jo March did most of her reading in the attic of her house.) So the Lincoln Highway set up a dialectic with my house, not least because Point Lobos was over the fireplace and my mother’s most romantic, and often repeated, memories were of California and Hawaii: she was never able to travel much during most of her short life, except to the New Jersey beaches and to New England where she went to college and still had friends. Her stories were the other side of my father’s silences, though he too had a trove of stories, set pieces with all the bitter absurdity of those in Joseph Heller’s *Catch-22* (Heller 1961). Drafted in World War II, and then again in the Korean War, my father, Edwin DeHaven Grosholz, spent 7 years of his life crossing the great Pacific again and again in destroyers and tankers, seeking refuge from his terror and displacement in alcohol, at sea. And though he made it back home, like Odysseus, he was often there but not there, sitting in his armchair reading through tome after tome of Naval History and smoking the cigarettes that eventually bore him away again.

Geometry starts with the house and field and town center, as we find it in Euclid’s *Elements*, for Euclidean geometry is the study of *figures*, finitely delimited and delimiting. Here is a sampling of his definitions, from Book I of the *Elements*. “3. The extremities of a line are points. 4. A straight line is a line which lies evenly with the points on itself. 5. A surface is that which has length and breadth only. 6. The extremities of a surface are lines. 13. A boundary is that which is an extremity of anything. 14. A figure is that which is contained by any boundary or boundaries.” And then he gives us the circle, various triangles, the square, and the oblong (rectangle), the rhombus, and various trapezia (Euclid 1956: 153–154). This is the world of childhood: the yard is a rectangle; the house is a closed figure, a set of rectangles and triangles (the walls and roof) hemming in a cuboid; the lane is a bounded straight line; the center of town is a square.

But the road goes ever, ever on. It is a line that continues indefinitely, like the river, like the train tracks that allow the train to glide so quickly and smoothly, in a straight line at a constant speed (inertial motion!) as the train whistle turns into a lament. So Ella Fitzgerald once sang “The Blues in the Night” like no one else, echoing the Doppler Effect, the very sound of departure, in the minor key and falling inflections of the melody:

…Now the rain’s a-fallin’
Hear the train a-callin’ “whoo-ee!”
My Mama done tol’ me
Ah-whooee-ah-whooee ol’ clickety-clack’s
Ah-echoin’ back the blues in the night…

The house is a finite figure, straight from the pages of Euclid, and thus a figure of finitude; but the road, or river, is not a figure, for "it flows along forever." Or rather, it is a figure after all, thanks to the linguistic blue shift given to the term 'figure' by the ambiguities of English: it is a figure of the infinite.

Ironically, we see a foreshadowing of the expansion of geometry into the infinite in the seventeenth and nineteenth centuries in the last of Euclid's definitions, which he added to clarify the peculiar status of parallel lines. "23. Parallel straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction" (Euclid 1956: 154). This definition doesn't limit itself to line segments; it involves co-planar lines that are "produced indefinitely" and yet never meet. So we are invited to think about what happens to a line as it goes on and on. In the seventeenth century, Desargues, inspired by optics and a novel theory of perspective, was one of the founders of projective geometry, and proposed that *all* co-planar lines intersect: parallel lines just intersect at infinity, 'the point at infinity.' And Leibniz, following the work of Desargues and Pascal, focused on space itself as an object whose structure is revealed by studying the transformations of figures, and noting what features remain invariant among the transformations: thus one could think of all the conic sections as variants of the circle.

Indeed, in 1679, Leibniz briefly considered the possibility of spherical geometry (the easiest of the non-Euclidean geometries to understand, because navigation on the spherical earth more or less exemplifies it), based on the analogy between all lines in projective geometry intersecting (some in the point at infinity) and all geodesics on a spherical surface intersecting (a geodesic is the shortest distance between two points on a spherical surface and thus the analogue to a straight line in 'flat' Euclidean geometry). However, he veered off in another direction, and left the explicit formulation of non-Euclidean geometry on a surface of constant positive curvature to the Hungarian mathematician Janos Bolyai in the nineteenth century, following upon the work of Euler and Gauss (Chemla 1998; Debuiche 2013). Nikolai Lobachevsky worked out the non-Euclidean geometry on a surface of constant negative curvature around the same time, entirely independently. Bernhard Riemann, building on the work of his teacher Carl Friedrich Gauss, came up with the generalized notion of a 2-dimensional surface (generalizable to *n*-dimensional surfaces) which launches geometry into the realm of topology (Gray 1989).

If only Hitler had been red-shifted or carried away by an eagle, or everyone had decided to stop fighting, in 1940, or 1945, or 1950, and my father could have kept on sailing east past Japan (with all the people in Hiroshima and Nagasaki still there, waving him along) and Korea (with its citizens persuasively renouncing the genocidal Communism of the mid-twentieth century, so that perhaps we wouldn't have had the war in Viet Nam, and would have invented a more human socialism that would now balance and counter the trends of international capitalism), around the south coast of India and then of Africa, and just come back along a geodesic to my mother and me. My mother used to sing me this lullaby, by Alfred, Lord Tennyson.

Sweet and low, sweet and low,
    Wind of the Western sea.
Blow, blow, breathe and blow,
    Wind of the Western sea.
Over the rolling waters go,
Come from the dying moon, and blow,
   Blow him again to me;
While my little one, while my pretty one, sleeps.

Sleep and rest, sleep and rest,
    Father will come to thee soon;
Rest, rest, on Mother's breast,
   Father will come to thee soon;
Father will come to his babe in the nest,
Silver sails all out of the west
   Under the silver moon:
Sleep, my little one, sleep, my pretty one, sleep.

Like a baby on its mother's breast, one can always dream of the unrealized possibles, while remaining grateful for some of the things that materialize and spiritualize reality, like milk. The house governs the poetics of space (inflected by time—and eventually Riemann's geometry provides a model for Einstein's space-time); the road and river govern a poetics of time (inflected by space—for we must all go home again, whether we can or cannot, in fact or in imagination, sooner or later). And what child does not thrill to the romance of departure, which is after all what eventually he or she prepares to do: depart from the house of childhood, aided and abetted by romance.

We might surmise that music is a middle term between poetry and mathematics. It is also true that the visual arts (from painting to architecture) can serve as a middle term, but I discovered that later in life, on my travels around Europe in my 20s and 30s, and here we are still concerned with childhood. Music is first of all a temporal art: a melody must be sung in time. However, sheer time only sweeps everything along and away; so something must temper and counter the sweep for music to be music, and mathematics helps to explain that tempering and counterpoint. The kind of music in play here is song, which requires a poem; mathematics also helps to explain how the flow of forgettable prose becomes a poem that everyone knows by heart, which is passed along from parent to child.

Music requires a scale; the scale that is the basis of European classical music arrays seven distinct notes (C D E F G A B) between middle C and high C, which define an octave. The octave is the interval between one musical pitch and another which is half (or double) its frequency. Sound, as we noted above apropos the Doppler effect, is composed of sound waves that travel through space with determinate frequency and wavelength. The two notes that define the interval of an octave sound the same to the human ear: they ring together, ring true, due to their closely related harmonics. Even before Euclid, Pythagoras (who died ca. 500 BCE) studied the musical scale in relation to the ratio between the lengths of vibrating strings needed to produce them: two strings of the same length have the same pitch and the interval is unison; if one is exactly half the other, the interval is the octave; and if one is two-thirds of the other, the interval is a perfect fifth. So we make our way up the scale from middle C, and miraculously end up at C again—high C; it is a natural periodicity that maps higher notes to lower notes. We can read it vertically, right off the two staves joined by a brace, the top staff marked by the treble clef and the lower staff by the bass clef, with middle C right in between the two staves. Smaller sopranos (I was one), singing in the church choir or the school choir, took our bearings from middle C, despaired of low C (it was left to the baritones) and tried not to be screechy at high C.

However, if we look again at our musical notation, there is a second kind of periodicity which is horizontal and provided not by nature but by convention: the bar or measure, and the time signature, which is also called the meter signature or the measure signature. We all recognize the locution ‘three-four time,’ ¾, which tells the musician that each bar contains the equivalent of three quarter notes: it is the time signature we waltz to, and hear that beat (**dah** dum dum) repeated in every bar. Smaller ballroom dancing class students (yes, I was one of them too) did our best not to trip our partners or to trip over our own feet, as they swept across the floor, on and on, would the music never cease? That’s to say, the time signature imposes a small periodicity on music (only a bar long); but a song requires something more, and that something comes from the poem, another kind of imposed periodicity.

Four-four time presides over many of the hymns in the Episcopal hymnal that I grew up with as a child, and so does the following metrical pattern: iambic tetrameter/ iambic trimeter/iambic tetrameter/iambic trimeter. Here is one of my favorite examples, which begins a well-known hymn written by Isaac Watts; other stanzas will turn up later on.

Oh God, our help in ages past,
Our hope for years to come,
Our shelter from the stormy blast,
And our eternal home.

What’s with all the Greek and Latin? An iamb is a metrical foot (analogous to the musical bar) with two syllables, the first stressed and the second unstressed; this is a good choice for English poets because in the “music” of English we stress certain syllables (a habit inherited from Anglo-Saxon/Old English), and we tend to alternate the stresses as we talk. If you put four iambs together, you get tetrameter, and three iambs, unsurprisingly, trimeter. Five iambs give us blank verse, iambic pentameter! In the composition of a hymn, the alternation of tetrameter and trimeter (united in their iambic natures) is underscored by rhyme: the trimeter lines share one full—though here it is a bit slanted—rhyme (come/home) and the tetrameter lines another (past/blast). These superadded periodicities define the line and the stanza: the word stanza in Italian means ‘room.’

This brings us to another kind of horizontal musical periodicity: groups of notes that are recognizable as a melody. Each stanza in the poem by Isaac Watts mentioned above, when it is sung as a hymn, repeats the melody of the first stanza. (So too, throughout a symphony, the signature theme surfaces again and again, each time re-created differently by different instruments and different harmonies and dissonances, which themselves play on the vertical periodicities of our musical system.) Thus, music and poetry keep the river of time from bearing all things away, by turning it into a house composed of stanzas. Or perhaps it is a houseboat? If you are sitting in a little room on a houseboat, sailing along in a straight line at a constant speed, and singing with your eyes closed, it is hard to tell whether you are at rest or moving: more on inertial motion later.

So music is a middle term between mathematics and poetry. The back yard is a middle term between the house and the wild. The attic is a middle term between today and the past. But what is a middle term? Why is it so important for our human efforts to understand the world, and each other, and to find meaning in life? We have inherited two different versions of a middle term, one from arithmetic and one from logic. We owe to Aristotle, and to Euclid, the useful notion of a middle term, though they are both indebted to Plato's analogy of the Divided Line (Plato, *Republic*, Book VI), which we will encounter later.

In his most famous syllogism, AAA-1, Aristotle formally introduces the notion of a valid deductive argument form, with two premises supporting the conclusion.

All M is P
All S is M

—
Therefore, All S is P

If for S, M and P we substitute concepts that make the premises true, the very form of the argument necessitates the truth of the conclusion. We find this in the *Prior Analytics* I.2, 24b 18-20 (Aristotle 1947). This is an astonishing insight, though like arithmetic it formulates and confirms our ordinary experience, when we argue and count: the modality of necessity arises in human discourse and then demands acknowledgment. One of the important features of the AAA-1 Syllogism is that it exhibits M as the middle term, which brings S (the subject term) and P (the predicate term) into rational relation, thus guiding the investigation of the truth of the standard logical proposition 'S is P' to a search for middle terms.

For Euclid, the middle term is re-conceptualized and re-imagined in the theory of proportions, which links the study of number to the study of geometry in Book V of the *Elements* (Euclid 1956). Greek mathematicians carefully segregated the terms that are paired in a ratio: they must be of the same kind. Thus, numbers are paired only with numbers, line segments with line segments, areas with areas. Moreover, all magnitudes must be finite, since Aristotle viewed infinity as a source of contradiction; thus ratios never involve anything resembling infinitesimals or infinites, as inimical to reason. The theory of proportions, however, allows for the comparison of heterogeneous ratios (pairing different kinds of things). In an expression like A:B::C:D we can substitute numbers for A and B, and line lengths for C and D, and assert a similitude, but not an identity: a proportion, for the Greeks, was emphatically not an equation, and a ratio was not a number. In a case like the one just given, there can be no middle term. The theory of proportions thus allows for the discernment and management of irrational magnitudes, like the square root of 2 discoverable on the hypotenuse of the right triangle whose 'legs' (the two other sides of the triangle) are both 1, but it blocks the investigation of infinitesimalistic reasoning, and makes the treatment of fractions and irrational magnitudes as numbers difficult to conceptualize. Those developments must wait for the seventeenth century, though they are prefigured by medieval insights (Sylla 1984)

However, if all the terms involved in the proportion are the same kind of thing, then a Greek mathematician can investigate the case where A:B::B:C and ask, if we know what A and C are, how can we determine B? Here, in a different sense, B is the middle term bringing A and C into rational relation. So the theory of proportions only partly solves the problem of how to understand disparate things: there are proportions that capture similitudes between (for example) relations between numbers and relations between line segments, but there is no middle term to be found between a number and a line segment. Geometry only comes into novel relation with arithmetic in the seventeenth century, thanks to a broad range of conceptual innovations; one might say that the polynomial plays the role of middle term between arithmetic things and geometrical things, but then the notion of middle term has been strongly revised, and depends on the abstract ambiguity of the polynomial. Does the equation $x^2 + y^2 = 1$ stand for an infinite set of pairs of numbers (x, y), or does it stand for a circle, a certain kind of set of points on the plane? It depends on what field you think the polynomial is defined over: is it $\mathbf{R}^2$? Is it the points on the plane? This raises the question of whether you are willing to identify $\mathbf{R}^2$ with the plane, an identification we make so often that we forget to examine it. In *An Introduction to Differentiable Manifolds and Riemannian Geometry*, William M. Boothby reminds us to be more careful, since $\mathbf{R}^2$ stands for a whole spectrum of possible mathematical items: vector spaces, metric spaces, topological spaces, and finally (but with some qualification) Euclidean space. He cautions that we must usually decide from context which one is intended (Boothby 1975: 1–5) We also forget that the implied identification of $\mathbf{R}^2$ with the plane radically revises the notion of number, a process that begins with Descartes' analytic geometry in the seventeenth century, encompasses prolonged debate about the nature of irrational and transcendental numbers, and attains a certain clarification in the work of Cantor in the nineteenth century, which is then clouded a bit in twentieth century debates over the axioms of set theory. That is, we can think of the polynomial as a kind of middle term between arithmetic and geometry only because it introduces a certain ambiguity into the meaning of both number and space. But the ambiguity is not vicious; rather, it is productive.

Many people hope that mathematics provides a setting where they can escape from the shadow of ambiguity, but ambiguity works its magic in mathematics as well as in poetry (Grosholz 2007, 2016). One of my favorite books of literary criticism is William Empson's *Seven Types of Ambiguity*, where he shows in the context of the English literary canon that artless ambiguity cancels out meaning, but artful ambiguity deepens and enhances meaning (Empson 1946/1966). The English language arises as a marriage between Anglo-Saxon/Old English and Norman French, blessed by lavish sprinkings of Latin. Within a period of less than a hundred years, it emerges with astonishing celerity, rapidity and quickness (see?!) from the households of stranded Norman conqueror-courtiers, who did not bring many women with them, and high-born Old English-speaking ladies who wanted to keep up their social position, in a context where literacy was first of all the business of a Catholic church conducting education and communication in Latin. So it is a marriage both figuratively and literally (Baugh and Cable 2012). Thus we have a particularly rich vocabulary where poets can (consciously or unconsciously) delve into the history of the language, that rich soil where etymologies extend their tangled roots downwards and seek the light upwards, to flower as brilliant ambiguities.

Here is an example, drawn from my reflections when some of my poems were translated into French by Alain Madeleine-Perdrillat (Ostovani et al. 2007). I came to realize in a new way how much linguistic and cultural information lies submerged in my language, and so too in my poems. Often, linguistic information includes knowing the first, second, third and *n*th dictionary meanings of a word, and of course a poet especially plays on this ambiguity that arises within the historical etymology of a word. Because English is such a metamorphic language, there are two or three words for everything and also unlimited opportunities for puns (the funny part of ambiguity), as well as accretions and intersections of meaning. One of the poems in the book I created with the artist Farhad Ostovani (Yves Bonnefoy's favorite collaborator in the last 20 years of his life), *Leaves/Feuilles*, was inspired by the olive trees of the Mediterranean, so often planted on terraces that rise up the steep hillsides, trees that often live more than a millennium.

Farhad Ostovani, for Yves Bonnefoy's translation of Giacomo Leopardi's 'A Silvia,' 2006. Lithograph

An olive tree can live a thousand years,
Drawing its silver leaves and oval fruit
From stony terraces, fretting the wind
In registers of sun-inflected shadow.

But we, my love, who count the terraces
Rising to meet the stories of the sky,
Who cultivate the olive groves, who hear
The interruption in the trees as music

And weep responsive to those minor chords,
Can live only a century, no more.
Although I love you, you are just a man,
And the great silver sun is just a star.

When Alain Madeleine-Perdrillat sent me his first draft of a translation, it made me realize the extent of the ambiguity that I (and English) had stored in the word "fretting."

Un olivier peut vivre bien mille ans,
Tirant ses fruits ovales et ses feuilles argentées
De terrasses de pierre, ajourant le vent
Selon les registres d'ombre ensoleillée.

Mais nous, mon âme, qui nombrons les terrasses
S'élevant vers les étapes du ciel, qui cultivons
Les champs d'oliviers, nous qui entendent
L'interruption dans les arbres comme si c'était musique,

Et qui pleurons, sensibles a ces accords mineurs,
À peine pouvons-nous vivre cent ans, pas plus.
Bien que je t'aime, tu n'es qu'un homme seulement,
Et le grand soleil d'argent n'est qu'une étoile.

I did my best to explain to him in a letter: "This poem is built around the 'conceit' that the olive trees are like musical instruments played by the wind. The wind is like the hand that passes over a harp, or like the hand that touches the strings of a violin and the bow that passes over the strings. Indeed, there is an elaborate pun *(jeu de mots*) on the word 'fret' which I am sure cannot be reproduced in French—but the point is to try to make all the double-entendres support a musical metaphor, as well as an architectural metaphor, as they do in English." Here are all the meanings!

Fret: Any of the ridges of wood or metal set across the finger board of a lute or similar instrument which help the fingers to stop the strings at the correct points. The origins of this word, c. 1500, may be from Old French, *frete*, meaning ring or ferule. There is a Middle English verb, *freten*, which means to bind or fasten.

Fret: An intransitive verb meaning to be regretful or to worry. A transitive verb meaning to corrode or wear away. Those who fret may moan, like the wind in the trees, *accords mineurs*. The origin of this word is the Old English word *fretan*, which means to devour, feed upon, consume, often used of monsters and Vikings! It may be related to Old French *froter*, to rub, wipe, beat, or thrash. After 1200, it takes on a figurative use, meaning to worry, consume or vex, and acquires its intransitive use around 1550: to fret oneself, to fret.

Fret: An interlaced, angular design: fretwork, ornamental work consisting of interlacing parts, especially work in which the design is formed by perforation. Also, any pattern of light and dark. This is captured by the French verb *ajourer*, which means to pierce or perforate, so it is not surprising that the French translator would choose this meaning, since it derives from the Old French *frete* (late fourteenth c.) which means interlaced work, trellis work.

However, by choosing the architectural meaning, one loses the musical and emotional meaning, which are key to the poem. The poet, writing the poem, is fretting: the poem is melancholy. The musical ambiguities continue in the words 'registers,' 'inflected,' 'count,' 'interruption,' 'minor chords.' In English, we often talk of music in terms of numbers: any one of a collection of songs or dances is a number. Numbers in the plural may refer to metrical feet or verses in a poem, and musical periods, bars or measures, or repeated groups of notes. I think I also visualized the terraces, and the stories of the sky, as a kind of grand staff, as on the musical page. Interruption evokes rests, in music: the silences, the rests, play an indispensable role in the creation of music; interruptions play an indispensable role in the music of life. The minor chords, of course, are the melancholy music that the wind plays in the trees, and that life plays in us: "Although I love you, you are just a man,/And the great silver sun is just a star." Even stars are mortal. So here was the final version, from *Feuilles/Leaves*.

Un olivier peut vivre mille ans,
Tirant ses fruits ovales et ses feuilles argentées
De terrasses de pierre, faisant geindre le vent
Selon les registres du soleil et de l'ombre.

Mais nous, mon amour, nous qui comptons les terrasses
S'étageant vers les portées du ciel,
Nous qui soignons les oliviers et entendons
Une musique dans les soupirs des arbres,

Et qui pleurons, sensibles à ces accords mineurs,
À peine pouvons-nous vivre cent ans, pas plus.
Bien que je t'aime, tu n'es qu'un homme
Et le grand soleil d'argent n'est qu'une étoile.

What has happened to the child? Here I turn to the poem "Halfway Down," by A. A. Milne, which I also learned from a brightly colored vinyl record and can and do frequently sing.

Halfway down the stairs
Is a stair
Where I sit.
There isn't any
Other stair
Quite like
It.

I'm not at the bottom,
I'm not at the top;
So this is the stair
Where
I always
Stop.

Halfway up the stairs
Isn't up
And isn't down.
It isn't in the nursery,
It isn't in the town.
And all sorts of funny thoughts
Run round my head.
"It isn't really
Anywhere!
It's somewhere else
Instead!"

It is from his collection *When We Were Very Young* (Milne 1961). I remember that stair, a middle term between the upstairs and the downstairs, between my room and the world outside, a place where no adult would ever sit. I used to sit there too, imagining. It had a family resemblance to my windowsill, where I waited expectantly for Peter Pan to appear on the threshold between my house and Neverland, and to the back of my closet, where I supposed that one day the back wall would become a door to Narnia. Thus the imagination turns ordinary places into a middle term between the house and terrifying, seductive, gorgeous fairyland. "It isn't really/Anywhere!/It's somewhere else/Instead!"

So when I first read Keats' "La Belle Dame sans Merci" and Yeats' "Song of the Wandering Aengus," and, years later, Yves Bonnefoy's memoir *L'Arrière pays* (Bonnefoy 2003), I recognized the protagonists immediately, young mortals entranced by the transient light of a shade, the beam that lights up and makes somber whatever it rests upon: "… a glimmering girl/With apple blossoms in her hair,/Who called me by my name and ran,/And faded through the brightening air," with whom one hopes to "… pluck till time and times are done/The silver apples of the moon,/The golden apples of the sun." Using Stith Thompson's celebrated *Motif-Index of Folk-Literature*, we might classify them with Walter Map's King Herla and Washington Irvings' Rip Van Winkle under F377, "Supernatural Lapse of Time in Fairyland." The combined proximity and inaccessibility of the other world does something odd to temporality; often those who briefly stray into fairyland and then return home find that centuries have passed (Thompson 1955–1958). This might remind you of the ‘Twin Paradox’ that stems from Special Relativity Theory: when one of two identical twins goes off in a spaceship at close to the speed of light, he returns home a few years later to discover that his brother is an old man.

Not only do these tales bend time, they cast a shadow on the real world, the world we think we live in. The shadow is not mere illusion; those moments halfway between, those ambiguous places that confound the usual categories, places where no grownup would ever sit (except perhaps some poets, or some mathematicians), reveal something crucial. Why does Keats’ pale knight-at-arms go on loitering there by the lake, where the sedge has withered and no birds sing? Yeats offers a suggestion in another poem, “The Wild Swans at Coole,” in which he finally abjures Maude Gonne; and so does Shakespeare, in the two great soliloquies Prospero utters in *The Tempest* (Act 4, scene 1 and Epilogue), where he abjures poetry (Shakespeare 1987). In both poems, we have the earth, the sea/lake, the heavens, envisioned upwards, all evoked in heart-breakingly beautiful language, flawlessly metered by one of the two inventors of blank verse in the second case, flawlessly metered and rhymed in the first case: unforgettable. Yet they renounce, making the words in which they renounce unforgettable, un-forego-able. Spinoza exhorts us to see all things *sub specie aeternitatis*: and this is what great poetry does, even or especially when it describes the passage of time, or when it performs loss, since it insists on the absolute value and meaning of what passes and what is lost, whilst expressing and performing its persistence.

Yves Bonnefoy suggests something similar, in his poem “To the Voice of Kathleen Ferrier,” the great English contralto who died tragically in 1953 at the age of 41, which I offer in my own translation (Bonnefoy 2001; Deitz 2013: 106).

All gentleness and irony converged
For this farewell of crystal and low clouds,
Thrustings of a sword played upon silence,
Light that glanced obscurely on the blade.

I celebrate the voice blended with gray
That falters in the distances of singing
As if beyond pure form another song’s
Vibrato rose, the only absolute.

O light and light’s denial, smiling tears
That shine upon both anguish and desire,
True swan, upon the water’s dark illusion,
Source, when evening deepens and descends.

You seem to be at home on either shore,
Extremes of happiness, extremes of pain.
And there among the luminous gray reeds
You seem to draw upon eternity.

And there is the wild swan again, though the woodland paths are dry, drifting “on the still water,/mysterious, beautiful.” The act of renunciation engenders lines of poetry that one can only call immortal, as the poet in reverie returns to the house of childhood. So I end this chapter with a poem published not long ago in the *Hudson Review*, which takes us back to Forrest Lane, between Lancaster Pike and Old Eagle School Road and the Main Line Railway.

Where I Went, and Cannot Come Again

That crabapple tree is gone, that used to blossom, no,
To burst like a low budget, pale pink Vesuvius
Halfway between our back door and the neighbors'
Who spoke Italian when the dark-eyed grandmothers appeared.

So too the ash, whose canopy embowered, overshadowed
The lawn with lunar-eclipse shadows, and the small magnolia
Whose open flowers filled up like bowls of alabaster
With April rain, their lips rimmed gold with pollen.

The dogwood's gone I used to climb, the sailor's mast
Blue spruce that lifted almost to the rooftop,
Even the wind-stunted Japanese pine that slanted
Sideways to shelter daylilies, marigolds and tulips.

Even the late-planted holly bush, that lent an air
Of Christmas to the suite of rhododendrons
That screened the front door, and the Christmas trees
Planted year by year against the uproar of the Lincoln highway.

All of them are gone now, and the house is bare,
Mere office space for the adjacent Catholic parish.
Mother, where's the garter snake once hidden in violets,
Periwinkle, hyacinth, all of them blue, all scattered?

Father, where's the porch you built and screened, each nail
Carefully marked and measured in blue pencil? Where's the girl
Who used to slam those doors in helpless anger, and returns
Now to name the vanished trees and close, more gently, the unopened doors?

The title of my poem is borrowed from A. E. Housman, whose poems were turned into songs again and again, though oddly he seemed indifferent to the musicians. (Grosholz 2008). We will encounter it later: it is there and not there, the land of lost content, the blue remembered hills, and, as the first line announces, the record of an air.

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