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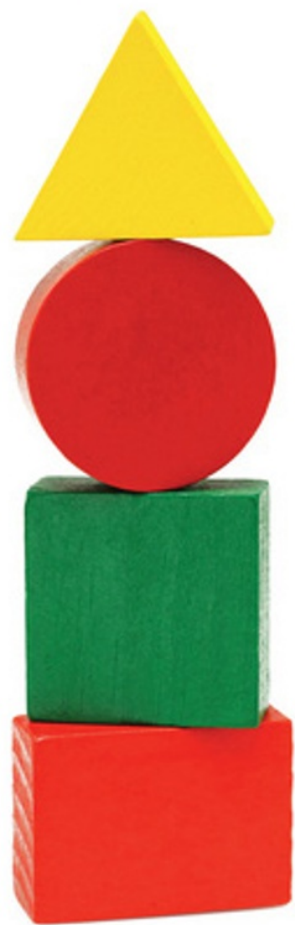
DUMMIES[®]

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- Score high on the mathematics section of the ACT
- Prepare for exam day with helpful tips and strategies
- Grasp algebra, geometry, and trigonometry
- Take three practice tests modeled from questions off the latest ACT tests

Mark Zegarelli

Math tutor and author of SAT Math For Dummies



ACT[®] ***Math***
FOR
DUMMIES[®]

by Mark Zegarelli



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John Wiley & Sons, Inc.

ACT® Math For Dummies®

Published by
John Wiley & Sons, Inc.
111 River St.
Hoboken, NJ 07030-5774
www.wiley.com

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Published simultaneously in Canada

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Library of Congress Control Number: 2011927309

ISBN 978-1-118-00154-7 (pbk); ISBN 978-1-118-07738-2 (ebk); ISBN 978-1-118-07739-9 (ebk); ISBN 978-1-118-07740-5 (ebk)

Manufactured in the United States of America

10 9 8 7 6 5 4 3 2



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Mark Zegarelli is the author of *Basic Math and Pre-Algebra For Dummies* (Wiley), *SAT Math For Dummies* (Wiley), and five other books on basic math, calculus, and logic. He holds degrees in both English and math from Rutgers University and is a math tutor and teacher.

Mark lives in San Francisco, California, and Long Branch, New Jersey.

Dedication

For my good friend David Feaster — as always, with love, laughter, and light.

Author's Acknowledgments

This is my seventh *For Dummies* book, and my seventh positive experience working with a first-rate team of editors who make the process of writing so very pleasurable and productive. Thanks so much to my Wiley editors Chrissy Guthrie, Jessica Smith, and Lindsay Lefevere. More thanks to technical editors Sarah Weaver and Amy Nicklin for keeping me on track, accurate, and finely tuned to the needs of readers who are studying for the ACT.

And thanks to Annie Koziol, Steven Satyricon, and all the rest of the friendly folks at Maxfield's House of Caffeine for their contribution in helping to make this book good to the last drop.

Publisher's Acknowledgments

We're proud of this book; please send us your comments at <http://dummies.custhelp.com>. For other comments, please contact our Customer Care Department within the U.S. at 877-762-2974, outside the U.S. at 317-572-3993, or fax 317-572-4002.

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Contents at a Glance

Introduction	1
Part I: Getting into the Act: An Overview of ACT Math Basics	5
Chapter 1: Reviewing ACT Math Basics	7
Chapter 2: Boosting Your Test-Taking Skills	17
Chapter 3: Discovering Some Problem-Solving Strategies	25
Part II: Building Your Pre-Algebra and Elementary Algebra Skills	37
Chapter 4: Starting with the Basics: Pre-Algebra	39
Chapter 5: It's Elementary Algebra, My Dear Watson	65
Chapter 6: Practice Problems for Pre-Algebra and Elementary Algebra	85
Part III: Digging In to Intermediate Algebra and Coordinate Geometry	101
Chapter 7: Moving to Intermediate Algebra	103
Chapter 8: Coordinating a Path through Coordinate Geometry	125
Chapter 9: Practice Problems for Intermediate Algebra and Coordinate Geometry	147
Part IV: Visualizing Plane Geometry and Trigonometry	167
Chapter 10: Plain Talk about Plane Geometry	169
Chapter 11: Trig and Beyond: ACT Advanced Math Topics	193
Chapter 12: Practice Problems for Geometry, Trig, and Advanced Math	213
Part V: Tracking Your Progress with Practice Tests	233
Chapter 13: Practice Test 1	235
Chapter 14: Practice Test 1: Answers and Explanations	251
Chapter 15: Practice Test 2	267
Chapter 16: Practice Test 2: Answers and Explanations	283
Chapter 17: Practice Test 3	301
Chapter 18: Practice Test 3: Answers and Explanations	317
Part VI: The Part of Tens	331
Chapter 19: Ten Key Differences Between the ACT and the SAT	333
Chapter 20: Ten Items to Check Off before Taking the ACT	337
Index	341

Table of Contents

***Introduction*..... 1**

About This Book.....	1
Conventions Used in This Book.....	2
Foolish Assumptions.....	2
How This Book Is Organized.....	2
Part I: Getting into the Act: An Overview of ACT Math Basics.....	2
Part II: Building Your Pre-Algebra and Elementary Algebra Skills.....	3
Part III: Digging In to Intermediate Algebra and Coordinate Geometry.....	3
Part IV: Visualizing Plane Geometry and Trigonometry.....	3
Part V: Tracking Your Progress with Practice Tests.....	4
Part VI: The Part of Tens.....	4
Icons Used in This Book.....	4
Where to Go from Here.....	4

***Part I: Getting into the Act: An Overview of ACT Math Basics*..... 5**

Chapter 1: Reviewing ACT Math Basics..... 7

Getting an Overview of ACT Math.....	7
Taking Four Key Steps to ACT Success.....	8
Sharpening your basic math skills.....	8
Getting comfortable with your calculator.....	9
Solidifying your ACT-specific math skills.....	9
Taking practice tests under timed conditions.....	9
What Should I Study? Knowing What's on the ACT.....	9
Taking care of the basics in pre-algebra.....	10
Basic arithmetic.....	10
Number sequences.....	10
Factors and multiples.....	10
Percents, ratios, and proportions.....	10
Powers (exponents) and square roots (radicals).....	10
Data and graphs.....	11
Basic statistics and probability.....	11
Moving on to elementary algebra.....	11
Evaluating, simplifying, and factoring expressions.....	11
Solving equations with one or more variables.....	12
Focusing on intermediate algebra.....	12
Taking a look at inequalities.....	12
Working with systems of equations.....	12
Understanding direct and inverse proportionality.....	13
Examining quadratic equations.....	13
Finding information about functions.....	13



Working with coordinate geometry.....	13
Graphing linear functions	13
Recognizing quadratic functions	14
Transforming functions.....	14
Grappling with higher-order polynomial functions and circles.....	14
Reviewing plane geometry.....	14
Lines and angles.....	14
Triangles.....	14
Quadrilaterals.....	15
Circles.....	15
Solid geometry.....	15
Dealing with trigonometry and other advanced topics	15
Trigonometry.....	15
Matrices.....	15
Logarithms	15
Imaginary and complex numbers.....	16
You're Off the Hook: Discovering What the ACT Doesn't Cover	16

Chapter 2: Boosting Your Test-Taking Skills 17

Two-Timing the Test: Taking Two Passes to Answer Questions.....	17
To Guess or Not to Guess	18
Answering Multiple-Choice Questions.....	19
Considering the five answer choices.....	19
Plugging and playing	20
Calculating Your Way to Success: Calculators and the ACT	21
When should I use a calculator?	22
What kind of calculators can I use?.....	22
What kind of calculators can't I use?.....	23
How do I use my calculator?	23

Chapter 3: Discovering Some Problem-Solving Strategies..... 25

Identifying Basic Math Skills You Need to Know.....	25
Knowing how the four basic operations operate	26
Keeping your negatives straight	26
Evaluating with the order of operations.....	27
Making peace with fractions.....	28
Getting Comfortable with Formulas	28
Reviewing the formulas you need to know.....	28
Pre-algebra	29
Elementary algebra.....	29
Intermediate algebra.....	29
Coordinate geometry.....	29
Plane geometry.....	30
Trigonometry and other topics.....	31
Working with your arsenal of formulas.....	32
Solving for a missing value	32
Using two different formulas to solve one problem	33
Working with unfamiliar formulas	33
Sorting Through Word Problems	34
Jotting down the numbers.....	34
Sketching out problem information	35

***Part II: Building Your Pre-Algebra and Elementary Algebra Skills*..... 37**

Chapter 4: Starting with the Basics: Pre-Algebra39

Getting Back to Basics: Answering Basic Arithmetic Questions	39
Reviewing the four basic operations.....	39
Following along with number sequences	40
Checking out inequalities	41
Evaluating a number's absolute value	42
Understanding Factors and Multiples.....	43
Finding factors.....	44
Generating lists of multiples.....	45
Perfecting Your Knowledge of Percents	46
Knowing basic percent conversions	46
Taking advantage of a quick trick for calculating some percents	47
Calculating straightforward percent problems	47
Untangling complex percent problems.....	48
Handling the ups and downs of percent increase and decrease.....	49
Using Ratios and Proportions to Compare Quantities	50
Working with Powers and Square Roots	52
Harnessing the power of exponents.....	52
Be square: Squaring and square roots.....	53
Squares	53
Square roots	53
Real-World Math: Studying Statistics.....	54
Presenting data in tables and graphs.....	54
Organizing information in tables.....	54
Representing data with graphs	55
Getting a better-than-average grasp of averages	57
Finding the meaning of the mean.....	57
Centering on the median.....	59
Discovering what's so popular about the mode	59
Looking at Likelihood: Probability	60
Learning to count.....	60
Counting the outcomes of independent events	60
Counting the outcomes of dependent events	61
Determining probability.....	61

Chapter 5: It's Elementary Algebra, My Dear Watson65

Knowing the Two Big Ideas of Algebra	65
Every step you take: Keeping equations balanced.....	65
Walkin' a lonely road: Isolating the variable	66
Discovering Six Choice Words about Algebra	67
Express Yourself: Working with Algebraic Expressions.....	68
Evaluating expressions	68
Simplifying expressions	69
Combining like terms.....	69
Removing parentheses	69
Getting FOILed.....	70
Simplifying expressions with exponents.....	71
Factoring expressions	71
Finding common factors	72
Unearthing three useful ways to factor squares and cubes	72

Solving Everything but the Kitchen Sink	74
Approaching rational equations rationally	74
Rooting out ways to solve square root equations.....	75
Gaining absolute confidence with absolute value.....	76
Exposing variables in the exponent	78
Taking extra care with extra variables.....	80
Lost in Translation: Translating Words into Equations	82
Chapter 6: Practice Problems for Pre-Algebra and Elementary Algebra	85
Practice Problems	85
Solutions to Practice Problems	92

Part III: Digging In to Intermediate Algebra and Coordinate Geometry 101

Chapter 7: Moving to Intermediate Algebra	103
Knowing More or Less about Inequalities.....	103
Becoming a pro at inequalities.....	104
Separate but unequal: Solving inequalities with absolute value	106
Beating the System: Dealing with Systems of Equations.....	108
Solving systems of equations systematically.....	108
Finding values with substitution.....	109
Combining equations.....	109
Working word problems using a system of equations.....	111
Keeping Things in Proportion: Direct and Inverse Proportionality.....	114
Maintaining a balance with direct proportions	114
Turning things around with inverse proportions.....	115
Working with Quadratic Equations and the Roots of Polynomials.....	116
Factoring to solve quadratic equations	117
Choosing the quadratic formula when all else fails	118
Making Connections with Functions.....	119
Using functions as models	119
Defining relationships with functional notation and evaluation	122
Getting to know domain and range	123
Arriving at the domain event.....	123
Feeling at home with the range.....	124
Chapter 8: Coordinating a Path through Coordinate Geometry.....	125
Reviewing Some Basics of Coordinate Geometry.....	125
Graphing Linear Functions.....	126
Lining up some line segment skills	126
Finding coordinates with the midpoint formula	127
Going the whole ten yards with the distance formula	127
Measuring the slope of a line	128
Applying the slope-intercept form.....	130
Working with parallel and perpendicular lines.....	132
Giving Your Quads a Workout: Graphing Quadratic Functions	133
Searching for the qualities of a quadratic	134
Trick 1: Checking the direction of your parabola	134
Trick 2: Looking at your parabola's vertex.....	135
Trick 3: Keeping an eye on variable c	136

Calculating two features of parabolas.....	138
Accessing the axis of symmetry.....	138
Changing direction at the vertex.....	139
Solving and graphing quadratic inequalities.....	139
Recognizing Three Transformations.....	140
Reversing graphs with reflections.....	141
Movin' on up (or down): Vertical shift.....	142
Shifting horizontally.....	142
Taking on Advanced Equations.....	143
Raising the bar with higher-order polynomials.....	143
Going 'round and 'round with circles.....	146
Chapter 9: Practice Problems for Intermediate Algebra and Coordinate Geometry.....	147
Intermediate Algebra and Coordinate Geometry Practice Problems.....	147
Solutions to Intermediate Algebra and Coordinate Geometry Practice Problems.....	158
 Part IV: Visualizing Plane Geometry and Trigonometry.....	167
Chapter 10: Plain Talk about Plane Geometry.....	169
Knowing Your Angles.....	169
Angles around one point.....	170
Angles and parallel lines.....	171
Interior angles in a polygon.....	172
Working with Triangles.....	173
Finding the area of a triangle.....	173
Answering questions containing right triangles.....	175
Taking advantage of the Pythagorean theorem.....	175
Recognizing some common right triangles.....	176
Breezing through Your Work with Quadrilaterals.....	178
Squares.....	179
Rectangles.....	180
Parallelograms.....	181
Trapezoids.....	182
Wheeling and Dealing with Circles.....	183
Rounding up the basic circle formulas.....	183
Understanding tangent lines.....	184
Making sense of arc length.....	185
Striking a few chords.....	187
Examining Solid Geometry.....	188
Focusing on cubes and boxes.....	188
Incorporating spheres into your geometric repertoire.....	190
Figuring the volume of other solids (prisms, cylinders, pyramids, and cones)...	191
Prisms and pyramids.....	191
Cylinders and cones.....	192
Chapter 11: Trig and Beyond: ACT Advanced Math Topics.....	193
Trigonometry: Watching the Sines and Taking a Few Tangents.....	193
Picking out the opposite, the adjacent, and the hypotenuse.....	194
Knowing how to SOH CAH TOA.....	195
Feeling radiant with radian measure.....	197
Graphing trig functions.....	198
Identifying trig identities.....	200

Reloading the Matrix.....	201
Adding and subtracting matrices	201
Making sense of matrix multiplication.....	203
Multiplying a matrix by a real number	203
Multiplying two matrices	203
Determining the meaning of determinants.....	206
Logging Some Hours with Logarithms.....	207
Imagining the Possibilities with Imaginary Numbers	209
Chapter 12: Practice Problems for Geometry, Trig, and Advanced Math	213
Geometry, Trig, and Advanced Math Practice Problems.....	213
Solutions to Geometry, Trig, and Advanced Math Practice Problems.....	223
<i>Part V: Tracking Your Progress with Practice Tests</i>	<i>233</i>
Chapter 13: Practice Test 1.....	235
Mathematics Test Answer Sheet	237
Mathematics Test	239
Chapter 14: Practice Test 1: Answers and Explanations	251
Mathematics Test	251
Answer Key for Practice Test 1.....	265
Chapter 15: Practice Test 2.....	267
Mathematics Test Answer Sheet	269
Mathematics Test	271
Chapter 16: Practice Test 2: Answers and Explanations	283
Mathematics Test	283
Answer Key for Practice Test 2.....	299
Chapter 17: Practice Test 3.....	301
Mathematics Test Answer Sheet	303
Mathematics Test	305
Chapter 18: Practice Test 3: Answers and Explanations	317
Mathematics Test	317
Answer Key for Practice Test 3.....	330
<i>Part VI: The Part of Tens.....</i>	<i>331</i>
Chapter 19: Ten Key Differences Between the ACT and the SAT.....	333
Chapter 20: Ten Items to Check Off before Taking the ACT.....	337
<i>Index.....</i>	<i>341</i>

Introduction

Almost half of all graduating students in the United States (an estimated 45 percent) take the ACT. If you're among this lucky bunch, you may be looking toward the future with a combination of anticipation and dread. You're anticipating college, with its new experiences and first tastes of freedom, but you're dreading the hoops you have to jump through to get there. Of course, the ACT is one of these hoops. If you do well on this test, you're propelled to your first-choice college; if you do poorly, maybe not.

You're not alone. And I'm here to help. As the author of *Basic Math and Pre-Algebra For Dummies* (Wiley) and *SAT Math For Dummies* (Wiley), I've already helped thousands of would-be college students get through the arduous testing process and move into the exciting life that awaits them in college.

If you're willing to keep the faith and do the work in front of you, I'm confident that you can be among those who succeed on the ACT and take one step forward into your dreams. Let this book be your guide.

About This Book

Although you certainly want to do well on all four sections of the ACT (as well as the optional writing test), this book focuses exclusively on math. The reason I'm playing favorites is simple: For many students (and possibly you, too), math is the toughest section of the test. Because your composite score on the ACT is based in part on your math score, you don't want math to drag down an otherwise good score.

The math you need to know to do well on the ACT is basically what's covered in high school: pre-algebra, elementary algebra, intermediate algebra, coordinate geometry, plane geometry, and trigonometry. If you're thinking of college, the good news is that you've probably studied a lot of this material already. The bad news is that you may have forgotten it or never felt entirely comfortable with it in the first place.

This situation is perfectly normal, and most students who aren't math geniuses feel roughly the same as you. So your task is to find a realistic strategy to pull together the stuff you already know — even if you're not currently confident that you know it — and tie in the more advanced topics that may have eluded you. Luckily, you've come to the right place.

Everything in this book is designed to take the small amount of math you may remember and build on it so you can use it to tackle typical questions that appear on the ACT. If you've laid the groundwork in school and you're willing to practice, I'll get you through the test.

My approach in this book is simple: practice, practice, and more practice. The focus is specifically on the types of questions that appear over and over again on the ACT. Even if math isn't your strong suit, when you become comfortable with this core material, you stand a much better chance of getting the kind of ACT score you want to see. And the best part is that you don't necessarily have to read this book from cover to cover. You can hop and skip around to whatever topics you need to work on most.

Conventions Used in This Book

Here are a few conventions to keep in mind as you make your way through this book:

- ✔ Variables (such as x and y) and newly defined terms are in *italics*.
- ✔ Keywords in bulleted lists, action parts of numbered steps, and answers in the practice tests are in **bold**.
- ✔ Web sites are in monofont.
- ✔ I alternate the range of the answer choices throughout the book. In one question, you see Choices (A), (B), (C), (D), and (E), and then in the one after it you see Choices (F), (G), (H), (J), (K). Because the ACT itself uses this pattern, I want you to become familiar with it before test day. And, yes, test-designers skip the letter *I*. Why? Probably to avoid confusion because it looks like the number 1.

Foolish Assumptions

I'm going out on a limb here, but if you've bought this book, my first assumption is that you or somebody you know is thinking about taking the ACT. If not, you can certainly use it to improve your knowledge of high school math. And beyond that, it also makes a useful door-stop or something to stick under the leg of a *very* wobbly table.

My second assumption is that you've taken a year of algebra, whether in high school, junior college, or some other place. You don't actually have to feel like you *know* algebra. In fact, the point of this book is to build these very skills. But if you've at least sat through an algebra class, you probably know a lot more than you think you know. Really!

How This Book Is Organized

ACT Math For Dummies is divided into six parts, each of which focuses on a different math topic you may encounter on the ACT. In this section, I give you a basic overview of these parts so you have a better idea of where you'd like to begin.

Part I: Getting into the Act: An Overview of ACT Math Basics

Chapter 1 provides an introduction to ACT math, giving you the most basic information about the test. It also includes a road map of the rest of the book. In Chapter 2, I discuss test-taking skills that aren't strictly math-related but are still necessary to do well on the ACT. In particular, I focus on some ways to work with multiple-choice questions, including plugging in answers. I also show you how to use your calculator to your advantage, rule out answers and guess wisely, and know when to jump over questions that look long or difficult.

In Chapter 3, I discuss a variety of problem-solving strategies. These include looking for a pattern by drawing a chart or picture, memorizing basic math ideas, working with math formulas, setting up word problems, and making a plan to answer difficult questions.

Part II: Building Your Pre-Algebra and Elementary Algebra Skills

In Part II, I begin helping you build your math skills. In Chapter 4, for example, you review the basics of pre-algebra: number sequences; inequalities; absolute value; factors and multiples; percents, ratios, and proportions; powers and square roots; tables and graphs; basic statistics; and probability.

In Chapter 5, you focus on elementary algebra (Algebra I), including the following: understanding basic algebra concepts and vocabulary; evaluating, simplifying, and factoring expressions; solving a variety of equations, including rational equations and equations with absolute values, square roots, and exponents; and working with equations that have more than one variable.

Chapter 6 provides 30 practice questions, covering the material from Chapters 4 and 5. Each question includes a detailed explanation of how to answer that question.

Part III: Digging In to Intermediate Algebra and Coordinate Geometry

Part III adds on to the math skills you gain in Part II. Chapter 7 discusses intermediate algebra (Algebra II). You study these topics: inequalities, systems of equations, direct and inverse proportionality, quadratic equations, and functions.

Chapter 8 covers coordinate geometry, which usually is introduced in Algebra I and is continued in Algebra II. I discuss the following topics: graphing points and lines, the distance and midpoint formulas, the slope-intercept form of a line, parallel and perpendicular lines, quadratic functions, transformations, higher-order polynomial functions, and circles on the xy -graph.

Chapter 9 offers 30 practice questions focusing on the topics covered in Chapters 7 and 8, along with answers and step-by-step explanations for each.

Part IV: Visualizing Plane Geometry and Trigonometry

Part IV completes the math skills you need to know to get a good score on the ACT. Chapter 10 focuses on plane geometry: lines and angles; triangles, including the area formula and the Pythagorean theorem; quadrilaterals such as squares, rectangles, parallelograms, and trapezoids; circles, including the formulas for area and circumference, tangents, arc length, and chords; and solid geometry.

In Chapter 11, I cover a variety of advanced math topics: trigonometry, including trig ratios, radian measure, the graphing of trig functions, and basic identities; matrices, including matrix operations and determinants; logarithms; and imaginary and complex numbers.

Chapter 12 tests your understanding of the math you discover in Chapters 10 and 11 with 30 practice questions and fully worked-out explanations.

Part V: Tracking Your Progress with Practice Tests

Part V includes three full-length ACT mathematics tests. Chapters 13, 15, and 17 each contain a different test. Each has 60 questions that should be completed in 60 minutes. In Chapters 14, 16, and 18, you find the answer keys and detailed explanations of how to answer each question.

Part VI: The Part of Tens

In Part VI, I present two top-ten lists that provide useful information about the ACT. In Chapter 19, I outline ten key differences between the ACT and the SAT. And in Chapter 20, I give you a checklist of small but vital things to remember leading up to your test day.

Icons Used in This Book

Throughout this book, I use the following four icons to help you keep track of the different kinds of information. Here's what each icon means:



This icon highlights essential information that you need to know, such as key ideas or formulas. Spending a few extra minutes studying these important points is time well spent. You also can use these icons to skim through a chapter you've already studied. Doing so helps solidify your understanding.



Each tip provides an insightful way to approach a question. You may find it especially helpful as a quick way to cut through a seemingly complicated problem.



This icon is like a flashing red light, drawing your attention to a sticky or subtle point that may trip you up if you're not aware of it. Take an extra moment to slow down and make sure you understand the point being made so it doesn't trip you up on the test.



When you see this icon, you know you're ready to put pencil to paper. This book includes dozens of examples of ACT questions, showing you how to answer them from start to finish. Use these examples to better understand a specific type of problem and then refer to them when answering the practice questions.

Where to Go from Here

I've written this book as a complete reference to ACT math. You can start anywhere you like, dip in, clarify your understanding, and then hop back out with ease.

If you're completely unfamiliar with the ACT, I recommend that you begin with Chapter 1 before branching out. In that chapter, I outline the basics of the math portion of the ACT and provide an overview of the topics that are covered on the test.

If you're ready to get studying, flip to whatever chapter discusses the topic you need help with most. Chapters 4 through 12 provide a thorough review of the specific math skills that the ACT covers. You can focus on these chapters until you feel ready to take one of the practice tests in Part V. Alternatively, you can take a practice test first to find out which types of questions you can answer easily and which hang you up.

Part I

Getting into the Act: An Overview of ACT Math Basics

The 5th Wave

By Rich Tennant



"You're driving 650 miles from Boston to Cleveland. Gas is \$3.25 a gallon, your car gets 35 miles to the gallon, and your average speed is 50 mph. What's the formula for figuring out why you would want to go to Cleveland?"

In this part. . .

In Part I, you get an overview of the types of math problems you can expect on the ACT. I outline the math skills you need most, show you some helpful test-taking skills, and discuss some useful problem-solving strategies.

Chapter 1

Reviewing ACT Math Basics

In This Chapter

- ▶ Beginning with an overview of ACT math
 - ▶ Identifying four important steps to success
 - ▶ Studying the six math topics tested on the ACT
 - ▶ Figuring out what's not tested
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All across the United States — and especially in the Midwest, South, and Rocky Mountain states — high school juniors and seniors prepare for one of the most action-packed Saturdays of their lives. No, I don't mean the prom, graduation day, or a really excellent date with someone their parents don't know about and wouldn't approve of if they did. No, they're preparing for something even more fun than that: the ACT.

And if you believe any of this fun business, you're in for a real treat: I also have a fortune in gold from the prince of a country whose name you can't pronounce all ready to deposit into your bank account. Yeah, right.

Well, okay, some of this is true: You *will* be taking the ACT (which I figured out the moment you picked up this book). In this chapter, I begin the work of helping you get ready for the most fun part of all, the 60-minute math test.

I start this chapter with a look at what's on the math portion of the ACT. Then I provide a bit of advice about four general ways to improve your score. Finally, I list the six math topics that the ACT tests you on, including a breakdown of the specific skills within each topic that you should focus on to do well.

The chapter ends with a sigh of relief as you discover the math that you don't need to worry about on the ACT. (However, the teacher of your current math class may still want you to know something about it, so don't flush it from your memory just yet!)

Getting an Overview of ACT Math

The ACT contains five separate tests, which are always presented in this order:

1. English
2. Mathematics
3. Reading
4. Science
5. Writing (optional)

This book focuses exclusively on the second test, the ACT mathematics test. This math portion lasts for 60 fun-filled minutes and contains 60 questions. Generally speaking, questions appear roughly in order of difficulty.

The test is scored on a scale of 1 (lowest) to 36 (highest). Additionally, you receive three subscores in the following three separate areas

- ✓ Pre-algebra and elementary algebra
- ✓ Intermediate algebra and coordinate geometry
- ✓ Plane geometry and trigonometry

Subscores are tallied on a scale of 1 (lowest) to 18 (highest). Note that, flying in the face of common sense, your three subscores will *not* add up to your math score.



On the ACT, test graders don't take off points for wrong answers — that is, you won't be penalized for guessing. So keep an eye on the time and, just before your time is up, fill in answers to all 60 questions. Make as many educated guesses as you can, of course. But even wild guessing won't hurt your score, so fill in those answers before time's up!

Taking Four Key Steps to ACT Success

I like to give the following four important pieces of advice to those students who want to improve their ACT math scores:

- ✓ Sharpen your basic math skills.
- ✓ Get comfortable using your calculator.
- ✓ Solidify your ACT-math-specific skills.
- ✓ Take practice tests under timed conditions.

In this section, I give you an overview of these four steps. The rest of the book is devoted, one way or another, to working on them.

Sharpening your basic math skills

By the basics, I mean the math calculations that precede pre-algebra. For example:

- ✓ Multiplication tables up to 9×9
- ✓ Adding, subtracting, multiplying, and dividing fractions
- ✓ Converting percents to decimals and vice versa
- ✓ Converting common percents, such as 10%, 20%, 25%, 50%, and 75%, to fractions and vice versa
- ✓ Working with negative numbers
- ✓ Knowing the order of operations (PEMDAS: parentheses, exponents, multiplication and division, addition and subtraction)

When I say *know this stuff*, I mean that you should know it stone cold — the way you know your own name. If you're wasting precious time trying to remember 7×8 or calculate $-3 - 5$, you'll benefit greatly from investing just a few hours to work with a set of flash cards designed to help you strengthen these skills. In Chapter 3, I cover these topics to refresh your memory.

Getting comfortable with your calculator

You may already be well aware (and grateful!) that calculators are allowed on the ACT. Moreover, because virtually everyone will be using one, you should consider a calculator not just optional but mandatory.

Generally speaking, the kind of calculator you need for the ACT is at least one notch above the basic calculator you may use to keep track of spending or add up the calories in that big lunch you just ate — you need either a scientific calculator or a graphing calculator.

If you already own a good calculator and are reasonably adept at using it, just use this quick rule of thumb to make sure it's up to speed: Check to see whether it can do trigonometric calculations like $\sin x$. If it can, it's probably just fine; if not, you probably need to think about an upgrade well before the ACT so you have time to practice with it.

In Chapter 2, I give you some specifics about your calculator. And for those of you who are considering or who already own a graphing calculator, I recommend *TI-83 Plus Graphing Calculator For Dummies* (Wiley) and *TI-89 Graphing Calculator For Dummies* (Wiley), which are both written by C. C. Edwards.

Solidifying your ACT-specific math skills

Studying for the ACT should be mostly a review of skills that are covered in your math classes. However, no matter how hard you work in your classes, the material you covered two or three years ago may not be fresh in your mind. So focused preparation for the ACT can really pay off.

Parts II, III, and IV (Chapters 4 through 12) provide a detailed review of the math that shows up most on the ACT. You can work through dozens of example problems and answer 90 practice questions specifically related to those topics.

Taking practice tests under timed conditions

Time is money, and money changes everything. So it's not surprising that time changes everything — especially on the ACT, where you have only 60 minutes to answer 60 math questions. Purposefully working under low-stakes time pressure adds a useful dimension to your study, especially as you get closer to your test date when the stakes will be higher.

Part V of this book — Chapters 13 through 18 — contains three complete practice tests (and answers!) for you to try out. I recommend that you take them under real test conditions. In other words, take them in one hour with the calculator you plan to use on the test and no additional help. You may start out taking the first test as a benchmark before you begin working on the rest of the problems in the book. Or, if you prefer, save all three tests until you feel confident answering questions with no time pressure — then start the clock running and see how you do.

What Should I Study? Knowing What's on the ACT

The ACT covers six overall topics in math: pre-algebra, elementary algebra, intermediate algebra, coordinate geometry, plane geometry, and trigonometry. In this section, I break down all these topics into manageable bits and discuss the individual skills included in each. Parts II, III, and IV (Chapters 4 through 12) cover this material in depth, with plenty of example questions and practice problems.

Taking care of the basics in pre-algebra

Pre-algebra includes a variety of topics that prepare you for algebra. In this section, I discuss the specific pre-algebra skills that show up most on the ACT. And in Chapter 4, I focus on these types of questions, providing plenty of example questions and showing you how to answer them.

Basic arithmetic

You obviously need to know the four operations: addition, subtraction, multiplication, and division. You also want to feel comfortable working with negative numbers, fractions, and decimals. I cover some of this material in this book, but if you feel that you need a more thorough review, pick up *Basic Math and Pre-Algebra For Dummies* (Wiley) by Yours Truly.

Number sequences

A *number sequence* is a list of numbers arranged in a pattern. Here's an example:

2, 5, 8, 11, 14, 17 . . .

In this case, each number in the sequence is 3 greater than the number before it. An ACT question may ask you to find the next number or a missing number in a number sequence.

Factors and multiples

When one natural number is divisible by another, the smaller number is a *factor* of the greater number, and the greater number is a *multiple* of the smaller number. For example, 12 is divisible by 4, so

- ✓ 4 is a factor of 12.
- ✓ 12 is a multiple of 4.

To answer an ACT question, you may need to find all the factors of a number or the greatest common factor or the least common multiple among several numbers.

Percents, ratios, and proportions

Like fractions and decimals, *percents* are a mathematical way of representing part of a whole. For example, 50 percent of something is half of it. A *ratio* is a mathematical comparison. For instance, if you have twice as many brothers as sisters, the ratio of brothers to sisters is 2 to 1, or 2:1. A *proportion* is an equation using two ratios. ACT questions may ask you to calculate something using percents and ratios, or you may have to set up a proportion to answer a question.

Powers (exponents) and square roots (radicals)

When you take a number to a *power*, you multiply that number by itself repeatedly. For example: 3^4 (read *three to the fourth power*) = $3 \times 3 \times 3 \times 3 = 81$. In this case, 3 is the *base* (the number multiplied) and 4 is the *exponent* (the number of times the base is multiplied).

And when you take a *root* (also called a *radical*) of a number, you find a result that can be multiplied by itself repeatedly to produce the number you started with. The most common root is the *square root* — a result which, when multiplied by itself, produces the number you started with. For example, $\sqrt{25}$ (read *the square root of 25*) = 5 because $5 \times 5 = 25$.

Powers and square roots are common math operations, and they show up a lot on all sorts of ACT questions.

Data and graphs

A *graph* is a visual representation of data. Common graphs include bar graphs, pie charts, line graphs, and pictograms. Graph reading is a basic but essential skill that you need for the ACT. A typical question may ask you to identify specific data given in a graph, or you may need to pull this data as a first step in a more complex calculation.

Basic statistics and probability

Statistics is the mathematical study of real-world information called *data sets* — lists of numbers that are objectively observed and recorded. Three common operations used on data sets are three types of averages called the *mean*, the *median*, and the *mode*. On the ACT, you need to know how to calculate all three.

Moving on to elementary algebra

Elementary algebra is essentially the algebra that's covered in an Algebra I class. In this section, I go over the highlights of what skills the ACT expects you to remember and work with when answering questions. Chapter 5 covers these topics in greater detail with lots of examples.

Evaluating, simplifying, and factoring expressions

An *expression* is any string of numbers and symbols that makes mathematical sense. In algebra, you can do three common things with expressions:

- ✔ **Evaluate:** To *evaluate* an expression, you plug in the value of each variable and change the expression to a number. Be sure to follow the order of operations (exponents in the order they occur left to right, multiplication and division in the order they occur left to right, and addition and subtraction in the order they occur left to right). For example, here's how you evaluate the expression $5x + 7$, given that $x = 4$:

$$\begin{aligned}5x + 7 \\ &= 5(4) + 7 \\ &= 20 + 7 \\ &= 27\end{aligned}$$

- ✔ **Simplify:** To *simplify* an expression, you remove parentheses and combine like terms to make the expression more compact. For example, here's how you simplify the expression $3(x + 6) + 2x$:

$$\begin{aligned}3(x + 6) + 2x \\ &= 3x + 18 + 2x \\ &= 5x + 18\end{aligned}$$

- ✔ **Factor:** To *factor* an expression, you find a factor that's common to each term in the expression and pull it out of the expression using parentheses. For example, here's how you factor $2x$ out of the expression $6x^2 - 10x$:

$$\begin{aligned}6x^2 - 10x \\ &= 2x(3x - 5)\end{aligned}$$

Easier ACT questions ask you to simply evaluate, simplify, or factor an expression. More difficult questions may require you to use these skills to handle more complex calculations.

Solving equations with one or more variables

Solving equations is the main point of algebra. You solve an equation by isolating the variable (commonly x) while keeping the equation in balance — that is, by making sure that in each step, you apply the exact same operation to both sides of the equation. Here are a few types of equations you need to know how to solve on the ACT:

- ✓ Equations with fractions (rational equations), such as $\frac{2x}{5} = \frac{x+1}{4}$
- ✓ Equations with square roots (radicals), such as $\sqrt{5x+3} - 2 = x$
- ✓ Equations with absolute values, such as $|3x - 6| = 10$
- ✓ Equations with variables in the exponent, such as $8^{x-1} = 16$

Typically, an equation with more than one variable, such as $ab + c = 10$, can't be solved for a number. However, you can solve an equation with more than one variable in terms of the other variables in the equation. For example, here's how you solve this equation for b in terms of a and c :

$$\begin{aligned}ab + 3c &= 10 \\ab &= 10 - 3c \\b &= \frac{10 - 3c}{a}\end{aligned}$$

An ACT question may ask you to solve an equation in terms of other variables. Additionally, this skill is useful when working with math formulas.

Focusing on intermediate algebra

Intermediate algebra is the focus of a high school Algebra II class. In this section, I outline the essential intermediate algebra skills you need to be successful on the ACT. Later on, in Chapter 7, you can gain a solid understanding of this material.

Taking a look at inequalities

An *inequality* is a statement telling you that two math expressions aren't equal. On the ACT, inequalities come in four basic varieties:

- ✓ Greater than ($>$)
- ✓ Less than ($<$)
- ✓ Greater than or equal to (\geq)
- ✓ Less than or equal to (\leq)

You solve inequalities using the same algebra rules you would use to solve equations — with the exception of a couple of twists (flip to Chapter 7 for details). The solution to an inequality is typically a range of answers expressed as a simpler inequality.

Working with systems of equations

A *system of equations* is made of two equations that are simultaneously true. On the ACT, a system of equations usually is limited to two variables. For example, take a look at this system:

$$\begin{aligned}3x + y &= 10 \\x - 5y &= -4\end{aligned}$$

You can solve a simple system of equations by the *substitution method*, isolating a variable in one equation and then plugging its equivalent into the other equation. For a more complicated system of equations, use the *elimination (or combination) method* by either adding or subtracting the two equations and solving the equation that remains.

Understanding direct and inverse proportionality

When two values, x and y , are *directly proportional*, a value, k , makes the following equation true:

$$\frac{x}{y} = k$$

Values that are directly proportional tend to rise and fall together. For example, when one value doubles, the other value also doubles.

When two values, x and y , are *inversely proportional*, a value, k , makes the following equation true:

$$xy = k$$

Values that are inversely proportional tend to rise or fall opposite of each other. For example, when one value is multiplied by 3, the other value is divided by 3.

Examining quadratic equations

A *quadratic equation* is an equation in the form $ax^2 + bx + c = 0$. You can solve a quadratic equation either by factoring or by using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The ACT almost certainly will have several questions that require you to work with quadratic equations.

Finding information about functions

A *function* is a mathematical connection between two values. Usually, the values are an input variable, x , and an output variable, y . In a function, when you know the value of x , the value of y is determined.

Typical ACT questions may ask you to use functions as models, to work with functional notation $f(x)$, or to find the domain or range of a function.

Working with coordinate geometry

Coordinate geometry is geometry that occurs on the xy -graph. This topic overlaps with material introduced in both Algebra I and Algebra II classes. Here, I give you an overview of the basic information from coordinate geometry that you need to review to do well on the ACT. I go over these ideas in greater detail in Chapter 8.

Graphing linear functions

A *linear function* is any function of the form $y = mx + b$. For example:

$$y = 3x + 5 \qquad y = \frac{5}{6}x - \frac{1}{3} \qquad y = -x$$

Linear functions, which produce a straight line when graphed, are common on the ACT. Some of the skills you need to feel comfortable with include mastering the distance and midpoint formulas, finding the slope of a line, using the slope-intercept form to solve problems, and working with parallel and perpendicular lines.

Recognizing quadratic functions

A *quadratic function* is in the form $y = ax^2 + bx + c$. For example:

$$y = 2x^2 + 11x + 9$$

$$y = x^2 + 4x + 4$$

$$y = x^2 - 1$$

On the graph, a quadratic function produces a *parabola* — a curve that looks roughly like an arch (or a U). On the ACT, a question may ask you to pair up a quadratic function with its graph. More difficult questions may require you to find the axis of symmetry or the vertex of a parabola or to solve a quadratic inequality.

Transforming functions

A *transformation* of a function is a small change that affects that function in a predictable way. Typical transformations include reflections across the x -axis and y -axis as well as vertical and horizontal shifts. An ACT question may ask you to compare two similar functions and select the equation that transforms one into the other. Or a question may provide a function and a transformation and ask you to produce the resulting graph.

Grappling with higher-order polynomial functions and circles

More difficult ACT questions may include higher-order polynomials, such as cubic equations of the form $y = ax^3 + bx^2 + cx + d$, and graphs of circles. These questions are rather uncommon and require only a basic familiarity with the concepts.

Reviewing plane geometry

Plane geometry is the focus of a typical high school geometry class. In this section, I discuss the geometry that you're likely to see on the ACT. Chapter 10 gives you a complete review of these topics.

Lines and angles

One common type of ACT question presents you with a figure that contains lines and angles and then asks you to find the value of a given angle. To answer this type of question, you need to know how to measure right angles, vertical angles, supplementary angles, the angles in a triangle, and the angles that result when two lines are parallel.

For example, an ACT question may show you a figure with some angles labeled and ask you to find the measure of an unlabeled angle. Or it may ask you to identify a pair of angles that are equal in measure.

Triangles

Virtually every ACT includes several questions about triangles. You may need to find the area of a triangle given the height and the base, use the Pythagorean theorem to work with right triangles, or work with the most common types of right triangles, such as the 3-4-5 triangle.

An ACT question may ask you to find the area of a triangle given the measurements of its height and base, or, turning this question around, it may ask you to find the height given the length of the base and the area of the triangle. ACT questions involving right triangles may ask you to identify the length of one side of a right triangle, given information about the other sides.