

LEARNING MADE EASY



Algebra II Essentials

for
dummies[®]
A Wiley Brand



The “must-know”
formulas and equations

What you need to know
to master Algebra II

Core topics in quick,
focused lessons

Mary Jane Sterling

Emerita Instructor
of Mathematics



Algebra II Essentials

by Mary Jane Sterling

for
dummies[®]
A Wiley Brand

Algebra II Essentials For Dummies®

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Contents at a Glance

Introduction	1
CHAPTER 1: Making Advances in Algebra.....	5
CHAPTER 2: Lining Up Linear Equations.....	15
CHAPTER 3: Making Quick Work of Quadratic Equations	23
CHAPTER 4: Rolling Along with Rational and Radical Equations	35
CHAPTER 5: Forging Function Facts.....	47
CHAPTER 6: Graphing Linear and Quadratic Functions	61
CHAPTER 7: Pondering Polynomials.....	75
CHAPTER 8: Being Respectful of Rational Functions.....	91
CHAPTER 9: Examining Exponential and Logarithmic Functions.....	107
CHAPTER 10: Getting Creative with Conics.....	121
CHAPTER 11: Solving Systems of Equations.....	135
CHAPTER 12: Taking the Complexity Out of Complex Numbers.....	155
CHAPTER 13: Ten (or So) Special Formulas.....	163
Index	167

Table of Contents

INTRODUCTION	1
About This Book	1
Conventions Used in This Book	2
Foolish Assumptions	2
Icons Used in This Book	2
Where to Go from Here	3
CHAPTER 1: Making Advances in Algebra	5
Bringing Out the Best in Algebraic Properties	5
Making short work of the basic properties	6
Organizing your operations	7
Enumerating Exponential Rules	8
Multiplying and dividing exponents	8
Rooting out exponents	9
Powering up exponents	10
Working with negative exponents	10
Assigning Factoring Techniques	10
Making two terms factor	11
Factoring three terms	12
Factoring four or more terms by grouping	13
CHAPTER 2: Lining Up Linear Equations	15
Getting the First Degree: Linear Equations	15
Solving basic linear equations	16
Eliminating fractions	16
Lining Up Linear Inequalities	17
Solving basic inequalities	18
Introducing interval notation	19
Absolute Value: Keeping Everything in Line	20
Solving absolute value equations	20
Seeing through absolute value inequality	21
CHAPTER 3: Making Quick Work of Quadratic Equations	23
Using the Square Root Rule When Possible	24
Solving Quadratic Equations by Factoring	24
Factoring quadratic binomials	25
Factoring quadratic trinomials	26

The Quadratic Formula to the Rescue	27
Realizing rational solutions.....	27
Investigating irrational solutions	27
Promoting Quadratic-like Equations.....	28
Solving Quadratic Inequalities	29
Keeping it strictly quadratic	30
Signing up for fractions	31
Increasing the number of factors	33
CHAPTER 4: Rolling Along with Rational and Radical Equations.....	35
Rounding Up Rational Equations and Eliminating Fractions	35
Making your least common denominator work for you.....	36
Proposing proportions for solving rational equations.....	38
Reasoning with Radicals	39
Squaring both sides of the equation.....	39
Taking on two radicals.....	40
Dealing with Negative Exponents.....	42
Factoring out a negative exponent as a greatest common factor.....	42
Solving quadratic-like trinomials.....	43
Fiddling with Fractional Exponents	44
Solving equations by factoring fractional exponents.....	44
Promoting techniques for working with fractional exponents.....	44
CHAPTER 5: Forging Function Facts	47
Describing Function Characteristics.....	47
Denoting function notation	48
Using function notation to evaluate functions.....	48
Determining Domain and Range.....	49
Delving into domain	49
Wrangling with range	50
Counting on Even and Odd Functions	51
Determining whether even or odd	52
Using even and odd functions in graphs	53
Taking on Functions One-to-One	53
Defining which functions are one-to-one	54
Testing for one-to-one functions	54
Composing Functions	55
Composing yourself with functions	55
Composing with the difference quotient.....	56

Getting into Inverse Functions.....	57
Finding which functions are inverses	58
Finding an inverse of a function.....	59
CHAPTER 6: Graphing Linear and Quadratic Functions	61
Identifying Some Graphing Techniques	61
Finding x- and y-intercepts	62
Reflecting on a graph's symmetry.....	62
Mastering the Graphs of Lines	64
Determining the slope of a line.....	64
Describing two line equations.....	65
Identifying parallel and perpendicular lines.....	67
Coming to Terms with the Standard Form of a Quadratic.....	67
Starting with “a” in the standard form.....	68
Following “a” with “b” and “c”.....	69
Eyeing a Quadratic’s Intercepts	69
Finding the one and only y-intercept	69
Getting at the x-intercepts	70
Finding the Vertex of a Parabola	71
Computing vertex coordinates.....	71
Linking up with the axis of symmetry	72
Sketching a Graph from the Available Information	72
CHAPTER 7: Pondering Polynomials.....	75
Sizing Up a Polynomial Equation.....	75
Identifying Intercepts and Turning Points.....	76
Interpreting relative value and absolute value	76
Dealing with intercepts and turning points	77
Solving for y-intercepts and x-intercepts.....	78
Determining When a Polynomial Is Positive or Negative	79
Incorporating a sign line	79
Recognizing a sign change rule	80
Solving Polynomial Equations.....	81
Factoring for roots	81
Taking sane steps with the rational root theorem	82
Putting Descartes in charge of signs	84
Finding Roots Synthetically	86
Using synthetic division when searching for roots.....	86
Synthetically dividing by a binomial	88

CHAPTER 8:	Being Respectful of Rational Functions	91
	Examining Rational Functions.....	91
	Deliberating on domain	92
	Investigating intercepts	92
	Assigning Roles to Asymptotes.....	93
	Validating vertical asymptotes	93
	Finding equations for horizontal asymptotes	94
	Taking vertical and horizontal asymptotes to graphs	94
	Getting the scoop on oblique (slant) asymptotes.....	96
	Discounting Removable Discontinuities.....	97
	Finding removable discontinuities by factoring.....	97
	Evaluating the removals.....	98
	Looking at Limits of Rational Functions.....	99
	Determining limits at function discontinuities	100
	Finding infinity.....	102
	Looking at infinity	104
 CHAPTER 9:	 Examining Exponential and Logarithmic Functions	 107
	Computing Exponentially	107
	Getting to the Base of Exponential Functions	108
	Classifying bases	108
	Introducing the more frequently used bases: 10 and e.....	110
	Exponential Equation Solutions.....	110
	Creating matching bases	111
	Quelling quadratic patterns.....	111
	Looking into Logarithmic Functions.....	113
	Presenting the properties of logarithms.....	113
	Doing more with logs than sawing	115
	Solving Equations Containing Logs	117
	Seeing all logs created equal.....	117
	Solving log equations by changing to exponentials	118
 CHAPTER 10:	 Getting Creative with Conics.....	 121
	Posing with Parabolas.....	122
	Generalizing the form of a parabola's equation	123
	Making short work of a parabola's sketch	124
	Changing a parabola's equation to the standard form.....	125

Circling around a Conic.....	126
Getting Eclipsed by Ellipses.....	127
Determining the shape	129
Finding the foci.....	130
Getting Hyped for Hyperbolas.....	130
Including the asymptotes	131
Graphing hyperbolas.....	132
CHAPTER 11: Solving Systems of Equations	135
Looking at Solutions Using the Standard	
Linear-Systems Form	136
Solving Linear Systems by Graphing.....	136
Interpreting an intersection.....	137
Tackling the same line	137
Putting up with parallel lines.....	137
Using Elimination (Addition) to Solve Systems	
of Equations	138
Finding Substitution to Be a Satisfactory Substitute.....	139
Variable substituting made easy.....	139
Writing solutions for coexisting lines	140
Taking on Systems of Three Linear Equations.....	141
Finding the solution of a system of three	
linear equations	141
Generalizing with a system solution.....	143
Increasing the Number of Equations.....	144
Intersecting Parabolas and Lines	146
Determining if and where lines and parabolas	
cross paths.....	147
Determining that there's no solution	149
Crossing Parabolas with Circles.....	150
Finding multiple intersections.....	150
Sifting through the possibilities for solutions	151
CHAPTER 12: Taking the Complexity Out of	
Complex Numbers.....	155
Simplifying Powers of i	156
Getting More Complex with Complex Numbers	157
Performing complex operations	157
Performing complex division by multiplying	
by the conjugate	158
Simplifying reluctant radicals	159

Unraveling Complex Solutions in Quadratic Equations	160
Investigating Polynomials with Complex Roots.....	160
Classifying conjugate pairs	161
Making use of complex zeros.....	161
CHAPTER 13: Ten (or So) Special Formulas.....	163
Using Multiplication to Add.....	163
Factoring in Factorial.....	164
Picking Out Permutations	164
Collecting Combinations.....	164
Adding n Integers	165
Adding n Squared Integers	165
Adding Odd Numbers.....	165
Going for the Geometric.....	166
Calculating Compound Interest.....	166
INDEX	167

Introduction

Here you are, perusing a book on the essentials of Algebra II. You'll find here, as Joe Friday (star of the old *Dragnet* series) said, "The facts, ma'am, just the facts." For those of you too young to remember *Dragnet*, just think of this essentials book as being the Twitter version — not too detailed but with all the necessary information. In this book, you find the information you need with enough examples to show you the processes, but not a bunch of nitty-gritty details that tend to get in the way.

About This Book

A book on Algebra II isn't a romance novel (although I do love math), and it isn't science fiction. You could think of this book as a cross between a travel guide and a mathematical laboratory manual. How do travel and math go together? Let me try some situations that may fit:

- » You just finished working through Algebra I and feel eager to embark on a new adventure.
- » You haven't worked with algebra in a while, but math has always been your strength, so you think that a little prepping with some basic concepts will bring you up to speed.
- » You're helping a friend or family member with Algebra II and want just the most necessary information — no frills or extra side-trips.

Even though I've pared the material in this book down to the basics, I haven't lost sight of the fact that other math areas are what drive Algebra II. Algebra is the passport to studying calculus, trigonometry, number theory, geometry, and all sorts of good mathematics. Algebra is basic, and the algebra you find here will help you grow your skills and knowledge so you can do well in math courses and possibly pursue other math topics.

Conventions Used in This Book

To help you navigate this book, I use the following conventions:

- » I *italicize* special mathematical terms and define them right then and there so you don't have to search around.
- » I use **boldface** text to indicate keywords in bulleted lists or the action parts of numbered steps. I describe many algebraic procedures in a step-by-step format and then use those steps in an example or two.

Foolish Assumptions

Algebra II is essentially a continuation of Algebra I, so I need to make some assumptions about readers of this book.

I assume that a person taking on Algebra II has a grasp of working with operations on signed numbers, simplifying radical expressions, and manipulating with rational terms. Another assumption I make is that your order of operations is in order. You should be able to work your way through algebraic equations and expressions using the ordering rules. I also assume that you know how to solve basic linear and quadratic equations and can make quick sketches of basic graphs. Even though I lightly cover these topics in this book, I assume that you have a general knowledge of the necessary procedures.

If you feel a bit over your head after reading through some chapters, you may want to refer to *Algebra I For Dummies*, 2nd Edition (Wiley), or *Algebra II For Dummies* (Wiley) for a more complete explanation of the basics. My feelings won't be hurt; I wrote those, too!

Icons Used in This Book

The icons that appear in this book are great for calling attention to the hot topics when doing algebra.



ALGEBRA
RULES

This icon provides you the rule or law or instruction on how to proceed whenever encountering the particular mathematical situation. The algebra rule given is “the law” — it always applies and always must be followed.



EXAMPLE

When you see the Example icon, you know that you’ll find the result of an attempt at working out an equation or concept. An example often has a hidden agenda — it shows you more of a process than a basic rule can get across by itself.



TIP

This icon is like the sign alerting you to the presence of something special to watch out for on your adventure. It can save you time and energy. Use this information to cut to the chase and avoid unnecessary detours.



REMEMBER

This icon helps you bring back information that you may have misplaced along the way. The information is needed to get you from here to the goal.



WARNING

This icon alerts you to common hazards and stumbling blocks that could trip you up — cause accidents or get you into trouble with the math police. Those who have gone before you have found that these items can cause a big problem — so pay heed.

Where to Go from Here

You can use the table of contents at the beginning of the book and the index in the back to navigate your way to the topic that you’re most interested in. You may want to start with some problem solving — in the form of equations or inequalities. If that’s the case, then look at Chapter 2 for linear equations and inequalities or Chapters 3 and 4 for quadratic and other degree equations. Chapter 5 is your destination if you want to see what constitutes a function and its characteristics. And specific functions such as linear and quadratics are found in Chapter 6; polynomials are found in Chapter 7, rationals in Chapter 8, and exponentials and logs in Chapter 9. I saved the imaginary for last, in Chapter 12. But you could stop off and look at conics in Chapter 10, if those curves are of interest. Also, systems of equations incorporate several types of functions, and you find them in Chapter 11.

And, if you're more of a freewheeling type of guy or gal, take your finger, flip open the book, and mark a spot. No matter your motivation or what technique you use to jump into this book, you won't get lost because you can go in any direction from there.

Enjoy!

- » Making algebra orderly with the order of operations and other properties
- » Enlisting rules of exponents
- » Focusing on factoring

Chapter 1

Making Advances in Algebra

Algebra is a branch of mathematics that people study before they move on to other areas or branches in mathematics and science. Algebra all by itself is esthetically pleasing, but it springs to life when used in other applications.

Any study of science or mathematics involves rules and patterns. You approach the subject with the rules and patterns you already know, and you build on those rules with further study. In this chapter, I recap for you the basic rules from Algebra I so that you work from the correct structure. I present these basics so you can further your study of algebra and feel confident in your algebraic ability.

Bringing Out the Best in Algebraic Properties

Mathematicians developed the rules and properties you use in algebra so that every student, researcher, curious scholar, and bored geek working on the same problem would get the same answer — no matter the time or place.

Making short work of the basic properties

The commutative, associative, and other such properties are not only basic to algebra, but also to geometry and many other mathematical topics. I present the properties here so that I can refer to them as I solve equations and simplify expressions in later chapters.



ALGEBRA
RULES

The commutative property

The *commutative property* applies to the operations of addition and multiplication. It states that you can change the order of the values in an operation without changing the final result:

$$\begin{array}{ll} a + b = b + a & \text{Commutative property of addition} \\ a \cdot b = b \cdot a & \text{Commutative property of multiplication} \end{array}$$

So you can be sure that $2 + 4 = 4 + 2$ and $8 \cdot 7 = 7 \cdot 8$.

The associative property



ALGEBRA
RULES

Like the commutative property (see the preceding section), the *associative property* applies to the operations of addition and multiplication. The *associative property* states that you can change the grouping of operations without changing the result:

$$\begin{array}{ll} a + (b + c) = (a + b) + c & \text{Associative property of addition} \\ a(b \cdot c) = (a \cdot b)c & \text{Associative property of multiplication} \end{array}$$

This property tells you that $3 + (8 + 5) = (3 + 8) + 5$ and that $-4 \cdot (8 \cdot 3) = (-4 \cdot 8) \cdot 3$.

The distributive property



ALGEBRA
RULES

The *distributive property* states that you can multiply each term in an expression within parentheses by the factor outside the parentheses and not change the value of the expression. It takes one operation — multiplication — and spreads it out over terms that you add to and subtract from one another:

$$\begin{array}{ll} a(b + c - d) = & \text{Distributing multiplication over} \\ a \cdot b + a \cdot c - a \cdot d & \text{addition and subtraction} \end{array}$$

For example, you can use the distributive property on the problem $12\left(\frac{1}{2} + \frac{2}{3} - \frac{3}{4}\right)$ to make your life easier. You distribute the 12 over the fractions by multiplying each fraction by 12 and then combining the results: $= 12 \cdot \frac{1}{2} + 12 \cdot \frac{2}{3} - 12 \cdot \frac{3}{4} = 6 + 8 - 9 = 5$.



ALGEBRA
RULES

Identities

The numbers 0 and 1 have special roles in algebra — as *identities*.

$a + 0 = 0 + a = a$ The *additive identity* is 0. Adding 0 to a number doesn't change that number; the number keeps its identity.

$a \cdot 1 = 1 \cdot a = a$ The *multiplicative identity* is 1. Multiplying a number by 1 doesn't change that number; the number keeps its identity.

Inverses



ALGEBRA
RULES

You find two types of *inverses* in algebra — additive inverses and multiplicative inverses:

- » A number and its *additive inverse* add up to 0.
- » A number and its *multiplicative inverse* have a product of 1.

The additive inverse of 6 is -6 , so $6 + (-6) = 0$. And the multiplicative inverse of 6 is $\frac{1}{6}$, so $6 \cdot \frac{1}{6} = 1$.

The multiplication property of zero



ALGEBRA
RULES

The *multiplication property of zero* (MPZ) states that if the product of $a \cdot b \cdot c \cdot d \cdot e \cdot f = 0$, at least one of the terms has to represent the number 0. The only way the product of two or more values can be 0 is for at least one of the values to actually be 0. If you multiply $(16)(467)(11)(9)(0)$, the result is 0. It doesn't really matter what the other numbers are — the 0 always wins.

Organizing your operations

When mathematicians switched from words to symbols to describe mathematical processes, their goal was to make dealing with problems as simple as possible; however, at the same time, they wanted everyone to know what was meant by an expression and for everyone to get the same answer to a problem. Along with

the special notation came a special set of rules on how to handle more than one operation in an expression.



ALGEBRA
RULES

The *order of operations* dictates that you follow this sequence:

1. Raise to powers or find roots.
2. Multiply or divide.
3. Add or subtract.



REMEMBER

If you have to perform more than one operation from the same level, work those operations moving from left to right. If any grouping symbols appear, perform the operation inside the grouping symbols first.

So, to do the problem $4 + 3^2 - 5 \cdot 6 + \sqrt{23 - 7} + \frac{14}{2}$, follow the order of operations:

1. The radical acts like a grouping symbol, so you subtract what's in the radical first to get $4 + 3^2 - 5 \cdot 6 + \sqrt{16} + \frac{14}{2}$.
2. Raise the power and find the root: $4 + 9 - 5 \cdot 6 + 4 + \frac{14}{2}$.
3. Do the multiplication and then the division: $4 + 9 - 30 + 4 + 7$.
4. Add and subtract, moving from left to right: $4 + 9 - 30 + 4 + 7 = 13 - 30 + 4 + 7 = -17 + 4 + 7 = -13 + 7 = -6$.

Enumerating Exponential Rules

Several hundred years ago, mathematicians introduced powers of variables and numbers called *exponents*. Instead of writing xxxxxxxx, you use the exponent 8 by writing x^8 . This form is easier to read and much quicker. The use of exponents expanded to being able to write fractions with negative exponents and radicals with fractional exponents. You find all the details in *Algebra I For Dummies*, 2nd Edition (Wiley).

Multiplying and dividing exponents



ALGEBRA
RULES

When two numbers or variables have the same base, you can multiply or divide those numbers or variables by adding or subtracting their exponents: