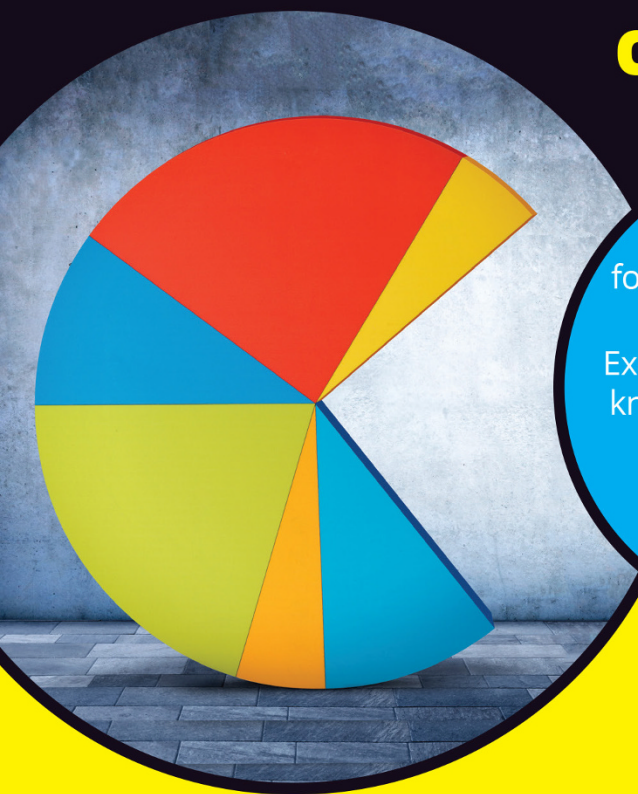


LEARNING MADE EASY



# Pre-Algebra Essentials

for  
**dummies**<sup>®</sup>  
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The “must-know”  
formulas and equations

Exactly what you need to  
know to ace pre-algebra

Concise coverage  
of key topics

**Mark Zegarelli**

Author of *Basic Math & Pre-Algebra  
Workbook For Dummies*





# Pre-Algebra Essentials

by Mark Zegarelli with  
Krista Fanning

for  
**dummies**<sup>®</sup>  
A Wiley Brand

# Pre-Algebra Essentials For Dummies®

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# Introduction

Why do people often enter preschool excited about learning how to count and leave high school as young adults convinced that they can't do math? The answer to this question would probably take 20 books this size, but solving the problem of math aversion can begin right here.

Remember, just for a moment, an innocent time — a time before math inspired panic attacks or, at best, induced irresistible drowsiness. In this book, I take you from an understanding of the basics to the place where you're ready to enter any algebra class and succeed.

## About This Book

Somewhere along the road from counting to algebra, most people experience the Great Math Breakdown. Please consider this book your personal roadside helper, and think of me as your friendly math mechanic (only much cheaper!). The tools for fixing the problem are in this book.

I've broken down the concepts into easy-to-understand sections. And because *Pre-Algebra Essentials For Dummies* is a reference book, you don't have to read the chapters or sections in order — you can look over only what you need. So feel free to jump around. Whenever I cover a topic that requires information from earlier in the book, I refer you to that section or chapter in case you want to refresh yourself on the essentials.

Note that this book covers only need-to-know info. For a broader look at pre-algebra, you can pick up a copy of *Basic Math & Pre-Algebra For Dummies* or the corresponding workbook.

# Conventions Used in This Book

To help you navigate your way through this book, I use the following conventions:

- » *Italicized* text highlights new words and defined terms.
- » **Boldfaced** text indicates keywords in bulleted lists and the action part of numbered steps.
- » `Mono font` text highlights web addresses.
- » Variables, such as  $x$  and  $y$ , are in italics.

# Foolish Assumptions

If you're planning to read this book, you're likely

- » A student who wants a solid understanding of the core concepts for a class or test you're taking
- » A learner who struggled with algebra and wants a reference source to ensure success in the next level
- » An adult who wants to improve skills in arithmetic, fractions, decimals, percentages, geometry, algebra, and so on for when you have to use math in the real world
- » Someone who wants a refresher so you can help another person understand math

My only assumption about your skill level is that you can add, subtract, multiply, and divide. So to find out whether you're ready for this book, take this simple test:

$$5 + 6 = \underline{\quad}$$

$$10 - 7 = \underline{\quad}$$

$$3 \times 5 = \underline{\quad}$$

$$20 \div 4 = \underline{\quad}$$

If you can answer these four questions, you're ready to begin.

# Icons Used in This Book

Throughout the book, I use three icons to highlight what's hot and what's not:



REMEMBER

This icon points out key ideas that you need to know. Make sure you understand before reading on! Remember this info even after you close the book.



TIP

Tips are helpful hints that show you the quick and easy way to get things done. Try them out, especially if you're taking a math course.



WARNING

Warnings flag common errors that you want to avoid. Get clear about where these little traps are hiding so you don't fall in.

## Where to Go from Here

You can use this book in a few ways. If you're reading this book without immediate time pressure from a test or homework assignment, you can certainly start at the beginning and keep on going through to the end. The advantage to this method is that you realize how much math you *do* know — the first few chapters go very quickly. You gain a lot of confidence as well as some practical knowledge that can help you later on, because the early chapters also set you up to understand what follows.

Or how about this: When you're ready to work, read up on the topic you're studying. Leave the book on your nightstand and, just before bed, spend a few minutes reading the easy stuff from the early chapters. You'd be surprised how a little refresher on simple stuff can suddenly cause more-advanced concepts to click.

If your time is limited — especially if you're taking a math course and you're looking for help with your homework or an upcoming test — skip directly to the topic you're studying. Wherever you open the book, you can find a clear explanation of the topic at hand, as well as a variety of hints and tricks. Read through the examples and try to do them yourself, or use them as templates to help you with assigned problems.





## IN THIS CHAPTER

- » Identifying four important sets of numbers
- » Reviewing addition, subtraction, multiplication, and division
- » Examining commutative, associative, and distributive operations
- » Knowing exponents, roots, and absolute values
- » Understanding how factors and multiples are related

# Chapter 1

## Arming Yourself with Math Basics

**Y**ou already know more about math than you think you know. In this chapter, you review and gain perspective on basic math ideas such as sets of numbers and concepts related to the Big Four operations (adding, subtracting, multiplying, and dividing). I introduce you (or reintroduce you) to properties and operations that will assist with solving problems. Finally, I explain the relationship between factors and multiples, taking you from what you may have missed to what you need to succeed as you move onward and upward in math.

## Understanding Sets of Numbers

You can use the number line to deal with four important *sets* (or groups) of numbers. Each set builds on the one before it:

- » **Counting numbers (also called natural numbers):** The set of numbers beginning 1, 2, 3, 4, . . . and going on infinitely

- » **Integers:** The set of counting numbers, zero, and negative counting numbers
- » **Rational numbers:** The set of integers and fractions
- » **Real numbers:** The set of rational and irrational numbers

Even if you filled in all the rational numbers, you'd still have points left unlabeled on the number line. These points are the irrational numbers.

An *irrational number* is a number that's neither a whole number nor a fraction. In fact, an irrational number can only be approximated as a *non-repeating decimal*. In other words, no matter how many decimal places you write down, you can always write down more; furthermore, the digits in this decimal never become repetitive or fall into any pattern. (For more on repeating decimals, see Chapter 5.)

The most famous irrational number is  $\pi$  (you find out more about  $\pi$  when I discuss the geometry of circles in Chapter 11):

$$\pi = 3.14159265358979323846264338327950288419716939937510...$$

Together, the rational and irrational numbers make up the *real numbers*, which comprise every point on the number line.

## The Big Four Operations

When most folks think of math, the first thing that comes to mind is four little (or not-so-little) words: addition, subtraction, multiplication, and division. I call these operations the *Big Four* all through the book.

### Adding things up

Addition is the first operation you find out about, and it's almost everybody's favorite. Addition is all about bringing things together, which is a positive thing. This operation uses only one sign — the plus sign (+).



REMEMBER

When you add two numbers together, those two numbers are called *addends*, and the result is called the *sum*.



TIP

Adding a negative number is the same as subtracting, so  $7 + -3$  is the same as  $7 - 3$ .

## Take it away: Subtracting

Subtraction is usually the second operation you discover, and it's not much harder than addition. As with addition, subtraction has only one sign: the minus sign ( $-$ ).



REMEMBER

When you subtract one number from another, the result is called the *difference*. This term makes sense when you think about it: When you subtract, you find the difference between a higher number and a lower one.



TIP

Subtracting a negative number is the same as adding a positive number, so  $2 - (-3)$  is the same as  $2 + 3$ . When you're subtracting, you can think of the two minus signs canceling each other out to create a positive.

## Multiplying

Multiplication is often described as a sort of shorthand for repeated addition. For example,

$$4 \times 3 \text{ means add 4 to itself 3 times: } 4 + 4 + 4 = 12$$

$$9 \times 6 \text{ means add 9 to itself 6 times: } 9 + 9 + 9 + 9 + 9 + 9 = 54$$



REMEMBER

When you multiply two numbers, the two numbers that you're multiplying are called *factors*, and the result is the *product*. In the preceding example, 4 and 3 are the factors and 12 is the product.

When you're first introduced to multiplication, you use the *times* sign ( $\times$ ). However, algebra uses the letter  $x$  a lot, which looks similar to the times sign, so people often choose to use other multiplication symbols for clarity.

## Arriving on the dot

In math beyond arithmetic, the symbol  $\cdot$  replaces  $\times$ . For example,

$$6 \cdot 7 = 42 \text{ means } 6 \times 7 = 42$$

$$53 \cdot 11 = 583 \text{ means } 53 \times 11 = 583$$

That's all there is to it: Just use the  $\cdot$  symbol anywhere you would've used the standard times sign ( $\times$ ).

## Speaking parenthetically



REMEMBER

In math beyond arithmetic, using parentheses *without* another operator stands for multiplication. The parentheses can enclose the first number, the second number, or both numbers. For example,

$$3(5) = 15 \text{ means } 3 \times 5 = 15$$

$$(8)7 = 56 \text{ means } 8 \times 7 = 56$$

$$(9)(10) = 90 \text{ means } 9 \times 10 = 90$$

However, notice that when you place another operator between a number and a parenthesis, that operator takes over. For example,

$$3 + (5) = 8 \text{ means } 3 + 5 = 8$$

$$(8) - 7 = 1 \text{ means } 8 - 7 = 1$$

## Doing division lickety-split

The last of the Big Four operations is division. *Division* literally means splitting things up. For example, suppose you're a parent on a picnic with your three children. You've brought along 12 pretzel sticks as snacks and want to split them fairly so that each child gets the same number (don't want to cause a fight, right?).

Each child gets four pretzel sticks. This problem tells you that

$$12 \div 3 = 4$$

As with multiplication, division also has more than one sign: the *division sign* ( $\div$ ) and the *fraction slash* (/) or *fraction bar* ( $\frac{\quad}{\quad}$ ). So some other ways to write the same information are

$$12/3 = 4 \text{ and } \frac{12}{3} = 4$$



REMEMBER

When you divide one number by another, the first number is called the *dividend*, the second is called the *divisor*, and the result is the *quotient*. For example, in the division from the earlier example, the dividend is 12, the divisor is 3, and the quotient is 4.