Samuli Siltanen

Step into the World of Mathematics

Math Is Beautiful and Belongs to All of Us

Translated by Lauri Snellman

Springer

Step into the World of Mathematics

Samuli Siltanen

Step into the World of Mathematics

Math Is Beautiful and Belongs to All of Us

Translated by Lauri Snellman

Springer

Samuli Siltanen
Department of Mathematics & Statistics
University of Helsinki
Helsinki, Finland

Translated by
Lauri Snellman
Espoo, Finland

This work has been published with the financial assistance of FILI – Finnish Literature Exchange.

ISBN 978-3-030-73342-1 ISBN 978-3-030-73343-8 (eBook)
https://doi.org/10.1007/978-3-030-73343-8

This Springer imprint is published by the registered company Springer Nature Switzerland AG.
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Mathematics—Our Invisible Friend

"What use will this ever be?" Every maths teacher has heard the question during lessons on logarithms, derivatives, or arcus tangents. Of course, we can cope with our everyday lives without using them. Even when we are enhancing a selfie, driving to a wedding in Hankasalmi, or clicking the cheapest flight to London on the Internet.

But what is happening under the hood? The Instagram filter raises the numerical values of the pixels into the power of one-half, and goes for a fast Fourier transform on the frequency domain to spice up the outlines of our selfie. The navigator knows our position, because the complex corrective terms given by Einstein's theory of relativity have been programmed into its GPS system. The prices of plane tickets change every second depending on how the clicks of the holiday fanatics having an adventure on the website change a statistical formula.

The math that runs our digital everyday lives is hidden into mobile apps and into workplace computer systems. Even the much talked-about AI runs only additions and multiplications and thus roughly emulates the functioning of the network of nerve cells in the brain. All of these

mathematical calculations are hidden, and the layman does not know about it—before reading this book!

In my book, I tell you about this hidden mathematics, introduce its uses and developers to you, and describe how new mathematics is developed. I do not require any mathematical skills from you, dear reader. You can view me as a travel writer, who describes the mysterious valleys and strange ways of life in a faraway country. You do not have to be an expert to enjoy my tales praising the landscapes of the world of mathematics.

These three fellows come up time to time in my story: the average, the intermediate model, and optimization.

You already know averages from your school grades and different statistics. That mathematical operation is modest and simple, but it is powerful! The influence of averages can be felt everywhere, just like the Force in Star Wars. Though you do not have to be a Jedi knight to use averages.

Those Finnish readers who have watched Kummeli probably remember *Kari ja Karvattomat*, a dance band. Its most swinging song goes in English: "*Intermediate model guy, intermediate model song, intermediate model face*". The song gets out of me this artistic interpretation: even though the artist does not sell the Helsinki Olympic Stadium full and the song does not compete for the Finnish Copyright Association's Award, the song can be OK and get the partygoers dancing. But what does Kummeli-Kari's song have to do with mathematics?

A *mathematical model* is a mathematical formula that imitates some phenomenon of life. Robert Malthus' rule for predicting the population of the world is an example of them. The Malthusian growth model determines the time for the doubling of the human population. The prediction holds good for decade-long observations, but over 1000 of years it leads to predictions of 17 people over every square

metre on Earth, including the seas. Of course, this cannot really happen, so there is something wrong with the model. The predictive accuracy of the Malthusian model becomes very poor in the long run. The same holds true for every other mathematical model too: they are inaccurate but useful within suitably fixed limits. That is why I sometimes call models *intermediate models,* when I want to remind of their limited validity.

Computer modelling is for the most part based on optimization, which aims at finding the best possible circumstances. For example, a bicycle designer may have fixed a maximum weight for the bike and a goal of finding the best shape for its frame. The bike should take the maximum stress, be easy to manufacture, and also look cool to sell a lot. One can imagine a *penalty* that depends on the shape of the frame to aid in the design process. The penalty grows bigger, when a proposed design goes against the design criteria. The mathematical solution to the problem resembles a hiker skating in a mountain range of icebergs and looking for a way to the bottom of the valley.

I will also tell about how we researchers create new mathematics. This might be a surprise to the proud granny I met on a train, whose grandchild "has already read *all of math*". She could have been thinking that all of mathematics is already finished and collected in some leather-bound book, and the brisk kid hoovers it into his fair-haired head at school. Dear grandma, mathematics is a living science and more of it is born every day! Your grandchild might see it as liberating: there is always more nice stuff to learn!

We all need mathematics every day, without knowing it. But you can learn to know it! Welcome to the world of mathematics. It is reliable, surprising, and most beautiful.

What Is in the Book?

I tell about my own background in Chap. 1. I have liked calculating and solving problems since I was a kid. When I grew up, I ended up first as developing medical imaging technologies for industrial companies and then as a maths professor. I see mathematics everywhere.

Chapter 2 is a gentle introduction to the main theme of the book, *mathematical models.* They are digital helpers that make our thinking more effective just like escalators make moving into higher floors more effective. Mathematical models try to imitate some living phenomenon as accurately as possible by using different kinds of calculations. The models devour data about our world, our messages, our pictures, our thermometers, and our choices of music. They edit and supplement the data. They guide our vacuum cleaners, our timetables, and our feelings.

In Chap. 3, I introduce *climate models* and models for gravity waves that are rippling in the universe. We would not know anything about them without mathematical modelling. I also write about how mathematics could help us to solve the big problems facing mankind. Natural science and technology are a most natural field for mathematics, because "the book of nature is written in the language of mathematics", as Galilei said.

Chapter 4 is dedicated to the use of mathematics as an aid to medicine. I use statistical research and imaging as examples. I especially concentrate on my own field of study of X-ray tomography, which gives doctors a very accurate 3-D map of the internal organs of the patient.

In Chap. 5, I move to a more controversial topic. I ask, if mathematics can describe human thoughts, wants, creativity, or even love? There are already signs of this, when an AI

plays chess better than a human being and reaches poetic heights when translating literature.

Chapter 6 is dedicated to the idea that everyone has a right to basic mathematical skills and that everyone should have an equal opportunity to work in mathematics.

I tried to write about mathematics in an easy to approach and fun way. If you hit on a passage you find difficult, please move onwards in the text. Easier stuff can be found there.

Contents

1

My Adventures in the World of Mathematics

I'll Reveal It Straight Away I'm thoroughly absorbed by mathematics. I feel that I'm a lamb leg that has been marinading for decades. The garlic of the marinade has been replaced with logarithms, and the olive oil with fractions.

In this chapter I'll tell you about my mathematical background, so that you'll get a better idea of the world I'm inviting you into.

1.1 Primary School

As a small boy, I was a natural science nerd living in Töölö. I did electric experiments on the differences between direct and alternating currents and on voltages. A little electric motor from a slide projector went faster on a 9 volt battery than on a 4.5 volt one. Adding a small adjustable resistor or a potentiometer made possible a smooth adjustment of the speed. Great! Replacing the battery with a 230 volt mains current didn't give an even faster speed, but burnt the fuse and broke the engine.

S. Siltanen, *Step into the World of Mathematics*,
https://doi.org/10.1007/978-3-030-73343-8_1

On the other hand, the engine of the slide projector didn't work at all on batteries. It required a mains current. Did adding a potentiometer to this circuit slow down the rotation? In a way yes, because the fan stopped, burning the fuse and Mum's nerves with it.

I got a chemistry set for Christmas. After I've gone through the tricks in the manual, it was time to improvise. Adding baking soda to vinegar led to nice bubbles, which I tried to stop by putting a bottle stopper on the test tube. I cleaned up broken glass from my room after the explosion, and Mum wasn't happy on this time either.

My other experiments included putting together a rocket boat out of fuel torn from sparkler sticks and dropping rotating "paper landers" from a seventh-floor balcony. The result was a half a metre flame spouting from toy a boat standing still and parachute strings that were tangled by rotation.

Kids and youngsters: be careful with electricity and fire! It was just luck that I didn't hurt myself with my experiments.

My physics and mathematics experiments taught me a lot of things that don't work. My tests in the field of mathematics gave me many experiences of success, and maybe therefore math became my calling.

As a primary school pupil, I found somewhere a method for calculating square roots with a pen and paper. To my great delight, I determined the square root of 2 more accurately than Dad's calculator (that is, with 13 decimals 14,142,135,623,731). Dad worked at Taivallahti primary school, which was my school. He brought old math textbooks home from there. I solved all their exercises out of the sheer joy of calculating. I felt great happiness when I was able to study power and logarithmic functions years before they were taught in class.