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Pre-Calculus Workbook

3rd Edition

by Mary Jane Sterling

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Pre-Calculus Workbook For Dummies®, 3rd Edition

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Contents at a Glance

Introduction	1
 Part 1: Setting the Foundation: The Nuts and Bolts of Pre-Calculus	5
CHAPTER 1: Preparing for Pre-Calculus	7
CHAPTER 2: Real Numbers Come Clean	25
CHAPTER 3: Controlling Functions by Knowing Their Function	39
CHAPTER 4: Searching for Roots	75
CHAPTER 5: Exponential and Logarithmic Functions	95
 Part 2: Trig Is the Key: Basic Review, the Unit Circle, and Graphs	113
CHAPTER 6: Basic Trigonometry and the Unit Circle	115
CHAPTER 7: Graphing and Transforming Trig Functions	137
 Part 3: Digging into Advanced Trig: Identities, Theorems, and Applications	155
CHAPTER 8: Basic Trig Identities	157
CHAPTER 9: Advanced Trig Identities	175
CHAPTER 10: Solving Oblique Triangles	193
 Part 4: Polar Coordinates, Cones, Solutions, Sequences, and Finding Your Limits	209
CHAPTER 11: Exploring Complex Numbers and Polar Coordinates	211
CHAPTER 12: Conquering Conic Sections	229
CHAPTER 13: Finding Solutions for Systems of Equations	265
CHAPTER 14: Spotting Patterns in Sequences and Series	301
CHAPTER 15: Previewing Calculus	315
 Part 5: The Part of Tens	329
CHAPTER 16: Ten Plus Parent Graphs	331
CHAPTER 17: Ten Missteps to Avoid	341
 Index	345

Table of Contents

INTRODUCTION	1
About This Book	1
Foolish Assumptions	2
Icons Used in This Book	2
Beyond the Book	3
Where to Go from Here	3
 PART 1: SETTING THE FOUNDATION: THE NUTS AND BOLTS OF PRE-CALCULUS	 5
CHAPTER 1: Preparing for Pre-Calculus	7
Reviewing Order of Operations: The Fun in Fundamentals	8
Keeping Your Balance While Solving Equalities	10
When Your Image Really Counts: Graphing Equalities and Inequalities	12
Graphing with two points	12
Graphing by using the slope-intercept form	13
Graphing inequalities	14
Using Graphs to Find Distance, Midpoint, and Slope	15
Finding the distance	15
Calculating the midpoint	16
Discovering the slope	16
Answers to Problems on Fundamentals	19
 CHAPTER 2: Real Numbers Come Clean	 25
Solving Inequalities	25
Expressing Inequality Solutions in Interval Notation	28
Radicals and Exponents — Just Simplify!	30
Getting Out of a Sticky Situation, or Rationalizing	33
Answers to Problems on Real Numbers	35
 CHAPTER 3: Controlling Functions by Knowing Their Function	 39
Using Both Faces of the Coin: Even and Odd	40
Leaving the Nest: Transforming Parent Graphs	42
Quadratic functions	42
Square root functions	42
Absolute value functions	43
Cubic functions	43
Cube root functions	44
Steeper or flatter	44
Translations	46
Reflections	46
Combinations of transformations	46
Graphing Rational Functions	49
Piecing Together Piecewise Functions	52
Combining Functions	54

Evaluating Composition of Functions	55
Working Together: Domain and Range	57
Unlocking the Inverse of a Function: Turning It Inside Out	59
Answers to Problems on Functions.	61
CHAPTER 4: Searching for Roots	75
Factoring a Factorable Quadratic	75
Solving a Quadratic Polynomial Equation	78
Completing the square	78
Quadratic formula	79
Solving High-Order Polynomials	80
Factoring by grouping	80
Determining positive and negative roots: Descartes' Rule of Signs	81
Counting on imaginary roots	81
Getting the rational roots	81
Finding roots through synthetic division	82
Using Roots to Create an Equation	84
Graphing Polynomials	85
Answers to Problems on Roots and Degrees	89
CHAPTER 5: Exponential and Logarithmic Functions	95
Working with Exponential Functions	95
Eagerly Engaging Edgy Logarithmic Solutions	98
Making Exponents and Logs Work Together	101
Using Exponents and Logs in Practical Applications	103
Answers to Problems on Exponential and Logarithmic Functions	106
PART 2: TRIG IS THE KEY: BASIC REVIEW, THE UNIT CIRCLE, AND GRAPHS	113
CHAPTER 6: Basic Trigonometry and the Unit Circle	115
Finding the Six Trigonometric Ratios	115
Solving Word Problems with Right Triangles	118
Unit Circle and the Coordinate Plane: Finding Points and Angles	121
Finding Ratios from Angles on the Unit Circle	124
Solving Trig Equations	127
Making and Measuring Arcs	129
Answers to Problems on Basic Trig and the Unit Circle	131
CHAPTER 7: Graphing and Transforming Trig Functions	137
Getting a Grip on Periodic Graphs	137
Parent Graphs and Transformations: Sine and Cosine	138
Tangent and Cotangent: More Family Members	141
Generations: Secant and Cosecant	143
Answers to Problems on Graphing and Transforming Trig Functions	147

PART 3: DIGGING INTO ADVANCED TRIG: IDENTITIES, THEOREMS, AND APPLICATIONS	155
CHAPTER 8: Basic Trig Identities	157
Using Reciprocal Identities to Simplify Trig Expressions	157
Simplifying with Pythagorean Identities	159
Discovering Even-Odd Identities	160
Simplifying with Co-Function Identities	162
Moving with Periodicity Identities	163
Tackling Trig Proofs (Identities)	165
Answers to Problems on Basic Trig Identities	167
CHAPTER 9: Advanced Trig Identities	175
Simplifying with Sum and Difference Identities	175
Using Double-Angle Identities	178
Reducing with Half-Angle Identities	180
Changing Products to Sums	181
Expressing Sums as Products	182
Powering Down: Power-Reducing Formulas	184
Answers to Problems on Advanced Trig Identities	186
CHAPTER 10: Solving Oblique Triangles	193
Solving a Triangle with the Law of Sines: ASA and AAS	194
Tackling Triangles in the Ambiguous Case: SSA	195
Conquering a Triangle with the Law of Cosines: SAS and SSS	197
Using Oblique Triangles to Solve Practical Applications	198
Figuring Area	201
Answers to Problems on Solving Triangles	202
PART 4: POLAR COORDINATES, CONES, SOLUTIONS, SEQUENCES, AND FINDING YOUR LIMITS	209
CHAPTER 11: Exploring Complex Numbers and Polar Coordinates	211
Performing Operations with and Graphing Complex Numbers	212
Round a Pole: Graphing Polar Coordinates	215
Changing to and from Polar	217
Graphing Polar Equations	220
Archimedean spiral	220
Cardioid	220
Rose	220
Circle	220
Lemniscate	220
Limaçon	221
Answers to Problems on Complex Numbers and Polar Coordinates	223

CHAPTER 12: Conquering Conic Sections	229
A Quick Conic Review	230
Going Round and Round with Circles	230
The Ups and Downs: Graphing Parabolas	232
Standing tall: Vertical parabolas	233
Lying down on the job: Horizontal parabolas	235
The Fat and the Skinny: Graphing Ellipses	237
Short and fat: The horizontal ellipse	237
Tall and skinny: The vertical ellipse	239
No Caffeine Required: Graphing Hyperbolas	241
Horizontal hyperbolas	241
Vertical hyperbolas	244
Identifying Conic Sections	246
Conic Sections in Parametric Form and Polar Coordinates	248
Parametric form for conic sections	248
Changing from parametric form to rectangular form	250
Conic sections on the polar coordinate plane	251
Answers to Problems on Conic Sections	253
CHAPTER 13: Finding Solutions for Systems of Equations	265
A Quick-and-Dirty Technique Overview	266
Solving Two Linear Equations with Two Variables	266
The substitution method	267
The elimination method	268
Not-So-Straight: Solving Nonlinear Systems	269
One equation that's linear and one that isn't	269
Two nonlinear equations	270
Systems of rational equations	271
Systems of More Than Two Equations	272
Graphing Systems of Inequalities	274
Breaking Down Decomposing Partial Fractions	276
Working with a Matrix	278
Getting It in the Right Form: Simplifying Matrices	281
Solving Systems of Equations Using Matrices	283
Gaussian elimination	283
Inverse matrices	285
Cramer's Rule	287
Answers to Problems on Systems of Equations	289
CHAPTER 14: Spotting Patterns in Sequences and Series	301
General Sequences and Series: Determining Terms	301
Working Out the Common Difference: Arithmetic Sequences and Series	303
Simplifying Geometric Sequences and Series	305
Expanding Polynomials Using the Binomial Theorem	308
Answers to Problems on Sequences, Series, and Binomials	310

CHAPTER 15: Previewing Calculus	315
Finding Limits: Graphically, Analytically, and Algebraically	316
Graphically	316
Analytically	318
Algebraically	319
Knowing Your Limits	321
Calculating the Average Rate of Change	322
Determining Continuity	323
Answers to Problems on Calculus	326
 PART 5: THE PART OF TENS	 329
CHAPTER 16: Ten Plus Parent Graphs	331
Squaring Up with Quadratics	331
Cueing Up for Cubics	332
Rooting for Square Roots and Cube Roots	333
Graphing Absolutely Fabulous Absolute Value Functions	334
Flipping over Rational Functions	334
Exploring Exponential Graphs and Logarithmic Graphs	335
Seeing the Sine and Cosine	336
Covering Cosecant and Secant	337
Tripping over Tangent and Cotangent	338
Lining Up and Going Straight with Lines	339
 CHAPTER 17: Ten Missteps to Avoid	 341
Going Out of Order (of Operations)	341
FOILing Binomials Incorrectly	342
Breaking Up Fractions Badly	342
Combining Terms That Can't Be Combined	342
Forgetting to Flip the Fraction	342
Losing the Negative (Sign)	343
Oversimplifying Roots	343
Executing Exponent Errors	343
Ignoring Extraneous	344
Misinterpreting Trig Notation	344
 INDEX	 345

Introduction

I hope that you'll find this workbook to be a big help with pre-calculus. If you've gotten this far in your math career, congratulations! Many students choose to stop their math education after they complete Algebra II, but not you!

If you've picked up this book (and obviously you have, given that you're reading this sentence!), maybe some of the concepts in pre-calculus are giving you a hard time, or perhaps you just want more practice. Maybe you're deciding whether you even want to take pre-calculus at all. This book fits the bill for all those reasons. And it's here to encourage you on your pre-calculus adventure.

You'll find this workbook chock-full of valuable practice problems and explanations. In instances where you feel you may need a more thorough explanation, please refer to *Pre-Calculus For Dummies* by Mary Jane Sterling (Wiley). This book, however, is a great stand-alone workbook if you need extra practice or want to just brush up in certain areas.

About This Book

Don't let pre-calculus scare you. When you realize that you already know a whole bunch from Algebra I and Algebra II, you'll see that pre-calculus is really just using that old information in a new way. And even if you're scared, I'm here with you, so no need to panic. Before you get ready to start this new adventure, you need to know a few things about this book.

This book isn't a novel. It's not meant to be read in order from beginning to end. You can read any topic at any time, but it's structured in such a way that it follows the "normal" curriculum. This is hard to do, because most states don't have state standards for what makes pre-calculus pre-calculus. Looking at a good sampling of curriculums, though, this should be a good representation of a pre-calculus course.

Here are two suggestions for using this book:

- » Look up what you need to know when you need to know it. The index and the table of contents direct you where to look.
- » Start at the beginning and read straight through. This way, you may be reminded of an old topic that you had forgotten (anything to get those math wheels churning inside your head). Besides, practice makes perfect, and the problems in this book are a great representation of the problems found in pre-calculus textbooks.

For consistency and ease of navigation, this book uses the following conventions:

- » Math terms are *italicized* when they're introduced or defined in the text.
- » Variables are *italicized* to set them apart from letters.
- » The symbol used when writing imaginary numbers is a lowercase i .

Foolish Assumptions

I don't assume that you love math the way I do, but I do assume that you picked this book up for a reason of your own. Maybe you want a preview of the course before you take it, or perhaps you need a refresher on the topics in the course, or maybe your kid is taking the course and you're trying to help him to be more successful.

Whatever your reason, I assume that you've encountered most of the topics in this book before, because for the most part, they review what you've seen in algebra or geometry.

Icons Used in This Book

Throughout this book you'll see icons in the margins to draw your attention to something important that you need to know.



EXAMPLE

You see this icon when I present an example problem whose solution I walk you through step by step. You get a problem and a detailed answer.



TIP

Tips are great, especially if you wait tables for a living! These tips are designed to make your life easier, which are the best tips of all!



TECHNICAL
STUFF

The material following this icon is wonderful mathematics; it's closely related to the topic at hand, but it's not absolutely necessary for your understanding of the material being presented. You can take it or leave it — you'll be fine just taking note and leaving it behind as you proceed through the section.



REMEMBER

The Remember icon is used one way: It asks you to remember old material from a previous math course.



WARNING

Warnings are big red flags that draw your attention to common mistakes that may trip you up.

Beyond the Book

No matter how well you understand the concepts of algebra, you'll likely come across a few questions where you don't have a clue. Be sure to check out the free Cheat Sheet for a handy guide that covers tips and tricks for answering pre-calculus questions. To get this Cheat Sheet, simply go to www.dummies.com and enter "Pre-Calculus Workbook For Dummies" in the Search box.

The online practice that comes free with this book contains over 300 questions so you can really hone your pre-calculus skills! To gain access to the online practice, all you have to do is register. Just follow these simple steps:

1. **Register your book or ebook at Dummies.com to get your PIN. Go to www.dummies.com/go/getaccess.**
2. **Select your product from the dropdown list on that page.**
3. **Follow the prompts to validate your product, and then check your email for a confirmation message that includes your PIN and instructions for logging in.**

If you do not receive this email within two hours, please check your spam folder before contacting us through our Technical Support website at <http://support.wiley.com> or by phone at 877-762-2974.

Now you're ready to go! You can come back to the practice material as often as you want — simply log on with the username and password you created during your initial login. No need to enter the access code a second time.

Your registration is good for one year from the day you activate your PIN.

Where to Go from Here

Pick a starting point in the book and go practice the problems there. If you'd like to review the basics first, start at Chapter 1. If you feel comfy enough with your algebra skills, you may want to skip that chapter and head over to Chapter 2. Most of the topics there are reviews of Algebra II material, but don't skip over something because you think you have it under control. You'll find in pre-calculus that the level of difficulty in some of these topics gets turned up a notch or two. Go ahead — dive in and enjoy the world of pre-calculus!

If you're ready for another area of mathematics, look for a couple more of my titles: *Trigonometry For Dummies* and *Linear Algebra For Dummies*.

1

**Setting the
Foundation:
The Nuts
and Bolts of
Pre-Calculus**

IN THIS PART . . .

Pre-calculus is really just another stop on the road to calculus. The chapters in this part begin with a review of the basics: using the order of operations, solving and graphing equations and inequalities, and using the distance and midpoint formulas. Some new material pops up in the form of interval notation, so be sure and check that out. As you move on to real numbers you find yourself focusing on radicals. Everything you ever wanted to know about functions is covered here: graphing and transforming parent graphs, dealing with rational functions, and piecewise functions. You'll see how to perform operations on functions and how to find the inverse. Then you move on to solving higher-degree polynomials using techniques like factoring, completing the square, and the quadratic formula. You also find out how to graph these complicated polynomials. Lastly, you discover exponential and logarithmic functions and what you're expected to know about them.

IN THIS CHAPTER

- » Brushing up on the order of operations
- » Solving equalities
- » Graphing equalities and inequalities
- » Finding distance, midpoint, and slope

Chapter 1

Preparing for Pre-Calculus

Pre-calculus is the stepping stone for calculus. It's the final stepping stone after all those years of math: algebra I, geometry, algebra II, and trigonometry. Now all you need is pre-calculus to get to that ultimate goal — calculus. And as you may recall from your algebra II class, you were subjected to much of the same material you saw in algebra and even pre-algebra (just a couple steps up in terms of complexity — but really the same stuff). Pre-calculus begins with certain concepts that you need to be successful in any mathematics course.

If you feel you're already an expert at everything algebra, feel free to skip past this chapter and get the full swing of pre-calculus going. If you have any doubts or concerns, however, you may want to review; read on.



TIP

If you don't remember some of the concepts discussed in this chapter, or even in this book, you can pick up another *For Dummies* math book for review. The fundamentals are important. That's why they're called fundamentals. Take the time now to review and save yourself frustration and possible math errors in the future!

Reviewing Order of Operations: The Fun in Fundamentals

You can't put on your sock after you put on your shoe, can you? At least, you shouldn't! The same concept applies to mathematical operations. There's a specific order as to which operation you perform first, second, third, and so on. At this point, it should be second nature, but because the concept is so important (especially when you start doing more complex calculations), a quick review is worth it, starting with everyone's favorite mnemonic device.



TIP

Please excuse who? Oh, yeah, you remember this one — my dear Aunt Sally! The old mnemonic still stands, even as you get into more complicated problems. Please Excuse My Dear Aunt Sally is a mnemonic for the acronym PEMDAS, which stands for

- » Parentheses (including absolute value, brackets, fraction lines, and radicals)
- » Exponents (and roots)
- » Multiplication and Division (from left to right)
- » Addition and Subtraction (from left to right)

The order in which you solve algebraic problems is very important. Always compute what's in the parentheses first, then move on to the exponents, followed by the multiplication and division (from left to right), and finally, the addition and subtraction (from left to right).



TECHNICAL
STUFF

You should also have a good grasp on the properties of equality. If you do, you'll have an easier time simplifying expressions. Here are the properties:

- » **Reflexive property:** $a = a$. For example, $4 = 4$.
- » **Symmetric property:** If $a = b$, then $b = a$. For example, if $2 + 8 = 10$, then $10 = 2 + 8$.
- » **Transitive property:** If $a = b$ and $b = c$, then $a = c$. For example, if $2 + 8 = 10$ and $10 = 5 \cdot 2$, then $2 + 8 = 5 \cdot 2$.
- » **Commutative property of addition:** $a + b = b + a$. For example, $3 + 4 = 4 + 3$.
- » **Commutative property of multiplication:** $a \cdot b = b \cdot a$. For example, $3 \cdot 4 = 4 \cdot 3$.
- » **Associative property of addition:** $a + (b + c) = (a + b) + c$. For example, $3 + (4 + 5) = (3 + 4) + 5$.
- » **Associative property of multiplication:** $a \cdot (b \cdot c) = (a \cdot b) \cdot c$. For example, $3 \cdot (4 \cdot 5) = (3 \cdot 4) \cdot 5$.
- » **Additive identity:** $a + 0 = a$. For example, $4 + 0 = 4$.
- » **Multiplicative identity:** $a \cdot 1 = a$. For example, $-18 \cdot 1 = -18$.
- » **Additive inverse property:** $a + (-a) = 0$. For example, $5 + (-5) = 0$.
- » **Multiplicative inverse property:** $a \cdot \frac{1}{a} = 1$, as long as $a \neq 0$. For example, $-2 \cdot \left(-\frac{1}{2}\right) = 1$.

» **Distributive property:** $a(b + c) = a \cdot b + a \cdot c$. For example, $5(4 + 3) = 5 \cdot 4 + 5 \cdot 3$.

» **Multiplicative property of zero:** $a \cdot 0 = 0$. For example, $4 \cdot 0 = 0$.

» **Zero product property:** If $a \cdot b = 0$, then $a = 0$ or $b = 0$. For example, if $x(2x - 3) = 0$, then $x = 0$ or $(2x - 3) = 0$.

Following are a couple examples so you can see the order of operations and the properties of equality in action before diving into some practice questions.



EXAMPLE

Q. Simplify: $\frac{6^2 - 4(3 - \sqrt{20 + 5})^2}{|4 - 8|}$

A. The answer is 5.

Following the order of operations, simplify everything in parentheses first. (Remember that radicals and absolute value bars act like parentheses, so do operations within them first before simplifying the radicals or taking the absolute value.)

$$= \frac{6^2 - 4(3 - \sqrt{25})^2}{|-4|}$$

Simplify the parentheses by taking the square root of 25 and subtracting the result from 3; find the absolute value of -4 :

$$= \frac{6^2 - 4(3 - 5)^2}{|-4|} = \frac{6^2 - 4(-2)^2}{4}$$

Now you can deal with the exponents by squaring the 6 and the -2 :

$$= \frac{36 - 4 \cdot 4}{4}$$

Although they're not written, parentheses are implied around the terms above and below a fraction bar. Therefore, you must simplify the numerator and denominator before dividing the terms following the order of operations:

$$= \frac{36 - 16}{4} = \frac{20}{4} = 5$$

Q. Simplify: $\frac{\left(\frac{1}{8} + \frac{1}{3}\right) + \frac{3}{8}}{\frac{1}{6} + \frac{1}{9}}$

A. The answer is 3.

Using the associative property and the commutative property of addition, rewrite the expression to make the fractions easier to add. Then add the fractions with the common denominators.

$$= \frac{\left(\frac{1}{8} + \frac{3}{8}\right) + \frac{1}{3}}{\frac{1}{6} + \frac{1}{9}} = \frac{\left(\frac{4}{8}\right) + \frac{1}{3}}{\frac{1}{6} + \frac{1}{9}}$$

Then reduce the resulting fraction and change the fractions in the numerator and denominator to equivalent fractions with common denominators:

$$= \frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{6} + \frac{1}{9}} = \frac{\frac{3}{6} + \frac{2}{6}}{\frac{3}{18} + \frac{2}{18}}$$

Adding the fractions, you get:

$$= \frac{\frac{5}{6}}{\frac{5}{18}}$$

To simplify the complex fraction, you multiply the numerator by the reciprocal of the denominator:

$$= \frac{\cancel{5}^1}{\cancel{6}_2} \cdot \frac{18^3}{\cancel{5}_1} = \frac{3}{1} = 3$$

1 Simplify: $\frac{3\sqrt{(4-6)^2 + [2-(-1)]^2}}{|-3-(-1)|}$

2 Simplify: $\frac{|-3|-|2|+(-1)}{|-7+2|}$

3 Simplify: $(2^3 - 3^2)^4 (-5)$

4 Simplify: $\frac{|5(1-4)+6|}{3\left(-\frac{1}{6}+\frac{1}{3}\right)-\frac{1}{2}}$

Keeping Your Balance While Solving Equalities

Just as simplifying expressions is a basic process in pre-algebra, solving for variables is the basis of algebra. And both are essential to the more complex concepts covered in pre-calculus.

Solving linear equations with the general format of $ax + b = c$, where a , b , and c are constants, is relatively easy using the properties of numbers. The goal, of course, is to isolate the variable, x .



REMEMBER

One type of equation you can't forget is the absolute value equation. The *absolute value* of a number is defined as its distance from 0. In other words, $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$. This definition is a piecewise function with two rules: one where the quantity inside the absolute value bars is positive and another where it's negative. To solve these equations, you must isolate the absolute value term and then set the quantity inside the absolute value bars to the positive and negative values (see the second example question that follows).

Check out the following examples or skip ahead to the practice questions if you think you're ready to tackle them.



EXAMPLE

Q. Solve for x : $3(2x - 4) = x - 2(3 - 2x)$

A. $x = 6$

First, using the distributive property, distribute the 3 and the -2 to get $6x - 12 = x - 6 + 4x$. Then combine like terms and solve using algebra, like so: $6x - 12 = 5x - 6$ giving you $x - 12 = -6$, and, finally, $x = 6$.

Q. Solve for x : $|x - 3| + (-16) = -12$

A. $x = 7$ or -1

Isolate the absolute value by adding 16 to each side, giving you $|x - 3| = 4$. One solution comes when you assume that the quantity inside the absolute value bars is positive: $x - 3 = 4$. This gives you the answer $x = 7$. The second solution comes from assuming that the quantity inside the absolute value bars is negative: $-(x - 3) = 4$. This becomes $-x + 3 = 4$, then $-x = 1$, and finally $x = -1$.

5 Solve: $3 - 6[2 - 4x(x + 3)] = 3x(8x + 12) + 27$

6 Solve: $\frac{x}{2} + \frac{x-2}{4} = \frac{x+4}{2}$

7 Solve: $|x - 3| + |3x + 2| = 4$

8 Solve: $3 - 4(2 - 3x) = 2(6x + 2)$

9 Solve: $2|x - 3| + 12 = 6$

10 Solve: $3(2x + 5) + 10 = 2(x + 10) + 4x + 5$

When Your Image Really Counts: Graphing Equalities and Inequalities

Graphs are visual representations of mathematical equations. In pre-calculus, you'll be introduced to many new mathematical equations and then be expected to graph them. You will have plenty of practice graphing these equations when you read the material involving the more complex equations. In the meantime, it's important to practice the basics: graphing linear equations and inequalities.



TECHNICAL
STUFF

The graphs of linear equations and inequalities exist on the *Cartesian coordinate system*, which is made up of two axes: the horizontal, or *x*-axis, and the vertical, or *y*-axis. Each point on the coordinate plane is called a *Cartesian coordinate pair* and has an *x* coordinate and a *y* coordinate. The notation for any point on the coordinate plane looks like this: (x, y) . A set of these ordered pairs that can be graphed on a coordinate plane is called a *relation*. The *x* values of a relation are its *domain*, and the *y* values are its *range*. For example, the domain of the relation $R = \{(2, 4), (-5, 3), (1, -2)\}$ is $\{2, -5, 1\}$, and the range is $\{4, 3, -2\}$.

You can graph a linear equation using two points or by using the slope-intercept form. The same can be used when graphing linear inequalities. These approaches are reviewed in the following sections.

Graphing with two points

To graph a line using two points, choose two numbers and plug them into the equation to solve for the range (*y*) values. After you plot these points (x, y) on the coordinate plane, you can draw the line through the points.

A nice alternative is to use the two intercepts, the points that fall on the x - or y -axes. To find the x -intercept ($x, 0$), plug in 0 for y and solve for x . To find the y -intercept ($0, y$), plug in 0 for x and solve for y . For example, to find the intercepts of the linear equation $2x + 3y = 12$, start by plugging in 0 for y : $2x + 3(0) = 12$. Then, using properties of numbers, solve for x and you get $x = 6$. So the x -intercept is $(6, 0)$. For the y -intercept, plug in 0 for x and solve for y : $2(0) + 3y = 12$ which give you $y = 4$. Therefore, the y -intercept is $(0, 4)$. At this point, you can plot those two points and connect them to graph the line, because, as you learned in geometry, two points make a line. See the resulting graph in Figure 1-1.

Graphing by using the slope-intercept form

The slope-intercept form of a linear equation gives a great deal of helpful information in a nice package. The equation $y = mx + b$ immediately gives you the y -intercept (b); it also gives you the slope (m). *Slope* is a fraction that gives you the rise over the run. To change equations that aren't written in slope-intercept form, you simply solve for y . For example, if you use the linear equation $2x + 3y = 12$, you start by subtracting $2x$ from each side: $3y = -2x + 12$. Next, you divide all the terms by 3 giving you $y = -\frac{2}{3}x + 4$. Now that the equation is in slope-intercept form, you know that the y -intercept is 4, and you can plot this point on the coordinate plane. Then you can use the slope to plot a second point. From the slope-intercept equation, you know that the slope is $-\frac{2}{3}$. This tells you that the rise is -2 and the run is 3. From the point $(0, 4)$, plot the point 2 down and 3 to the right. In other words, $(3, 2)$. Lastly, connect the two points to graph the line. The resulting graph is identical to Figure 1-1.

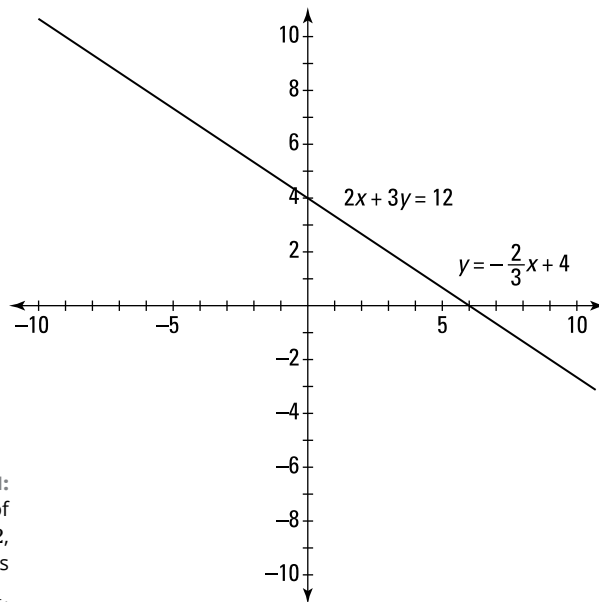


FIGURE 1-1:
Graph of
 $2x + 3y = 12$,
also known as
 $y = -\frac{2}{3}x + 4$.

Graphing inequalities



REMEMBER

Similar to graphing linear equations, graphing linear inequalities begins with plotting two points. However, because *inequalities* are used for comparisons — greater than, less than, or equal to — you have two more questions to answer after finding two points:

- » Is the line dashed ($<$ or $>$) or solid (\leq or \geq)?
- » Do you shade under the line ($y <$ or $y \leq$) or above the line ($y >$ or $y \geq$)?

Here's an example of graphing an inequality followed by a few practice questions.

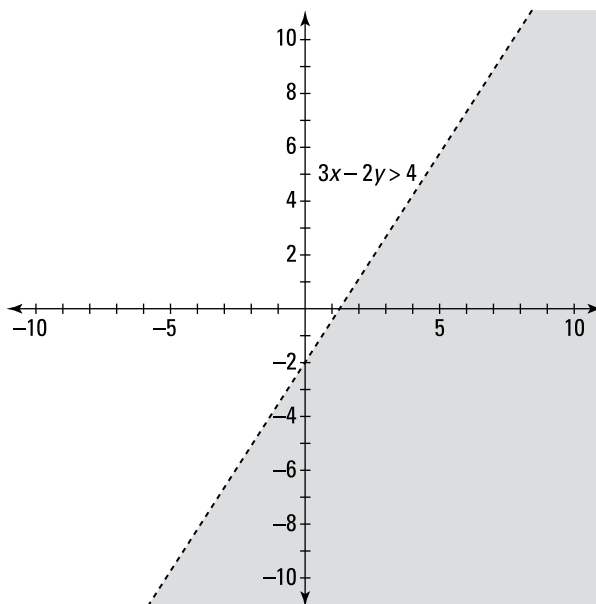


EXAMPLE

Q. Sketch the graph of the inequality: $3x - 2y > 4$

A. Put the inequality into slope-intercept form by subtracting $3x$ from each side of the equation to get $-2y > -3x + 4$ and then dividing each term by -2 to get $y < \frac{3}{2}x - 2$. (*Remember:* When you multiply or divide an inequality by a negative, you need to reverse the inequality.) From the resulting statement, you can find the y -intercept, -2 , and the slope, $\frac{3}{2}$. Use this information to graph two points by using the slope-intercept form. Next, decide the nature of the line (solid or dashed). Because the inequality is strict, the line is dashed. Graph the dashed line so you can decide where to shade. Because $y < \frac{3}{2}x - 2$ is a less-than inequality, shade below the dashed line, as shown in the following figure.

This method works only if the boundary line is first converted to slope-intercept form. An alternative is to graph the boundary line using any method and then use a sample point, such as $(0, 0)$, to determine which half-plane to shade.



11 Sketch the graph of $\frac{4}{3}(6x + 2y) = 16$.

12 Sketch the graph of $\frac{5x + 4y}{2} \geq 6$.

13 Sketch the graph of $4x + 5y \geq 2(3y + 2x + 4)$.

14 Sketch the graph of $x - 3y = 4 - 2y - y$.

Using Graphs to Find Distance, Midpoint, and Slope

Graphs are more than just pretty pictures. From a graph, it's possible to choose two points and then figure out the distance between them, the midpoint of the segment connecting them, and the slope of the line running through them. As graphs become more complex in both pre-calculus and calculus, you're asked to find and use all three of these pieces of information. Aren't you lucky?

Finding the distance



TECHNICAL
STUFF

Distance refers to how far apart two things are. In this case, you're finding the distance between two points. Knowing how to calculate distance is helpful for when you get to conics (see Chapter 12). To find the distance between two points (x_1, y_1) and (x_2, y_2) , use the following

formula: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Calculating the midpoint



TECHNICAL
STUFF

The *midpoint* is the middle of a segment. This concept also comes up in conics (see Chapter 12) and is ever so useful for all sorts of other pre-calculus calculations. To find the coordinates of the midpoint, M , of the points (x_1, y_1) and (x_2, y_2) , you just need to average the x and y values and express them as an ordered pair, like so: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$.

Discovering the slope



TECHNICAL
STUFF

Slope is a key concept for linear equations, but it also has applications for trigonometric functions and is essential for differential calculus. *Slope* describes the steepness of a line on the coordinate plane (think of a ski slope). Use this formula to find the slope, m , of the line (or segment) connecting the two points (x_1, y_1) and (x_2, y_2) : $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Note: Positive slopes move upward as you move from left to right. Negative slopes move downward as you move from left to right. Horizontal lines have a slope of 0, and vertical lines have an undefined slope.

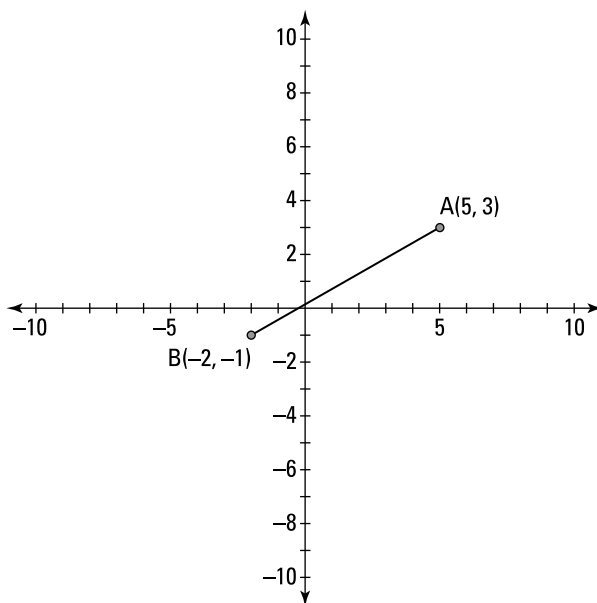
Following is an example question for your reviewing pleasure. Look it over and then try your hand at the practice questions.



EXAMPLE

Q. Find the distance, slope, and midpoint of \overline{AB} .

A. The distance is $\sqrt{65}$, the slope is $\frac{4}{7}$, and the midpoint is $\left(\frac{3}{2}, 1 \right)$.



Plug the x and y values into the distance formula and, following the order of operations, simplify the terms under the radical (keeping in mind the implied parentheses of the radical itself).

$$\begin{aligned} d &= \sqrt{(5 - (-2))^2 + (3 - (-1))^2} \\ &= \sqrt{7^2 + 4^2} = \sqrt{49 + 16} = \sqrt{65} \end{aligned}$$

Because 65 doesn't contain any perfect squares as factors, this is as simple as you can get.

To find the midpoint, plug the points into the midpoint formula, and simplify using the order of operations.

$$\begin{aligned} M &= \left(\frac{5 + (-2)}{2}, \frac{3 + (-1)}{2} \right) \\ &= \left(\frac{3}{2}, \frac{2}{2} \right) = \left(\frac{3}{2}, 1 \right) \end{aligned}$$

To find the slope, use the formula, plug in your x and y values, and use the order of operations to simplify.

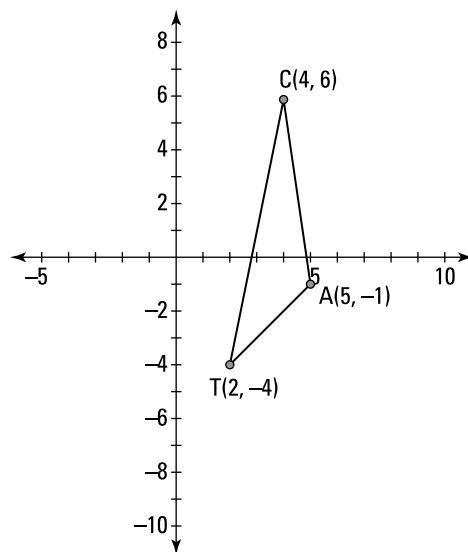
$$\begin{aligned} m &= \frac{-1 - 3}{-2 - 5} \\ &= \frac{-4}{-7} = \frac{4}{7} \end{aligned}$$

- 15 Find the length of segment CD , where C is $(-2, 4)$ and D is $(3, -1)$.

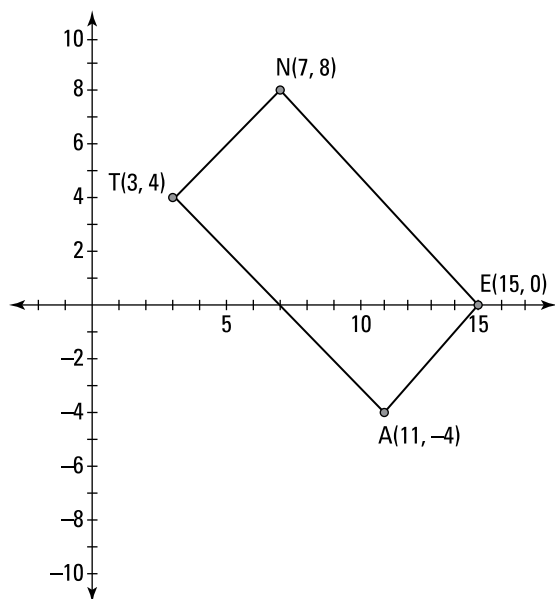
- 16 Find the midpoint of segment EF , where E is $(3, -5)$ and F is $(7, 5)$.

- 17 Find the slope of line GH , where G is $(-3, -5)$ and H is $(-3, 4)$.

- 18 Find the perimeter of triangle CAT .



- 19 Find the center of the rectangle $NEAT$.



- 20 Determine whether triangle DOG is a right triangle.

