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Calculus

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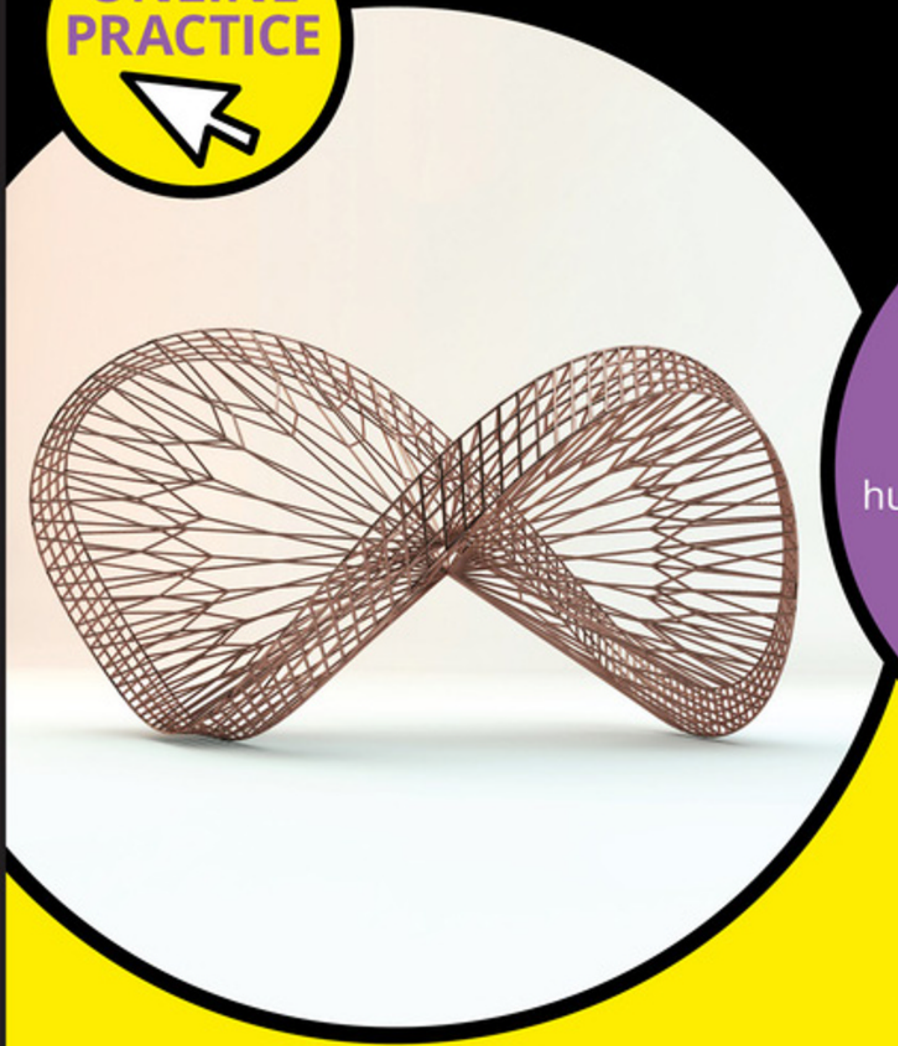

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Calculus Workbook

3rd Edition with Online Practice

by Mark Ryan

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Calculus Workbook For Dummies®, 3rd Edition with Online Practice

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Contents at a Glance

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Introduction

If you've already bought this book or are thinking about buying it, it's probably too late — too late, that is, to change your mind and get the heck out of calculus. (If you've still got a chance to break free, get out and run for the hills!) Okay, so you're stuck with calculus; you're past the point of no return. Is there any hope? Of course! For starters, buy this gem of a book and my other classic, *Calculus For Dummies* (also published by Wiley). In both books, you find calculus explained in plain English with a minimum of technical jargon. *Calculus For Dummies* covers topics in greater depth. *Calculus Workbook For Dummies,* 3rd Edition, gives you the opportunity to master the calculus topics you study in class or in *Calculus For Dummies* through a couple hundred practice problems that will leave you giddy with the joy of learning . . . or pulling your hair out.

In all seriousness, calculus is not nearly as difficult as you'd guess from its reputation. It's a logical extension of algebra and geometry, and many calculus topics can be easily understood when you see the algebra and geometry that underlie them.

It should go without saying that regardless of how well you think you understand calculus, you won't fully understand it until you get your hands dirty by actually doing problems. On that score, you've come to the right place.

About This Book

Calculus Workbook For Dummies, 3rd Edition, like *Calculus For Dummies,* is intended for three groups of readers: high school seniors or college students in their first calculus course, students who've taken calculus but who need a refresher to get ready for other pursuits, and adults of all ages who want to practice the concepts they learned in *Calculus For Dummies* or elsewhere.

Whenever possible, I bring calculus down to earth by showing its connections to basic algebra and geometry. Many calculus problems look harder than they actually are because they contain so many fancy, foreign-looking symbols. When you see that the problems aren't that different from related algebra and geometry problems, they become far less intimidating.

I supplement the problem explanations with tips, shortcuts, and mnemonic devices. Often, a simple tip or memory trick can make it much easier to learn and retain a new, difficult concept.

This book uses certain conventions:

- Variables are in *italics.*
- Important math terms are often in *italics* and defined when necessary.
- Extra-hard problems are marked with an asterisk. You may want to skip these if you're prone to cerebral hemorrhaging.

Like all *For Dummies* books, you can use this book as a reference. You don't need to read it cover to cover or work through all problems in order. You may need more practice in some areas than others, so you may choose to do only half of the practice problems in some sections or none at all.

However, as you'd expect, the order of the topics in *Calculus Workbook For Dummies*, 3rd Edition, follows the order of the traditional curriculum of a first-year calculus course. You can, therefore, go through the book in order, using it to supplement your coursework. If I do say so myself, I expect you'll find that many of the explanations, methods, strategies, and tips in this book will make problems you found difficult or confusing in class seem much easier.

Foolish Assumptions

Now that you know a bit about how I see calculus, here's what I'm assuming about you:

- You haven't forgotten all the algebra, geometry, and trigonometry you learned in high school. If you have, calculus will be *really* tough. Just about every single calculus problem involves algebra, a great many use trig, and quite a few use geometry. If you're really rusty, go back to these basics and do some brushing up. This book contains some practice problems to give you a little pre-calc refresher, and *Calculus For Dummies* has an excellent pre-calc review.
- You're willing to invest some time and effort in doing these practice problems. As with anything, practice makes perfect, and, also like anything, practice sometimes involves struggle. But that's a good thing. Ideally, you should give these problems your best shot before you turn to the solutions. Reading through the solutions can be a good way to learn, but you'll usually learn more if you push yourself to solve the problems on your own — even if that means going down a few dead ends.

Icons Used in This Book

The icons help you to quickly find some of the most critical ideas in the book.

REMEMBER

Next to this icon are important pre-calc or calculus definitions, theorems, and so on.

EXAMPLE

This icon is next to — are you sitting down? — example problems.

TIP

The tip icon gives you shortcuts, memory devices, strategies, and so on.

WARNING

Ignore these icons and you'll be doing lots of extra work and probably getting the wrong answer.

Beyond the Book

Look online at www.dummies.com to find a handy cheat sheet for *Calculus Workbook For Dummies*, 3rd Edition. Feel like you need more practice? You can also test yourself with online quizzes.

To gain access to the online practice, all you have to do is register. Just follow these simple steps:

1. **Register your book or ebook at Dummies.com to get your PIN. Go to www.dummies.com/go/getaccess.**
2. **Select your product from the dropdown list on that page.**
3. **Follow the prompts to validate your product, and then check your email for a confirmation message that includes your PIN and instructions for logging in.**

If you do not receive this email within two hours, please check your spam folder before contacting us through our Technical Support website at http://support.wiley.com or by phone at 877-762-2974.

Now you're ready to go! You can come back to the practice material as often as you want — simply log on with the username and password you created during your initial login. No need to enter the access code a second time.

Your registration is good for one year from the day you activate your PIN.

Where to Go from Here

You can go . . .

- To Chapter 1 — or to whatever chapter you need to practice.
- To *Calculus For Dummies* for more in-depth explanations. Then, because after finishing it and this workbook your newly acquired calculus expertise will at least double or triple your sex appeal, pick up *French For Dummies* and *Wine For Dummies* to impress Nanette or Jéan Paul.
- With the flow.
- To the head of the class, of course.
- Nowhere. There's nowhere to go. After mastering calculus, your life is complete.

1

Pre-Calculus Review

IN THIS PART . . .

Explore algebra and geometry for old times' sake.

Play around with functions.

Tackle trigonometry.

IN THIS CHAPTER

» Fussing with fractions

» Brushing up on basic algebra

» Getting square with geometry

Chapter 1

Getting Down to Basics: Algebra and Geometry

I know, I know. This is a *calculus* workbook, so what's with the algebra and geometry? Don't worry; I'm not going to waste too many precious pages with algebra and geometry, but these topics are essential for calculus. You can no more do calculus without algebra than you can write French poetry without French. And basic geometry (but not geometry proofs) is critically important because much of calculus involves real-world problems that include angles, slopes, shapes, and so on. So in this chapter — and in Chapter 2 on functions and trigonometry — I give you some quick problems to help you brush up on your skills. If you've already got these topics down pat, you can skip to Chapter 3.

In addition to working through the problems in Chapters 1 and 2 in this book, you may want to check out the great pre-calc review in *Calculus For Dummies,* 2nd Edition.

Fraction Frustration

Many, many math students hate fractions. I'm not sure why, because there's nothing especially difficult about them. Perhaps for some students, fraction concepts didn't completely click when they first studied them, and then fractions became a nagging frustration whenever they came up in subsequent math courses. Whatever the cause, if you don't like fractions, try to get over it. Fractions really are a piece o' cake; you'll have to deal with them in every math course you take.

You can't do calculus without a good grasp of fractions. For example, the very definition of the derivative is based on a fraction called the *difference quotient.* And, on top of that, the symbol for the derivative, $\frac{dy}{dx}$, is a fraction. So, if you're a bit rusty with fractions, get up to speed with the following problems — or else!

EXAMPLE

Q. Solve: $\frac{a}{b} \cdot \frac{c}{d} = ?$

A. $\boldsymbol{\frac{ac}{bd}}$. To multiply fractions, you multiply straight across. You do *not* cross-multiply!

Q. Solve: $\frac{a}{b} \div \frac{c}{d} = ?$

A. $\boldsymbol{\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}}$. To divide fractions, you flip the second one, and then multiply.

1 Solve: $\frac{5}{0} = ?$

2 Solve: $\frac{0}{10} = ?$

3 Does $\frac{3a+b}{3a+c}$ equal $\frac{a+b}{a+c}$? Why or why not?

4 Does $\frac{3a+b}{3a+c}$ equal $\frac{b}{c}$? Why or why not?

5 Does $\frac{4ab}{4ac}$ equal $\frac{ab}{ac}$? Why or why not?

6 Does $\frac{4ab}{4ac}$ equal $\frac{b}{c}$? Why or why not?

Misc. Algebra: You Know, Like Miss South Carolina

This section gives you a quick review of algebra basics like factors, powers, roots, logarithms, and quadratics. You absolutely *must* know these basics.

EXAMPLE

Q. Factor $9x^4 - y^6$.

A. $\mathbf{9x^4 - y^6 = (3x^2 - y^3)(3x^2 + y^3)}$. This is an example of the single most important factor pattern: $a^2 - b^2 = (a-b)(a+b)$. Make sure you know it!

Q. Rewrite $x^{2/5}$ without a fraction power.

A. $\mathbf{\sqrt[5]{x^2}}$ **or** $\mathbf{(\sqrt[5]{x})^2}$. Don't forget how fraction powers work!

7 Rewrite x^{-3} without a negative power.

8 Does $(abc)^4$ equal $a^4b^4c^4$? Why or why not?

9 Does $(a+b+c)^4$ equal $a^4+b^4+c^4$? Why or why not?

10 Rewrite $\sqrt[3]{\sqrt[4]{x}}$ with a single radical sign.

11 Does $\sqrt{a^2 + b^2}$ equal $a + b$? Why or why not?

12 Rewrite $\log_a b = c$ as an exponential equation.

13 Rewrite $\log_c a - \log_c b$ with a single log.

14 Rewrite $\log 5 + \log 200$ with a single log and then solve.

15 If $5x^2 = 3x + 8$, solve for x with the quadratic formula.

16 Solve: $|3x + 2| > 14$.

17 Solve: $-3^2 - x^0 + \sqrt{0} - |-1| - 1^0 - 0^1 = ?$

18 Simplify $\sqrt[3]{p^6 q^{15}}$.

19 Simplify $\left(\frac{8}{27}\right)^{-4/3}$.

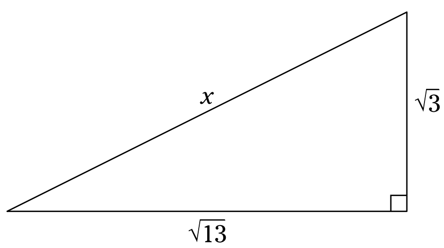
20 Factor $-x^{10}+16$ over the set of integers.

Geometry: When Am I Ever Going to Need It?

You can use calculus to solve many real-world problems that involve two- or three-dimensional shapes and various curves, surfaces, and volumes — such as calculating the rate at which the water level is falling in a cone-shaped tank or determining the dimensions that maximize the volume of a cylindrical soup can. So the geometry formulas for perimeter, area, volume, surface area, and so on will come in handy. You should also know things like the Pythagorean Theorem, proportional shapes, and basic coordinate geometry, like the midpoint and distance formulas.

EXAMPLE

Q. What's the area of the triangle in the following figure?



A. $\mathbf{\frac{\sqrt{39}}{2}}$.

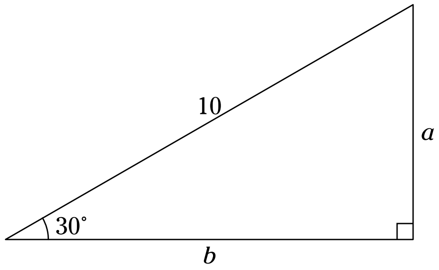
$$\begin{aligned} Area_{triangle} &= \frac{1}{2} base \cdot height \\ &= \frac{1}{2} \cdot \sqrt{13}\sqrt{3} \\ &= \frac{\sqrt{39}}{2} \end{aligned}$$

Q. How long is the hypotenuse of the triangle in the previous example?

A. $\mathbf{x = 4}$.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ x^2 &= a^2 + b^2 \\ x^2 &= \sqrt{13}^2 + \sqrt{3}^2 \\ x^2 &= 13 + 3 \\ x^2 &= 16 \\ x &= 4 \end{aligned}$$

21 Fill in the two missing lengths for the sides of the triangle in the following figure.



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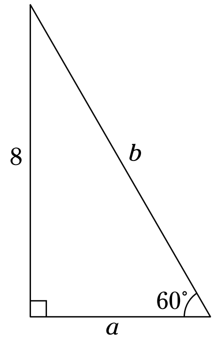
22 What are the lengths of the two missing sides of the triangle in the following figure?



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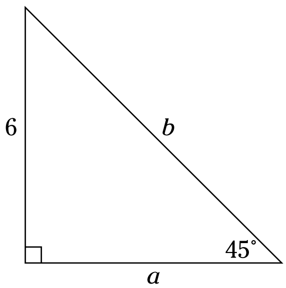
23 Fill in the missing lengths for the sides of the triangle in the following figure.



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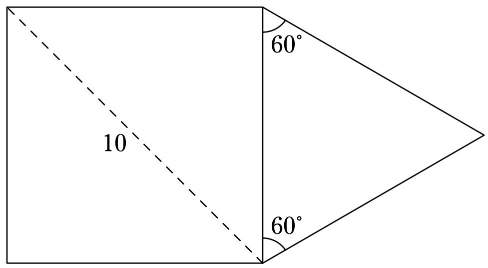
24 **a.** What's the total area of the pentagon in the following figure (the shape on the left is a square)?

b. What's the perimeter?



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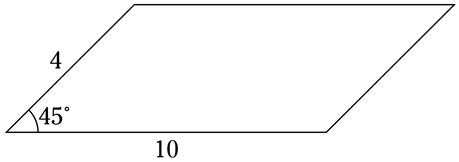
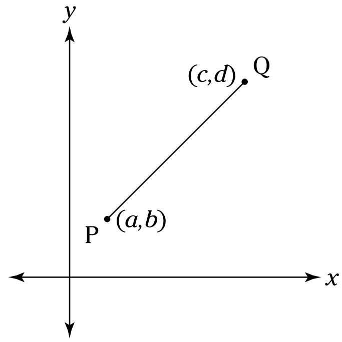
25 Compute the area of the parallelogram in the following figure.



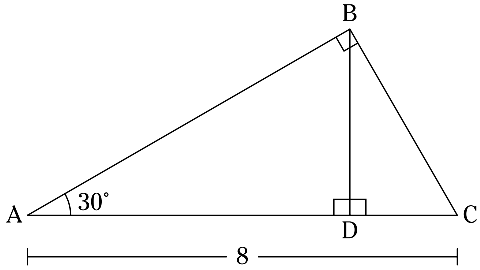
26 What's the slope of $\overline{PQ}$?



27 How far is it from P to Q in the figure from Problem 26?

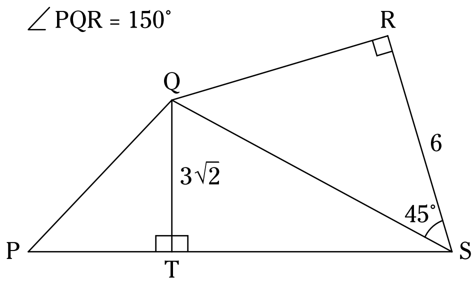
28 What are the coordinates of the midpoint of $\overline{PQ}$ in the figure from Problem 26?

29 What's the length of altitude of triangle *ABC* in the following figure?

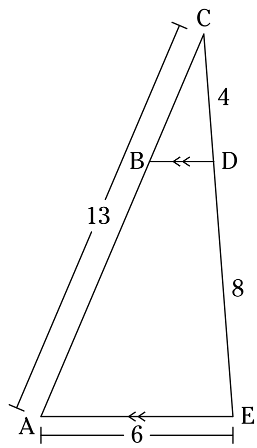


30 What's the perimeter of triangle *ABD* in the figure for Problem 29?

31 What's the area of quadrilateral *PQRS* in the following figure?

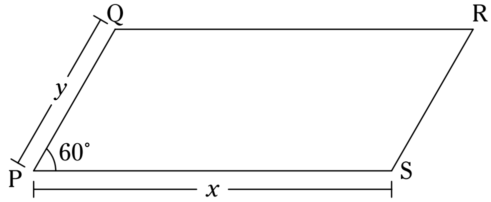


32 What's the perimeter of triangle *BCD* in the following figure?



33 What's the ratio of the area of triangle *BCD* to the area of triangle *ACE* in the figure for Problem 32?

34 In the following figure, what's the area of parallelogram *PQRS* in terms of *x* and *y*?



Solutions for This Easy, Elementary Stuff

1. Solve: $\frac{5}{0} = ?$ $\mathbf{\frac{5}{0}}$ **is undefined!** Don't mix this up with something like $\frac{0}{8}$, which equals zero. Here's a great way to think about this problem and fractions in general. Consider the following simple division or fraction problem: $\frac{8}{2} = 4$. Note the *multiplication* problem implicit here: 2 times 4 is 8. This multiplication idea is a great way to think about how fractions work. So in the current problem, you can consider $\frac{5}{0} =$ ____, and use the multiplication idea: 0 times ______ equals 5. What works in the blank? Nothing, obviously, because 0 times anything is 0. The answer, therefore, is undefined.

   Note that if you think about these two fractions as examples of slope $\left(\frac{rise}{run}\right)$, $\frac{5}{0}$ has a rise of 5 and a run of 0, which gives you a *vertical* line that has sort of an infinite steepness or slope (that's why it's undefined). Or just remember that it's impossible to drive up a vertical road, so it's impossible to come up with a slope for a vertical line. The fraction $\frac{0}{8}$, on the other hand, has a rise of 0 and a run of 8, which gives you a *horizontal* line that has no steepness at all and thus has the perfectly ordinary slope of zero. Of course, it's also perfectly ordinary to drive on a horizontal road.

2. Solve: $\frac{0}{10} = ?$ $\mathbf{\frac{0}{10} = 0}$. (See the solution to Problem 1 for more information.)

3. Does $\frac{3a+b}{3a+c}$ equal $\frac{a+b}{a+c}$? **No.** You can't cancel the 3s.

   WARNING

   You can't cancel in a fraction unless there's an unbroken chain of multiplication running across the entire numerator and the entire denominator — like with $\frac{4ab^2c(x+y)}{5apqr(x^2-y)}$ where you can cancel the *a*s (but only the *a*s). (Note that the addition and subtraction inside the parentheses don't break the multiplication chain.) But, you may object, can't you cancel $4x^2$ from the five terms in $\frac{8x^3-12x^2y+16x^5}{8x^2p-4x^2q^2}$, giving you $\frac{2x-3y+4x^3}{2p-q^2}$? Yes you can, but that's because that fraction can be factored into $\frac{4x^2(2x-3y+4x^3)}{4x^2(2p-q^2)}$, resulting in a fraction where there is an unbroken chain of multiplication across the entire numerator and the entire denominator. Then, the $4x^2$s cancel.

4. Does $\frac{3a+b}{3a+c}$ equal $\frac{b}{c}$? **No.** You can't cancel the 3as. (See the warning in Problem 3.) You can also just test this problem with numbers: Does $\frac{3\cdot 4+5}{3\cdot 4+6} = \frac{5}{6}$? No, they're not equal, and thus the canceling doesn't work.

5. Does $\frac{4ab}{4ac}$ equal $\frac{ab}{ac}$? **Yes.** You can cancel the 4s because the entire numerator and the entire denominator are connected with multiplication.

6. Does $\frac{4ab}{4ac}$ equal $\frac{b}{c}$? **Yes.** You can cancel the 4*a*s.

7 Rewrite x^{-3} without a negative power. $\boldsymbol{\frac{1}{x^3}}$.

8 Does $(abc)^4$ equal $a^4b^4c^4$? **Yes.** Exponents do distribute over multiplication.

9 Does $(a+b+c)^4$ equal $a^4+b^4+c^4$? **No!** Exponents do *not* distribute over addition (or subtraction).

TIP

When you're working a problem and can't remember the algebra rule, try the problem with numbers instead of variables. Just replace the variables with simple, round numbers and work out the numerical problem. (Don't use 0, 1, or 2 because they have special properties that can mess up your test.) Whatever works for the numbers will work with variables, and whatever doesn't work with numbers won't work with variables. Watch what happens if you try this problem with numbers:

$$(3+4+6)^4 \stackrel{?}{=} 3^4+4^4+6^4$$
$$13^4 \stackrel{?}{=} 81+256+1{,}296$$
$$28{,}561 \neq 1{,}633$$

10 Rewrite $\sqrt[3]{\sqrt[4]{x}}$ with a single radical sign. $\boldsymbol{\sqrt[12]{x}}$.

11 Does $\sqrt{a^2+b^2}$ equal $a+b$? **No!** The explanation is basically the same as for Problem 9. Consider this: If you turn the root into a power, you get $\sqrt{a^2+b^2}=\left(a^2+b^2\right)^{1/2}$. But because you *can't* distribute the power over addition, $\left(a^2+b^2\right)^{1/2} \neq \left(a^2\right)^{1/2}+\left(b^2\right)^{1/2}$, or $a+b$, and thus $\sqrt{a^2+b^2} \neq a+b$.

12 Rewrite $\log_a b = c$ as an exponential equation. $\boldsymbol{a^c = b}$.

13 Rewrite $\log_c a - \log_c b$ with a single log. $\boldsymbol{\log_c \frac{a}{b}}$.

14 Rewrite $\log 5 + \log 200$ with a single log and then solve.
$\mathbf{\log 5 + \log 200 = \log(5 \times 200) = \log 1{,}000 = 3}$.

REMEMBER

When you see "log" without a base number, the base is 10.

15 If $5x^2 = 3x+8$, solve for x with the quadratic formula. $\boldsymbol{x = \frac{8}{5}}$ **or** $\boldsymbol{-1}$.

Start by rearranging $5x^2 = 3x+8$ into $5x^2-3x-8=0$ because when solving a quadratic equation, you want just a zero on one side of the equation.

The quadratic formula tells you that $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. Plugging 5 into a, −3 into b, and −8 into c gives you $x=\frac{-(-3)\pm\sqrt{(-3)^2-4(5)(-8)}}{2\cdot 5}=\frac{3\pm\sqrt{9+160}}{10}=\frac{3\pm 13}{10}=\frac{16}{10}$ or $\frac{-10}{10}$, so $x=\frac{8}{5}$ or −1.

16 Solve: $|3x+2|>14$. $\boldsymbol{x<-\frac{16}{3} \cup x>4}$.

1. **Turn the inequality into an equation:**

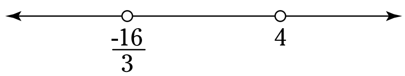
   $$|3x+2|=14$$

2. **Solve the absolute value equation.**

   $$\begin{aligned} 3x+2&=14 \\ 3x&=12 \\ x&=4 \end{aligned} \qquad \text{or} \qquad \begin{aligned} 3x+2&=-14 \\ 3x&=-16 \\ x&=-\frac{16}{3} \end{aligned}$$

3. **Place both solutions on a number line (see the following figure).**

   (You use hollow dots for > and <; if the problem had involved ≥ or ≤, you would use solid dots.)



4. **Test a number from each of the three regions on the line (left of the left dot, between the dots, and right of the right dot) in the original inequality.**

   For this problem you can use −10, 0, and 10.

   $$\begin{aligned} |3\cdot(-10)+2| &\overset{?}{>}14 \\ |-28| &\overset{?}{>}14 \\ 28 &\overset{?}{>}14 \end{aligned}$$

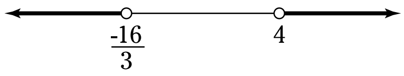
   True, so you shade the left-most region.

   $$\begin{aligned} |3\cdot(0)+2| &\overset{?}{>}14 \\ 2 &\overset{?}{>}14 \end{aligned}$$

   False, so you don't shade the middle region.

   $$\begin{aligned} |3\cdot(10)+2| &\overset{?}{>}14 \\ |32| &\overset{?}{>}14 \\ 32 &\overset{?}{>}14 \end{aligned}$$

True, so you shade the region on the right. The following figure shows the result. *x* can be any number where the line is shaded. That's your final answer.



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5. You may also want to express the answer symbolically.

Because *x* can equal a number in the left region *or* a number in the right region, this is an *or* solution which means *union* ($\cup$). When you want to include everything from both regions on the number line, you want the union of the two regions. So, the symbolic answer is

$$x < -\frac{16}{3} \ \cup \ x > 4$$

(You can write the above using the word "or" instead of the union symbol.) If only the middle region were shaded, you'd have an *and* or *intersection* problem ($\cap$). Using the above number line points, for example, you would write the middle-region solution like this:

$$x > -\frac{16}{3} \ \cap \ x < 4$$

(You can use the word "and" instead of the intersection symbol.) Note that in this solution (whether you use "and" or the intersection symbol) the two inequalities overlap or intersect in the middle region. You can avoid the intersection issue by simply writing the solution as

$$-\frac{16}{3} < x < 4$$

You say "to-*may*-to," I say "to-*mah*-to."

REMEMBER

While we're on the subject of absolute value, don't forget that $\sqrt{x^2} = |x|$. $\sqrt{x^2}$ does *not* equal $\pm x$.

17 Solve: $-3^2 - x^0 + \sqrt{0} - |-1| - 1^0 - 0^1 = ?$ **The answer is –12.**

Funny looking problem, eh? It's just meant to help you review a few basics. Take a look at the six terms:

Don't forget, $-3^2 = -9$. If you want to square a negative number, you have to put it in parentheses: $(-3)^2 = 9$. Next, anything to the zero power (including a variable) equals 1. That takes care of the second and fifth chunks of the problem. The square root of zero is just zero, of course, because zero squared equals zero. And you know that the absolute value of –1 is 1; you just have to be careful not to goof up with all those negative signs and subtraction signs. Finally, zero to any *positive* power equals zero. That does it:

$$\begin{aligned} &-3^2 - x^0 + \sqrt{0} - |-1| - 1^0 - 0^1 \\ &= -9 - 1 + 0 - 1 - 1 - 0 \\ &= -12 \end{aligned}$$

(18) Simplify $\sqrt[3]{p^6q^{15}}$. **The answer is p^2q^5.**

Most people prefer working with power rules to working with root rules, so that's the way I solve the problem here. First, rewrite the root as a power: $\sqrt[3]{p^6q^{15}} = \left(p^6q^{15}\right)^{1/3}$. Now, just distribute the power to the p^6 and the q^6, and then use the power-to-a-power rule:

$$\begin{aligned}&\left(p^6q^{15}\right)^{1/3}\\ &= \left(p^6\right)^{1/3}\left(q^{15}\right)^{1/3}\\ &= p^{6(1/3)}q^{15(1/3)}\\ &= p^2q^5\end{aligned}$$

(19) Simplify $\left(\frac{8}{27}\right)^{-4/3}$. **The answer is $\frac{81}{16}$.**

I'll give you the longer version of the solution and then show you a shortcut. First, use the definition of a negative exponent to rewrite the problem as $\frac{1}{\left(\frac{8}{27}\right)^{4/3}}$. Next, change the power to a root: $\frac{1}{\sqrt[3]{\frac{8}{27}}^{\,4}}$ (instead, you could first distribute the fraction power to the numerator and denominator). The rest shouldn't be too bad: $\frac{1}{\sqrt[3]{\frac{8}{27}}^{\,4}} = \frac{1}{\left(\frac{\sqrt[3]{8}}{\sqrt[3]{27}}\right)^4} = \frac{1}{\left(\frac{2}{3}\right)^4} = \frac{1}{\left(\frac{16}{81}\right)} = \frac{81}{16}$.

The shortcut is to use the fact that when you have a fraction raised to a negative power, you can flip the fraction and make the power positive, like this $\left(\frac{8}{27}\right)^{-4/3} = \left(\frac{27}{8}\right)^{4/3}$. Then proceed as follows: $\left(\frac{27}{8}\right)^{4/3} = \frac{27^{4/3}}{8^{4/3}} = \frac{\sqrt[3]{27}^{\,4}}{\sqrt[3]{8}^{\,4}} = \frac{3^4}{2^4} = \frac{81}{16}$.

(20) Factor $-x^{10}+16$ over the set of integers. $\mathbf{(4-x^5)(4+x^5)}$.

To factor $-x^{10}+16$, you use the oh-so-important a^2-b^2 rule. a^2-b^2 factors into $(a-b)(a+b)$. Make sure you know this factoring rule (and the corresponding FOILing rule, which is the factoring rule in reverse). Whenever you see a binomial with a subtraction sign (in the current problem, you have to switch the two terms to see the subtraction sign), ask yourself whether you can rewrite the binomial as $(\quad)^2-(\quad)^2$, in other words, as something squared minus something else squared. If you can, then the first blank is your a, and the second blank is your b. The binomial in this problem can be rewritten as $(4)^2-\left(x^5\right)^2$. Now just plug the 4 into the a and the x^5 into the b in $(a-b)(a+b)$, and you're done.

(21) Fill in the two missing lengths for the sides of the triangle. **$a = 5$ and $b = 5\sqrt{3}$.**

This is a 30°-60°-90° triangle.