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Years 9–10 Maths For Students

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Years 9–10 Maths FOR *STUDENTS*

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Contents at a Glance

Table of Contents

Introduction

In this book, I offer a refresher on some basic maths operations, such as addition, subtraction, multiplication and division, before moving on to the more advanced topic of algebra. So let me introduce you to algebra. This introduction is somewhat like what would happen if I were to introduce you to my friend Donna. I'd say, 'This is Donna. Let me tell you something about her.' After giving a few well-chosen tidbits of information about Donna, I'd let you ask more questions or fill in more details. In this book, you find some well-chosen topics and information, and I try to fill in details as I go along.

As you read this introduction, you're probably in one of two situations:

- You've taken the plunge and bought the book.
- You're checking things out before committing to the purchase.

In either case, you'd probably like to have some good, concrete reasons why you should go to the trouble of reading and finding out about algebra.

One of the most commonly asked questions in a mathematics classroom is, 'What will I ever use this for?' Some teachers can give a good, convincing answer. Others hem and haw and stare at the floor. My favourite answer is, 'Algebra gives you *power*.' Algebra gives you the power to move on to bigger and better things in mathematics. Algebra gives you the power of knowing that you know something that your neighbour doesn't know. Algebra gives you the power to be able to help someone else with an algebra task or to explain to others these logical mathematical processes.

Algebra is a system of symbols and rules that is universally understood, no matter what the spoken language. Algebra provides a clear, methodical process that can be followed from beginning to end. It's an organisational tool that is most useful when followed with the appropriate rules. What power! Some people like algebra because it can be a form of puzzle-solving. You solve a puzzle by finding the value of a variable. You may prefer Sudoku or crosswords, but it wouldn't hurt to give algebra a chance, too.

About This Book

This book isn't like a mystery novel; you don't have to read it from beginning to end. In fact, you can peek at how it ends and not spoil the rest of the story.

I divide the book into some general topics — from the beginning nuts and bolts to the important tool of factoring to equations, applications and geometry. So you can dip into the book wherever you want, to find the information you need.

Throughout the book, I use many examples, each a bit different from the others, and each showing a different twist to the topic. The examples have explanations to aid your understanding. (What good is knowing the answer if you don't know how to get the right answer yourself?)

The vocabulary I use is mathematically correct *and* understandable. So whether you're listening to your teacher or talking to someone else about algebra, you'll be speaking the same language.

Along with the *how*, I show you the *why*. Sometimes remembering a process is easier if you understand why it works and don't just try to memorise a meaningless list of steps.

I don't use many conventions in this book, but you should be aware of the following:

- When I introduce a new term, I put that term in *italics* and define it nearby (often in parentheses).
- I express numbers or numerals either with the actual symbol, such as 8, or the written-out word: *eight*. Operations, such as +, are either shown as this symbol or written as *plus*. The choice of expression all depends on the situation — and on making it perfectly clear for you.

The *sidebars* (those little grey boxes) are interesting but not essential to your understanding of the text. If you're short on time, you can skip the sidebars. Of course, if you read them, I think you'll be entertained.

Foolish Assumptions

I don't assume that you're as crazy about maths as I am — and you may be even *more* excited about it than I am! I do assume, though, that you have a mission here — to brush up on your basic skills, improve your maths grade, or just have some fun. I also assume that you have some experience with algebra — for example, full exposure for a year or so.

You may remember the first time algebra came up in your maths class. I can distinctly remember my first algebra teacher, Miss McDonald, saying, ‘This is an *n*.’ My whole secure world of numbers was suddenly turned upside down. I hope your first reaction was better than mine.

Wherever you are in your maths journey, or what aspect you need to improve on, never fear. Help is here!

Icons Used in This Book

The little drawings in the margin of the book are there to draw your attention to specific text. Here are the icons I use in this book:

To make everything work out right, you have to follow the basic rules of algebra (or mathematics in general). You can’t change or ignore them and arrive at the right answer. Whenever I give you an algebra rule, I mark it with this icon.

Paragraphs marked with the Remember icon help clarify a symbol or process. I may discuss the topic in another section of the book, or I may just remind you of a basic algebra rule that I discuss earlier.

The Tip icon isn’t life-or-death important, but it generally can help make your life easier — at least your life in maths and algebra.

The Warning icon alerts you to something that can be particularly tricky. Errors crop up frequently when working with the processes or topics next to this icon, so I call special attention to the situation so you won’t fall into the trap.

Where to Go From Here

If you want to refresh your basic skills or boost your confidence, start with Part I. If you’re ready to jump into the guts of algebra, or looking for some factoring practice and need to pinpoint which method to use with what, go to Part II. Part III is for you if you're ready to solve equations; you can find just about any type you're ready to attack. Part IV is where the good stuff is — applications and geometry — things to do with all those good solutions. The list in Part V is usually what you'd look at after visiting one of the other parts, but why not start there? It's a fun place!

Studying more advanced maths and algebra can give you some logical exercises, and thinking logically can help you with all aspects of life — at school and afterwards.

The best *why* for studying algebra is just that it's beautiful. Yes, you read that right. Algebra is poetry, deep meaning and artistic expression. Just look and you'll find it. Also, don't forget that it gives you *power*.

Enjoy the adventure!

Part I
Reviewing the Basics

getting started with
Years 9–10 Maths for
STUDENTS

In this part . . .

- Understand that maths can be a game — and that parents can play too.
- Work with addition and subtraction, and multiply with style and divide with ease.
- Get your head around negative numbers.
- Remember your fraction facts, and convert to and from percentages, decimals and fractions.
- Complete operations in the right order.

Chapter 1

Assembling Your Tools

In This Chapter

- Getting a refresher on whole numbers and fraction, decimals and percentages
- Starting to think about algebra
- Working out where geometry comes in
- Understanding the ways you can play with maths
- Focusing on how parents can help

One useful characteristic about numbers is that they're *conceptual,* which means that, in an important sense, they're all in your head. (This fact probably won't get you out of having to know about them, though — nice try!) For example, you can picture three of anything: three cats, three cricket balls, three cannibals, three planets. But just try to picture the concept of three all by itself, and you find it's impossible. Oh, sure, you can picture the numeral 3, but the *threeness* itself — much like love or beauty or honour — is beyond direct understanding. But when you understand the *concept* of three (or four, or a million), you have access to an incredibly powerful system for understanding the world: mathematics.

In this chapter, I run through some of the basics about working with whole and part numbers. I then provide an overview of some more advanced maths as I help you start to get your head around algebra and some aspects of geometry.

I then move on to some of the other tools you may like to assemble — including some tools for developing the right mindset towards maths and for connecting arguments and ideas using maths. I also provide some tools for parents helping their children work through their latest maths problem — especially valuable in the heat of the moment (you know, when the homework due tomorrow has just been discovered at the bottom of the schoolbag).

Working in with the Australian Curriculum

In recent years in Australia, the Australian Curriculum (AC) has been introduced, with the main objective of streamlining the curriculum for all subjects, across all states. In particular, the AC aims to ensure that students can move from state to state with minimal difference in subject content.

Schools can still use their discretion to order the topics within a subject and to organise different forms of assessment, but the core curriculum is to be the same across Australia. Through setting these consistent national standards, the AC aims to improve education outcomes for all Australian students. The website for the AC states that this then creates 'the base for future learning, growth and active participation in the Australian community'.

When focusing on maths in particular the website for the AC states:

> *The Australian Curriculum: Mathematics provides students with essential mathematical skills and knowledge in Number and Algebra, Measurement and Geometry, and Statistics and Probability. It develops the numeracy capabilities that all students need in their personal, work and civic life, and provides the fundamentals on which mathematical specialties and professional applications of mathematics are built.*

(See `www.australiancurriculum.edu.au` for more information.)

But the Australian Curriculum: Mathematics doesn't just improve specialised maths skills. Over many, many years, mathematics has evolved and changed. The introduction of digital technology and significant changes in the types of calculators available provide new ways for students to develop their mathematical thinking and reasoning. Focusing on developing skills in key areas can enable students to improve their understanding, fluency and problem-solving abilities.

Both the initial construction and continued development of the Australian Curriculum: Mathematics has ensured that considered links exist between different subject areas, so students can develop an understanding of where mathematics fits within all subject areas and, therefore, everyday scenarios. Because of this, hopefully, students everywhere can learn to see not only the beauty, but also the relevance in mathematics and mathematical concepts.

Starting with the Basics

Where would mathematics and algebra be without numbers? A part of everyday life, numbers are the basic building blocks of algebra. Numbers give you a value to work with. Where would civilisation be today if not for numbers? Without numbers to figure the distances, slants, heights and directions, the pyramids would never have been built. Without numbers to figure out navigational points, the Vikings would never have left Scandinavia. Without numbers to examine distance in space, humankind could not have landed on the moon.

Even the simple tasks and the most common of circumstances require a knowledge of numbers. Suppose that your mum asked you to figure out the amount of petrol it takes for her to get from home to school and then on to work and back each day. You need a number for the total kilometres between your home, school and work and another number for the total kilometres your car can run on a litre of petrol.

It's sometimes really convenient to declare, 'I'm only going to look at whole-number answers', because whole numbers do not include fractions or negatives. You could easily end up with a fraction if you're working through a problem that involves a number of cars or people. Who wants half a car or, heaven forbid, a third of a person?

Whole numbers: Adding, subtracting, multiplying and dividing

Adding things up and taking them away are the two most fundamental skills in arithmetic. If you master these skills — just two sides of the same coin — you'll find the rest of this book much, much easier than it would be without them. I provide a refresher on all the basics in Chapter 2.

In Chapter 2, I also cover multiplication and division. You may have memories of reciting times tables in earlier years at school. I was terrified of my maths teacher at primary school, even though I was good at my times tables. Every Friday, he spent half the lesson marching up and down barking out 20 questions from the times tables. We learnt them soon enough, but teaching by intimidation is hardly the method I'd recommend.

Instead, in Chapter 2 I show you the times tables and give you some games to play to remember them. The times tables usually only go up to ten, so I also show you how to work with bigger numbers.

Dividing is exactly the opposite of multiplying: You take a number of things and split them into equal piles. Armies are split up into divisions. So are Aussie Rules football leagues. So '92 divided by four' just means 'split up 92 into 4 piles and tell me how big the piles are'. Or, 'split up 92 into piles of 4, and tell me how many piles there are'. In Chapter 2, I show you some games to help you remember your division sums up to $100 \div 10$, and then show you how to do division when you have bigger numbers. Again, you just need to split up piles.

In Chapter 3, I move on to negative and positive numbers.

Parts of the whole: Fractions, decimals and percentages

Seeing how whole numbers fit together is relatively easy, but then suddenly the evil maths guys start throwing fractions and percentages at you — and things aren't so intuitive. Fractions (at least, proper fractions) are just numbers that are smaller than whole numbers — they follow the same rules as regular numbers but sometimes need a bit of adjusting before you can apply them to everyday situations.

I have two main aims for Chapter 4: To show you that fractions, decimals and percentages are nothing like as fearsome as you may believe; and to show you that fractions, decimals and percentages are all different ways of writing the same thing — therefore, if you understand one of them, you can understand all of them.

I won't promise that you'll emerge from Chapter 4 deeply in love with fractions, but I hope I can help you make peace with fractions so you can work through the questions likely to come up in exams and in real life.

In Chapter 5 I take you through the idea of order of operations, so you know what you're supposed to be doing first (adding, multiplying, whatever), for whole numbers and for fractions.

Moving On to Algebra

You've probably heard the word *algebra* on many occasions, and no doubt you know that it has something to do with mathematics. But what exactly is algebra? What is it really used for?

Parts II and III answer these questions and more, providing the straight scoop on what it's good for, how algebra is used, and what tools you need to make it happen. In this chapter, you find some of the basics necessary to more easily find your way through the different topics in Parts II and III. I also point you toward these topics.

In a nutshell, algebra is a way of generalising arithmetic. Through the use of variables (letters representing numbers) and formulas or equations involving those variables, you solve problems. The problems may be in terms of practical applications, or they may be puzzles for the pure pleasure of the solving. Algebra uses positive and negative numbers, integers, fractions, operations, and symbols to analyse the relationships between values. It's a systematic study of numbers and their relationship, and it uses specific rules.

Speaking in Algebra

Algebra and symbols in algebra are like a foreign language. They all mean something and can be translated back and forth as needed. It's important to know the vocabulary in a foreign language; it's just as important in algebra.

Here's some important vocabulary for your journey into algebra:

- An *expression* is any combination of values and operations that can be used to show how things belong together and compare to one another. $2x^2 + 4x$ is an example of an expression. You see distributions over expressions in Chapter 6.
- A *term*, such as $4xy$, is a grouping together of one or more *factors* (variables and/or numbers). Multiplication is the only thing connecting the number with the variables. Addition and subtraction, on the other hand, separate terms from one another. For example, the expression $3xy + 5x - 6$ has three *terms*.
- An *equation* uses a sign to show a relationship — that two things are equal. By using an equation, tough problems can be reduced to easier problems and simpler answers. An example of an equation is $2x^2 + 4x = 7$. See the chapters in Part III for more information on equations.
- An *operation* is an action performed upon one or two numbers or terms to produce a resulting number. Operations are addition, subtraction, multiplication, division, square roots, and so on. See Chapter 2 for more on operations.
- A *variable* is a letter representing some unknown; a variable always represents a number, but it *varies* until it's written in an equation or inequality. (An *inequality* is a comparison of two values.) Then the fate of the variable is set — it can be solved for, and its value becomes the solution of the equation. By convention, mathematicians usually assign letters at the end of the alphabet to be variables (such as x, y and z).
- A *constant* is a value or number that never changes in an equation — it's constantly the same. Five is a constant because it is what it is. A variable can be a constant if it is assigned a definite value. Usually, a variable representing a constant is one of the first letters in the alphabet. In the equation $ax^2 + bx + c = 0$, a, b and c are constants and

the x is the variable. The value of x depends on what a, b and c are assigned to be.

- An *exponent* is a small number written slightly above and to the right of a variable or number, such as the 2 in the expression 3^2. It's used to show repeated multiplication. An exponent is also called the *power* of the value. For more on exponents, see Chapter 6.

Taking aim at algebra operations

In algebra today, a variable represents the unknown. Before the use of symbols caught on, problems were written out in long, wordy expressions. Actually, using letters, signs and operations was a huge breakthrough. First, a few operations were used, and then algebra became fully symbolic. Nowadays, you may see some words alongside the operations to explain and help you understand, like having subtitles in a movie.

By doing what early mathematicians did — letting a variable represent a value, then throwing in some operations (addition, subtraction, multiplication, and division), and then using some specific rules that have been established over the years — you have a solid, organised system for simplifying, solving, comparing or confirming an equation. That's what algebra is all about: That's what algebra's good for.

What About Geometry?

Studying geometry is sort of a Dr Jekyll-and-Mr Hyde thing. You have the ordinary geometry of shapes (the Dr Jekyll part) and the strange world of geometry proofs (the Mr Hyde part).

Every day, you see various shapes all around you (triangles, rectangles, boxes, circles, balls and so on), and you're probably already familiar with some of their properties: area, perimeter, and volume, for example. In Chapters 15 and 16, you discover much more about these basic properties and then explore more advanced geometric ideas about shapes.

Geometry proofs are an entirely different sort of animal. They involve shapes, but instead of doing something straightforward like calculating the area of a shape, you have to come up with a mathematical argument that proves something about a shape. This process requires not only mathematical skills but verbal skills and logical deduction skills as well, and for this reason, proofs trip up many, many students. If you're one of these people and have already started singing the geometry-proof blues, you might even describe geometry proofs — like Mr Hyde — as monstrous. But I'm confident that, with the help of the chapters in Part IV, you'll have no trouble taming some of them.

Playing with Maths

Ideas are things you can play with, and maths is about ideas. Being able to approach maths as a game of ideas is another important tool for your toolbox.

When children make up stories, parents and teachers are usually delighted to listen. Children are brought up with books and stories, and watching them practise writing their own can be exciting. When children make up games, build things or start playing instruments, parents and teachers can see the connection to playing.

But with maths, people's vision can get clouded. It's easy to be concerned with *right* and *wrong* in math, while losing sight of the importance of playing with ideas. Yet playing is just as possible and just as useful in maths as it is in other areas of children's lives. These sections look at some examples of maths play, as they may happen in classrooms and at home.

Experimenting with symbols

Mathematical symbols have meaning, and you should really try to keep that meaning in mind as you work. Okay, I realise that it doesn't sound very playful, so stick with me here.

For example, think back to primary school and imagine a class full of second graders discussing how many muffins are in the partially filled muffin tin in Figure 1-1. Some of these second graders will count the muffins one by one. Others will see more sophisticated relationships. The teacher may help students record their thinking using arithmetic. For example:

- $3 + 3 + 1$
- $12 - 3 - 2$
- $\frac{1}{2}$ of $12 + 1$
- $2 + 2 + 3$
- $3 \times 3 - 2$

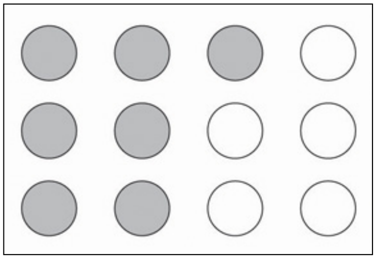

Figure 1-1: How many muffins? How did you count them?

TIP

Can you see how each of these expressions correctly counts the muffins? What could the child who wrote each expression have been thinking?

When you (or a second grader) are working on a task such as this one, you're playing with maths. You're imagining new relationships, trying something without fear of getting it wrong, and seeing a small corner of the world in new ways. Children do all these things when they play.

No *one answer* is involved when you ask, 'How can you count these muffins?' All of the ways of counting come up with seven muffins. That leaves you free to play with the ideas and see whether you can see what someone else sees in this situation.

Building models

Building is a form of play. Building with blocks, building sand castles, building a catapult to launch a cricket ball across the backyard (and hopefully not breaking any of next door's windows), and building a mathematical model are all different forms of the same playful human instinct to create.

When children make predictions, they're building mathematical models. For example, my son's bedtime went from 7.30 to 8.00 when he turned 8 years old. He took this to be a rule and began predicting future bedtimes at future ages, assuming that his bedtime would advance by a half hour each year. He extrapolated in both directions to determine that he will go to bed at 2.30 in the morning when he turns 21 and that he must have gone to bed at 6.00 when he was 4 years old. He built a model of the relationship between his bedtime and his age, and he played with that relationship.

Similar things happen in maths classrooms. Students identify patterns, they make predictions based on these patterns, and they check their predictions against reality.