LEARNING MADE EASY

3rd Edition

Pre-Calculus for dummies

A Wiley Brand

Make sense of logarithms and exponentials

Understand functions and solve linear equations

Graph algebraic and trig functions

Mary Jane Sterling

Pre-Calculus

3rd Edition

by Mary Jane Sterling

for dummies®
A Wiley Brand

Pre-Calculus For Dummies®, 3rd Edition

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Contents at a Glance

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Introduction

Welcome to *Pre-Calculus For Dummies,* a nondiscriminatory, equal-opportunity book. You're welcome to participate whether you are just looking for a quick review or, instead, need some serious prep work before tackling calculus. You may be reading this book for a few perfectly great reasons. Maybe your algebra is a bit rusty, and you want a review that focuses on the material you'll need for calculus. And then, perhaps your trigonometry preparation wasn't as thorough as you would have liked, and you want to get down-and-dirty with the basic topics. Regardless of why you opened up this book, you'll find material that will help you navigate the path that is pre-calculus on the way to the road that is calculus.

You may also be wondering, "Will I really need pre-calculus?" The answer is yes, if you're seriously contemplating taking a calculus course or two. You want to have the basics down cold so you can concentrate on the wonderful topics you'll be finding in calculus. And the concepts throughout this book are also used in many other real-world applications and mathematics arenas.

This book has one goal and one goal only — to teach you the topics covered in pre-calculus in as painless a way as possible. It's presented in words you can relate to and contains figures and pictures to further advance your understanding.

About This Book

This book is not necessarily meant to be read from beginning to end. It's structured in a way that you can flip to a particular chapter and get your needs met (this, that, or the other).

All vocabulary is mathematically correct and clear and explained when it's special to the particular topic. Being precise about mathematical terms is important; it makes the explanation of the subject quicker and clearer, and it gets everyone thinking the same thing about the topic.

Pre-calculus can be a special math arena, but the material covered in pre-calculus comes from many different areas: algebra, geometry, trigonometry, analytic geometry, and so forth. What you'll need from pre-calculus varies, depending on the calculus course or book you'll be using next. Because it can't be predicted exactly what you'll need to take away for the next book, you'll find pretty much every concept that may be under consideration. Just use this book according to your particular needs.

As you're contemplating what to do with all the subject areas covered in this book, consider the two alternatives:

- Look up only what you need to know, when you need to know it. This book is handy for this technique. Use the index, the table of contents, or better yet, the quickie contents at a glance found in the very front of this book to find what you need.
- Start at the beginning and read through the book, chapter by chapter. This approach is a good way to tackle these subject areas because the topics sometimes build on previous ones. Even if you're a math whiz and you want to skim through a section that you feel you know, you may be reminded of something that you forgot. It's recommended that you start at the beginning and slowly working your way through the material. The more practice you have, the better.

Foolish Assumptions

It can't be assumed that just because some of us absolutely love math that you share the same enthusiasm for the subject. It can be assumed, however, that you opened this book for a reason: You need a refresher on the subject, need to learn it for the first time, are trying to relearn it for some future endeavor, or have to help someone else understand it at home. It can also be assumed that you have been exposed, at least in part, to many of the concepts found in this subject because pre-calculus really just takes geometry and Algebra II concepts to the next level.

It has to be assumed that you're willing to do some work. Although pre-calculus isn't the end-all to math courses out there, it's still a higher level math course than some. You're going to have to work a bit, but you knew that, didn't you? And you're looking forward to the challenge!

And it's pretty clear that you're an adventurous soul and have chosen to take on this material, because pre-calculus is not necessarily a required topic. Maybe it's because you love math, or you have nothing better to do at this moment, or because the study of this material will enhance your next step in life. Obviously, you managed to get through some pretty complex concepts in geometry and Algebra II. If you made it this far, you'll make it even farther. This book is here to help!

Icons Used in This Book

Throughout the book you'll find little drawings (called *icons*) that are meant to draw your attention to something important or interesting to know.

TIP

When you see this icon, you know that it points out a way to make your life a lot easier. Easier is good.

REMEMBER

You'll see this icon when you come to an old idea that you should never forget. This is used when you need to recall a previously learned concept or a concept from a previous math course. It also calls attention to some detail being used in a special way.

WARNING

Think of Warnings as big stop signs. The presence of this icon alerts you to common errors or points out something that can be a bit tricky.

TECHNICAL STUFF

The material following this icon is wonderful mathematics; it's closely related to the topic at hand, and it usually introduces formulas and math techniques that are very specific to the current topic. Sometimes this will refer to an even more involved processes or formula that may or may not be needed at the time but is good to know. And you might even find the note going into further detail or explanation of an application or technical reference — for your future perusal.

Beyond the Book

In addition to the material in the print or ebook you're reading right now, this product also comes with some access-anywhere goodies on the web. No matter how well you understand the concepts of pre-calculus, you'll likely come across a few questions where you don't have a clue. To get this material, simply go to `www.dummies.com` and search for "*Pre-Calculus For Dummies* Cheat Sheet" in the Search box.

Where to Go from Here

If you have a really firm background in basic algebra skills, feel free to skip Chapter 1 and head right over to Chapter 2. If you want a brush-up, then it's suggested you read Chapter 1. In fact, everything in Chapter 2 is also a review except, perhaps, interval notation. So if you're really impatient or are a math genius, ignore everything until interval notation in Chapter 2. As you work through this book, keep in mind that many concepts in pre-calculus are take-offs from Algebra II, so don't make the mistake of completely skipping chapters because they sound familiar. They may sound familiar but are likely to include some brand-spanking-new material. Here's a brief list of sections that may sound familiar but include new concepts that you should pay attention to:

- Translating common functions
- Solving polynomials
- All the trig information
- Complex numbers
- Matrices

So where do you go from here? You get going straight into pre-calculus! Best of luck, and enjoy!

1

Getting Started with Pre-Calculus

IN THIS PART . . .

Sharpen algebraic skills.

Identify challenging areas in algebra and conquer those challenges.

Work from basic function types and graph their transformations.

Perform operations on real numbers and functions.

IN THIS CHAPTER

» Refreshing your memory on numbers and variables

» Accepting the importance of graphing

» Preparing for pre-calculus by grabbing a graphing calculator

Chapter 1
Pre-Pre-Calculus

Pre-calculus is the bridge (drawbridge, suspension bridge, covered bridge) between Algebra II and calculus. In its scope, you review concepts you've seen before in math, but then you quickly build on them. You see some brand-new ideas, but even those build on the material you've seen before; the main difference is that the problems get much more challenging (for example, going from linear systems to nonlinear systems). You keep on building until the end of the bridge span, which doubles as the beginning of calculus. Have no fear! What you find here will help you cross the bridge (toll free).

Because you've probably already taken Algebra I, Algebra II, and geometry, it's assumed throughout this book that you already know how to do certain things. Just to make sure, though, I address some particular items in this chapter in a little more detail before moving on to the material that is pre-calculus.

If there is any topic in this chapter that you're not familiar with, don't remember how to do, or don't feel comfortable doing, I suggest that you pick up another *For Dummies* math book and start there. If you need to do this, don't feel like a failure in math. Even pros have to look up things from time to time. Use these books like you use encyclopedias or the Internet — if you don't know the material, just look it up and get going from there.

Pre-Calculus: An Overview

Don't you just love movie previews and trailers? Some people show up early to movies just to see what's coming out in the future. Well, consider this section a trailer that you see a couple months before the *Pre-Calculus For Dummies* movie comes out! The following list, presents some items you've learned before in math, and some examples of where pre-calculus will take you next:

- **Algebra I and II:** Dealing with real numbers and solving equations and inequalities.

  Pre-calculus: Expressing inequalities in a new way called *interval notation.*

  You may have seen solutions to inequalities in set notation, such as $\{x \mid x > 4\}$. This is read, in inequality notation as $x > 4$. In pre-calculus, you often express this solution as an interval: $(4, \infty)$. (For more, see Chapter 2.)

- **Geometry:** Solving right triangles, whose sides are all positive.

  Pre-calculus: Solving non-right triangles, whose sides aren't always represented by positive numbers.

  You've learned that a length can never be negative. Well, in pre-calculus you sometimes use negative numbers for the lengths of the sides of triangles. This is to show where these triangles lie in the coordinate plane (they can be in any of the four quadrants).

- **Geometry/trigonometry:** Using the Pythagorean Theorem to find the lengths of a triangle's sides.

  Pre-calculus: Organizing some frequently used angles and their trig function values into one nice, neat package known as the *unit circle* (see Part 2).

  In this book, you discover a handy shortcut to finding the sides of triangles — a shortcut that is even handier for finding the trig values for the angles in those triangles.

- **Algebra I and II:** Graphing equations on a coordinate plane.

  Pre-calculus: Graphing in a brand-new way with the polar coordinate system (see Chapter 12).

  Say goodbye to the good old days of graphing on the Cartesian coordinate plane. You have a new way to graph, and it involves goin' round in circles. I'm not trying to make you dizzy; actually, polar coordinates can make you some pretty pictures.

- **Algebra II:** Dealing with imaginary numbers.

  Pre-calculus: Adding, subtracting, multiplying, and dividing complex numbers gets boring when the complex numbers are in rectangular form $(A + Bi)$. In pre-calculus, you become familiar with something new called *polar form* and use that to find solutions to equations you didn't even know existed.

All the Number Basics (No, Not How to Count Them!)

When entering pre-calculus, you should be comfy with sets of numbers (natural, integer, rational, and so on). By this point in your math career, you should also know how to perform operations with numbers. You can find a quick review of these concepts in this section. Also, certain properties hold true for all sets of numbers, and it's helpful to know them by name. I review them in this section, too.

The multitude of number types: Terms to know

Mathematicians love to name things simply because they can; it makes them feel special. In this spirit, mathematicians attach names to many sets of numbers to set them apart and cement their places in math students' heads for all time:

- **The set of natural or counting numbers: {1, 2, 3 . . .}**. Notice that the set of natural numbers doesn't include 0.
- **The set of whole numbers: {0, 1, 2, 3 . . .}**. The set of whole numbers does include the number 0, however.
- **The set of integers: {. . . –3, –2, –1, 0, 1, 2, 3 . . .}**. The set of integers includes positives, negatives, and 0.

  TIP

  Dealing with integers is like dealing with money: Think of positives as having it and negatives as owing it. This becomes important when operating on numbers (see the next section).
- **The set of rational numbers: the numbers that can be expressed as a fraction where the numerator and the denominator are both integers.** The word *rational* comes from the idea of a ratio (fraction or division) of two integers.

Examples of rational numbers include (but in no way are limited to) $\frac{1}{5}$, $-\frac{7}{2}$, and 0.23. A rational number is any number in the form $\frac{p}{q}$ where p and q are integers, but q is never 0. If you look at any rational number in decimal form, you notice that the decimal either stops or repeats.

Adding and subtracting fractions is all about finding a common denominator. And roots must be like terms in order to add and subtract them. For example, you can add $\sqrt{3}$ and $2\sqrt{3}$, but you can't add $\sqrt{3}$ and $\sqrt{6}$.

- **The set of irrational numbers: all numbers that can't be expressed as fractions.** Examples of irrational numbers include $\sqrt{2}$, $\sqrt[3]{4}$, and π.
- **The set of all real numbers: all the sets of numbers previously discussed.** For an example of a real number, think of a number . . . any number. Whatever it is, it's real. Any number from the previous bullets works as an example. The numbers that aren't real numbers are imaginary.

  Like telemarketers and pop-up ads on the Net, real numbers are everywhere; you can't get away from them — not even in pre-calculus. Why? Because they include all numbers except the following:

  - **A fraction with a zero as the denominator:** Such numbers don't exist and are called *undefined*.
  - **The square root of a negative number:** These numbers are part of *complex numbers*; the negative root is the *imaginary* part (see Chapter 12). And this extends to any even root of a negative number.
  - **Infinity:** Infinity is a concept, not an actual number.
- **The set of imaginary numbers: square roots of negative numbers.** Imaginary numbers have an imaginary unit, like *i*, 4*i*, and –2*i*. Imaginary numbers used to be considered to be made-up numbers, but mathematicians soon realized that these numbers pop up in the real world. They are still called imaginary because they're square roots of negative numbers, but they are a part of the language of mathematics. The imaginary unit is defined as $i = \sqrt{-1}$. (For more on these numbers, head to Chapter 12.)
- **The set of complex numbers: the sum or difference of a real number and an imaginary number.** Complex numbers appear like these examples: $3 + 2i$, $2 - \sqrt{2}i$, and $4 - \frac{2}{3}i$. However, they also cover all the previous lists, including the real numbers (3 is the same thing as $3 + 0i$) and imaginary numbers (2*i* is the same thing as $0 + 2i$).

  REMEMBER

  The set of complex numbers is the most complete set of numbers in the math vocabulary, because it includes real numbers (any number you can possibly think of), imaginary numbers (*i*), and any combination of the two.

The fundamental operations you can perform on numbers

From positives and negatives to fractions, decimals, and square roots, you should know how to perform all the basic operations on all real numbers. These operations include adding, subtracting, multiplying, dividing, taking powers of, and taking roots of numbers. The *order of operations* is the way in which you perform these operations.

TIP

The mnemonic device used most frequently to remember the order is PEMDAS, which stands for

1. **P**arentheses (and other grouping devices)
2. **E**xponents (and roots, which can be written as exponents)
3. **M**ultiplication and **D**ivision (whichever is first, from left to right)
4. **A**ddition and **S**ubtraction (whichever is first, from left to right)

REMEMBER

One type of operation that is often overlooked or forgotten about: absolute value. *Absolute value* gives you the distance from 0 on the number line. Absolute value should be included with the parentheses step, because you have to consider what's inside the absolute-value bars first (because the bars are a grouping device). Don't forget that absolute value is always positive or zero. Hey, even if you're walking backward, you're still walking!

The properties of numbers: Truths to remember

Remembering the properties of numbers is important because you use them consistently in pre-calculus. You may not often use these properties by name in pre-calculus, but you do need to know when to use them. The following list presents the properties of numbers:

- **Reflexive property: $a = a$.** For example, $10 = 10$.
- **Symmetric property: If $a = b$, then $b = a$.** For example, if $5 + 3 = 8$, then $8 = 5 + 3$.
- **Transitive property: If $a = b$ and $b = c$, then $a = c$.** For example, if $5 + 3 = 8$ and $8 = 4 \cdot 2$, then $5 + 3 = 4 \cdot 2$.
- **Commutative property of addition: $a + b = b + a$.** For example, $2 + 3 = 3 + 2$.

- **Commutative property of multiplication: $\boldsymbol{a \cdot b = b \cdot a}$.** For example, $2 \cdot 3 = 3 \cdot 2$.
- **Associative property of addition: $\boldsymbol{(a+b)+c = a+(b+c)}$.** For example, $(2+3)+4 = 2+(3+4)$.
- **Associative property of multiplication: $\boldsymbol{(a \cdot b) \cdot c = a \cdot (b \cdot c)}$.** For example, $(2 \cdot 3) \cdot 4 = 2 \cdot (3 \cdot 4)$.
- **Additive identity: $\boldsymbol{a + 0 = a}$.** For example, $-3 + 0 = -3$.
- **Multiplicative identity: $\boldsymbol{a \cdot 1 = a}$.** For example, $4 \cdot 1 = 4$.
- **Additive inverse property: $\boldsymbol{a + (-a) = 0}$.** For example, $2 + (-2) = 0$.
- **Multiplicative inverse property: $\boldsymbol{a \cdot \frac{1}{a} = 1}$.** For example, $2 \cdot \frac{1}{2} = 1$. (But remember, $a \neq 0$.)
- **Distributive property: $\boldsymbol{a(b+c) = a \cdot b + a \cdot c}$.** For example, $10(2+3) = 10 \cdot 2 + 10 \cdot 3 = 20 + 30 = 50$.
- **Multiplicative property of zero: $\boldsymbol{a \cdot 0 = 0}$.** For example, $5 \cdot 0 = 0$.
- **Zero-product property: If $\boldsymbol{a \cdot b = 0}$, then $\boldsymbol{a = 0}$ or $\boldsymbol{b = 0}$.** For example, if $x(x+2) = 0$, then $x = 0$ or $x + 2 = 0$.

REMEMBER

If you're trying to perform an operation that isn't on the previous list, then the operation probably isn't correct. After all, algebra has been around since 1600 BC, and if a property exists, someone has probably already discovered it. For example, it may look inviting to say that $10(2+3) = 10 \cdot 2 + 3 = 23$, but that's incorrect. The correct process and answer is $10(2+3) = 10 \cdot 2 + 10 \cdot 3 = 50$. Knowing what you can't do is just as important as knowing what you can do.

Visual Statements: When Math Follows Form with Function

Graphs are great visual tools. They're used to display what's going on in math problems, in companies, and in scientific experiments. For instance, graphs can be used to show how something (like real estate prices) changes over time. Surveys can be taken to get facts or opinions, the results of which can be displayed in a graph. Open up the newspaper on any given day and you can find a graph in there somewhere.

Hopefully the preceding paragraph answers the question of why you need to understand how to construct graphs. Even though in real life you don't walk around with graph paper and a pencil to make the decisions you face, graphing is vital in math and in other walks of life. Regardless of the absence of graph paper, graphs indeed are everywhere.

For example, when scientists go out and collect data or measure things, they often arrange the data as x and y values. Typically, scientists are looking for some kind of general relationship between these two values to support their hypotheses. These values can then be graphed on a coordinate plane to show trends in data. For example, a good scientist may show with a graph that the more you read this book, the more you understand pre-calculus! (Another scientist may show that people with longer arms have bigger feet. Boring!)

Basic terms and concepts

Graphing equations is a huge part of pre-calculus, and eventually calculus, so it's good to review the basics of graphing before getting into the more complicated and unfamiliar graphs you see later in the book.

Although some of the graphs in pre-calculus will look very familiar, some will be new — and possibly intimidating. This book will get you more familiar with these graphs so that you will be more comfortable with them. However, the information in this chapter is mostly information that you remember from Algebra II. You did pay attention then, right?

Each point on the coordinate plane on which you construct graphs — that is, a plane made up of the horizontal (x-) axis and the vertical (y-) axis, creating four quadrants — is called a coordinate pair (x, y), which is often referred to as a *Cartesian coordinate pair.*

TECHNICAL STUFF

The name *Cartesian coordinates* comes from the French mathematician and philosopher who invented all this graphing stuff, René Descartes. Descartes worked to merge algebra and Euclidean geometry (flat geometry), and his work was influential in the development of analytic geometry, calculus, and cartography.

A *relation* is a set (which can be empty, but in this book I only consider nonempty sets) of ordered pairs that can be graphed on a coordinate plane. Each relation is kind of like a computer that expresses x as input and y as output. You know you're dealing with a relation when the set is given in those curly brackets (like these: { }) and has one or more points inside. For example, $R = \{(2, -1), (3, 0), (-4, 5)\}$ is a relation with three ordered pairs. Think of each point as (input, output) just like from a computer.

The *domain* of a relation is the set of all the input values, usually listed from least to greatest. The domain of set R is $\{-4, 2, 3\}$. The *range* is the set of all the output values, also often listed from least to greatest. The range of R is $\{-1, 0, 5\}$. If any value in the domain or range is repeated, you don't have to list it twice. Usually, the domain is the x-variable and the range is y.

REMEMBER

If different variables appear, such as m and n, input (domain) and output (range) usually go alphabetically, unless you're told otherwise. In this case, m would be your input/domain and n would be your output/range. But when written as a point, a relation is always (input, output).

Graphing linear equalities and inequalities

When you first figured out how to graph a line on the coordinate plane, you learned to pick domain values (x) and plug them into the equation to solve for the range values (y). Then, you went through the process multiple times, expressed each pair as a coordinate point, and connected the dots to make a line. Some mathematicians call this the ol' *plug-and-chug method.*

After a bit of that tedious work, somebody said to you, "Hold on! You can use a shortcut." That shortcut involves an equation called *slope-intercept form*, and it's expressed as $y = mx + b$. The variable m stands for the slope of the line (see the next section), and b stands for the y-intercept (or where the line crosses the y-axis). You can change equations that aren't written in slope-intercept form to that form by solving for y. For example, graphing $2x - 3y = 12$ requires you to subtract $2x$ from both sides first to get $-3y = -2x + 12$. Then you divide every term by -3 to get

$$y = \frac{2}{3}x - 4$$

In the first quadrant, this graph starts at -4 on the y-axis; to find the next point, you move up two and right three (using the slope). Slope is often expressed as a fraction because it's rise over run — in this case $\frac{2}{3}$.

Inequalities are used for comparisons, which are a big part of pre-calculus. They show a relationship between two expressions (greater-than, less-than, greater-than-or-equal-to, and less-than-or-equal-to). Graphing inequalities starts exactly the same as graphing equalities, but at the end of the graphing process (you still put the equation in slope-intercept form and graph), you have two decisions to make:

- Is the line *dashed,* indicating $y <$ or $y >$, or is the line *solid,* indicating $y \leq$ or $y \geq$?
- Do you shade under the line for $y <$ or $y \leq$, or do you shade above the line for $y >$ or $y \geq$? Simple inequalities (like $x < 3$) express all possible answers. For inequalities, you show all possible answers by shading the side of line that works in the original equation.

For example, when graphing $y < 2x - 5$, you follow these steps:

1. Start off at –5 on the *y*-axis and mark a point.
2. Move up two and right one to find a second point.
3. When connecting the dots, you produce a straight dashed line through the points.
4. Shade on the bottom half of the graph to show all possible points in the solution.

Gathering information from graphs

After getting you used to coordinate points and graphing equations of lines on the coordinate plane, typical math books and teachers begin to ask you questions about the points and lines that you've been graphing. The three main things you'll be asked to find are the distance between two points, the midpoint of the segment connecting two points, and the exact slope of a line that passes through two points.

Calculating distance

TECHNICAL STUFF

Knowing how to calculate distance by using the information from a graph comes in handy in a big way, so allow this quick review of a few things first. *Distance* is how far apart two objects, or two points, are. To find the distance, *d*, between the two points (x_1, y_1) and (x_2, y_2) on a coordinate plane, for example, use the following formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

You can use this equation to find the length of the segment between two points on a coordinate plane whenever the need arises. For example, to find the distance between A(–6, 4) and B(2, 1), first identify the parts: $x_1 = -6$ and $y_1 = 4$; $x_2 = 2$ and $y_2 = 1$. Plug these values into the distance formula: $d = \sqrt{(2-(-6))^2 + (1-4)^2}$. This problem simplifies to $\sqrt{73}$.

Finding the midpoint

TECHNICAL STUFF

Finding the midpoint of a segment pops up in pre-calculus topics like conics (see Chapter 13). To find the midpoint of the segment connecting two points, you just average their *x* values and *y* values and express the answer as an ordered pair:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

You can use this formula to find the center of various graphs on a coordinate plane, but for now you're just finding the midpoint. You find the midpoint of the segment connecting the two points (see the previous section) by using the previous formula. This would give you

$$M = \left(\frac{-6+2}{2}, \frac{4+1}{2}\right), \quad \text{or} \quad \left(-2, \frac{5}{2}\right).$$

Figuring a line's slope

TECHNICAL STUFF

When you graph a linear equation, slope plays a role. The slope of a line tells how steep the line is on the coordinate plane. When you're given two points (x_1, y_1) and (x_2, y_2) and are asked to find the slope of the line between them, you use the following formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

If you use the same two points A and B from the previous sections and plug the values into the formula, the slope is $-\frac{3}{8}$.

Positive slopes always move up when going to the right or move down going to the left on the plane. Negative slopes either move down when going right or up when going left. (Note that if you moved the slope down and left, it would be negative divided by negative, which has a positive result.) Horizontal lines have zero slope, and vertical lines have undefined slope.

TIP

If you ever get the different types of slopes confused, remember the skier on the ski-slope:

- When he's going uphill, he's doing a lot of work (+ slope).
- When he's going downhill, the hill is doing the work for him (– slope).
- When he's standing still on flat ground, he's not doing any work at all (0 slope).
- When he hits a wall (the vertical line), he's done for and he can't ski anymore (undefined slope)!

Get Yourself a Graphing Calculator

It's highly recommended that you purchase a graphing calculator for pre-calculus work. Since the invention of the graphing calculator, the emphases and time spent on calculations in the classroom and when doing homework have changed because the grind-it-out computation isn't necessary. Many like doing the majority of the