Physicists on Wall Street and Other Essays on Science and Society

Jeremy Bernstein

Physicists on Wall Street and Other Essays on Science and Society

Springer

Jeremy Bernstein
New York
NY, USA

ISBN: 978-0-387-76505-1 e-ISBN: 978-0-387-76506-8
DOI: 10.1007/978-0-387-76506-8

Library of Congress Control Number: 2008931402

Printed on acid-free paper

9 8 7 6 5 4 3 2 1

springer.com

About the Author

Jeremy Bernstein has had a long and distinguished career in which he made major contributions in the fields of writing, teaching, and science. He is currently a professor emeritus of physics at the Stevens Institute of Technology in Hoboken, New Jersey. He was a staff writer for the *New Yorker* magazine from 1961 to 1995 and has written more than a dozen books on popular science and travel. He has won the AAS-Westinghouse Prize and (twice) the U.S. Steel-American Institute of Physics science writing prize. His book *Albert Einstein* was nominated for a National Book Award. He is also the recipient of the Brandeis Creative Arts Medal for non-fiction, the Britannica Award for the dissemination of knowledge, and the Germant Award, given by the American Institute of Physics for contributions that link physics to the arts and humanities. He has taught non-fiction writing at Princeton University. Professor Bernstein was born in Rochester, New York and was educated at Harvard University, where he received three degrees. His primary interests in physics research are in the areas of elementary particles and cosmology.

Preface

Everyone has their own way of learning. Mine has always been verbal. When I was a kid, I used to think out loud. I was caught at it once by a school janitor—a very nice man who suggested that what I was doing was a little strange. After that, whenever I wanted to learn something new or complicated, I'd begin by reading up on it and then lecturing to myself in my head. I still do this. If I get stuck I realize that I had better do some more reading or maybe ask an expert. Once I think I have it down pretty well, I write an essay. Almost never are these essays commissioned, and almost never do I know where, if anywhere, they are going to be published. For this reason a lot of my essays are too long to be published in most magazines or newspapers. It is very rare that you will get as much a 3,000 words in a magazine. The old *New Yorker*, for which I wrote for thirty-five years, was different. Many of the things I turned in emerged substantially longer than what they were in their original form. But this is incredibly rare. If the editing is good, an editor can preserve the essence of what you are saying while chopping the piece in half, or even more than half. Like most authors, I have learned to accept this if I respect the editor and the publication. But there is always a sense of loss.

There is a bit more opportunity with books of essays to take the time to get where you are going with an idea. The essays in this book are about science and scientists in a very broad sense. Each one reflects something about which I became extremely interested at various times. A couple of these essays, the ones that were published in trade journals, were published at their original length or longer. The rest have either never been published at all or have only been published partially. The subject matter is very eclectic. I hope the reader will find the choice interesting.

Jeremy Bernstein

Acknowledgments

"Options" first appeared in a somewhat different form in *Commentary* and "Heisenberg in Poland" and "Orion" did likewise in *The American Journal of Physics*.

Contents

Part I
Economists

Chapter 1
Options

Da könnt' mir halt der liebe Gott leid tun, die Theorie stimmt doch-Then I would have been sorry for the dear Lord—the theory is correct. (Einstein's response to a student who asked him what his feelings would be if experiment failed to confirm his theory of gravitation)

If you decide you don't have to get A's, you can learn an enormous amount in college.

—I. I. Rabi

In the spring of 1969 I got the somewhat lunatic idea that I wanted to go to the northwest frontier of Pakistan to see the high mountains—K-2, Nanga Parbat, and the like. As it happened, I had a colleague in physics who was a Pakistani, and he had a connection both with the University of Islamabad and the Ford Foundation, which had a program to send scholars to Pakistan to teach at the university. He arranged for me to become a Ford Foundation visiting professor. I told the foundation that I intended to drive to Pakistan and, after expressing considerable surprise, they agreed to give me what they would have had to spend on first-class airfare towards purchasing a specially modified Land Rover suitable for making such a trip. I then persuaded two friends to come with me and in something less than a month we drove to Pakistan by way of Greece, Turkey, Iran, and Afghanistan and over the Khyber Pass to Islamabad. Upon arriving, I learned that the university was closed for a month as a kind of punishment for some sort of student uprising. This gave us a full month to explore the frontier, places such as Chitral, Swat, Hunza, and Gilgit—now inaccessible, as is the rest of our route. At the end of the month my friends went back to France, and I moved into the Ford Foundation staff house in Islamabad to take up my teaching duties.

It was a pleasant, if somewhat lonely, existence as the sole resident of the staff house, apart from the staff, which included a driver for the car I had been assigned. But after about a month, one morning I heard a pair of English-speaking voices, male and female. Upon investigation it turned out to be another Ford professor and his wife. But this was not *any* professor. It was Marshall Stone. Although I had never met him, Stone was one of my heroes. He was one of the world's greatest mathematicians. He had taught at Harvard for many years and then, in 1946, he was

J. Bernstein, *Physicists on Wall Street and Other Essays on Science and Society*,

brought to the University of Chicago to create what became the leading school of mathematics in this country. Moreover, Stone was the teacher of my teacher at Harvard, George Mackey, who had interested me in the mathematical foundations of the quantum theory, some of which had been provided by Stone. Stone, who died in India in 1989 at the age of 85, was, incidentally, the son of the late Chief Justice of the Supreme Court, Harlan Fiske Stone. Now, here he was, in the Ford Foundation staff house accompanied by his rather recently acquired wife, Vila, a very attractive and voluble Yugoslavian.

It turned out that Stone, who was an inveterate traveler, had also come to Pakistan to visit the frontier. Thus, the three of us spend a good deal of time together, during which I told him what I knew about it. In the course of this, Vila mentioned that she had a daughter who lived in New York and that I might like to meet her. It took some time, but eventually I called. She knew who I was, and we became friends. Some years later, she told me that she was seeing someone with whom I might have things to talk about, since he was studying "derivatives." A derivative in calculus is the rate of change along a curve. It is one of the first things one learns in calculus. I assumed that this fellow was taking a first course in calculus, which did not seem to me to be much of a conversation piece. In actuality, he turned out to be an amiable chap whose name was Myron. I forget what we talked about, but I managed to steer the conversation away from any mention of derivatives. I have a dim recollection that, at one point, his date whispered in my ear that Myron was going to win the Nobel Prize someday. I am sure that I thought that if they were giving out Nobel Prizes for learning calculus, things were worse than I had imagined. But, as it turned out, she was right. In 1997, not long after our meeting, Myron Scholes and Robert Merton shared the Nobel Prize in economics. Scholes and Fischer Black, who had died two years earlier, had created what is known as the Black-Scholes equation, which they published in 1973. (Merton invented another approach to the same problem. We will attempt to explain all of this later.) It does indeed deal with derivatives—investment vehicles such as options on stocks or bonds, whose present value is derived from the projected future values of the financial commodities—stocks and bonds—that underlie them. The Black-Scholes equation, and its many adumbrations, is used to set the price of such options. It is, If you like, the Newton's Law or the Schrödinger equation of the whole field of financial engineering that makes these derivative markets operate.

I had more or less forgotten about all of this until I read an autobiographical memoir entitled *My Life as a Quant; Reflections on Physics and Finance* by Emanuel Derman. A "quant" is the rubric used on Wall Street and elsewhere to describe people who practice quantitative financial analysis—financial engineering—for which the Black-Scholes equation is a prototype. Physics comes into Derman's memoir because, although he became a professor in Columbia University's Department of Industrial Engineering and Operations Research and ran the financial engineering program there, he actually had a Ph.D. in physics from Columbia, which he obtained in 1973. He was one of the early POWS—Physicists on Wall Street—having joined the financial firm of Goldman Sachs in 1985. The first part of his book traces the somewhat unlikely steps that took him from his native Cape

Town, South Africa, first to Columbia and then via the AT&T Bell Labs, and elsewhere, to Wall Street. For reasons that will be explained later, the stops along his route are especially familiar to me. Indeed, although I do not have any specific memories, Delman notes in his book that our paths crossed at various times. He and I are both theoretical elementary particle physicists, and our world is not large.

Derman arrived in New York in 1966. He had taken an undergraduate degree in applied mathematics and theoretical physics at the University of Cape Town. As he notes, it is quite unusual to find a university in which someone could do an undergraduate degree in physics and have, apart from a one-year lab course, very little contact with experiments. Upon graduation he applied for scholarships to study in various universities outside South Africa and obtained one from Columbia. The physics department he found at Columbia in the 1960s was very familiar to me. I spent a lot of time there and at one point even had a visiting appointment. In Derman's day the department was still under the aegis of the noted Nobelist I. I. Rabi, whose standards were extremely high and, although Derman did not see this side of his character, could be very tough. To give an example; the economist Milton Friedman's son David was a post doc in the department. I have an ineluctable memory of a departmental Chinese lunch during which young Friedman was heard discoursing on every subject known to man. Rabi suddenly said, "Be quiet. We'll hear from you when you are older." That was the end of that. Rabi wasn't the only present and future Nobelist in that department. You had to be very good, and very determined, to survive in that atmosphere.

Because of his course background, Derman was at a disadvantage. Although he passed the qualifying examinations with high enough scores so that he had the option of working in theoretical physics—which, for some reason, was reserved only to students who were considered especially bright—it was decided that he did not have enough knowledge of modern physics (things like quantum mechanics and its applications) to begin a thesis. He had to spend two years taking courses in subjects that most of the other graduate students already knew. Once having completed this he could begin to look for a thesis advisor. The leading theorist in the department was Tsung Dao Lee, known universally as "T. D." T. D. was in his early forties when Derman arrived. He had shared the Nobel Prize with his collaborator, Chen-Ning Yang, a decade earlier. Derman had a shrewd eye for people. Here is what he wrote about T. D.

"With his Moses-by-Michelangelo persona, beams of light emerging from his forehead, T. D. radiated an intense purity. At first I imagined that his rigorous questioning was the by-product of a pure search for knowledge and truth. Later I thought I began to detect a latent glee with which he savaged the imperfections in other people's talks. He enjoyed disorienting them."[1] I am not so sure that it was a matter of enjoyment. T. D. simply could not stand listening to something he was sure was wrong. On the matter of a "pure search for knowledge" I once put the following proposition to a small group of physicists at lunch. I said suppose God

[1] We will refer to Derman's book *My Life as a Quant*, Wiley, Hoboken, 2004, hereafter as M. L. This quote can be found on p. 35 of M. L.

offered you a book of his own composition in which all the problems of physics were solved. Would you look at it knowing that it would end your role in discovery? Many of the physicists said they would. T. D. was not sure.

Few theoretical physics students at Columbia worked with T. D because T. D. accepted very few students—only the *wunderkinder*—so Derman did not try. He describes how he tried to connect with one of T. D.'s ex *wunderkind*, Gerald Feinberg. Some of the *wunderkinder* became faculty members at Columbia, and Feinberg was one. Derman's unsuccessful attempts to approach Feinberg are amusing to read, especially to me, as he was my best friend until his death in 1992 at the age of 58. I don't know how Derman's life would have turned out if he had been able to work with Feinberg, but he became the first graduate student of another of the *wunderkinder*, Norman Christ. Derman writes that because he and Christ were about the same age, surprisingly perhaps, he never found a comfortable way to relate to him; he was never even on a first-name basis with him. Nonetheless, he was able to produce a thesis, although this took another five years. He notes wryly that about 10 percent of his projected life span was spent getting a Ph.D. degree at Columbia. He wrote his thesis on what we refer to as "phenomenology"—applying some underlying theory to make a model that either predicts or explains an experimental result. In this instance one of the creators of the underlying theory was Steven Weinberg, who shared a Nobel Prize for this work. From the sound of it, Derman wrote a very respectable thesis that required him to learn to use the rather primitive computer facilities that were then available. One had to program machines with IBM punch cards—a very tedious and error-prone exercise. The thesis was good enough to get him a post-doctoral position at the University of Pennsylvania. In the meanwhile, he had married a young woman who had left Czechoslovakia after the Russian occupation. She was then also studying physics but later switched to biology.

The next several years were very difficult for the Dermans. Emanuel moved from one temporary job to another, usually in cities far from where his wife was. One of these jobs was in New York at the Rockefeller University. Life at the Rockefeller was pretty lush, and this time he was in the same city as his wife. However it became clear after his second year that his appointment was not going to be renewed. He even thought of giving up physics and going to medical school, but he could not quite bring himself to do it. Finally, he took an assistant professorship at the University of Colorado in Boulder. This was a real nightmare, since he was now separated geographically from both his wife and very young son. After one year he had had enough and took a job in the Business Analysis Systems Center at Bell Labs in Murray Hill, New Jersey, to which he could commute from New York. Derman's description of Bell Labs as he experienced it was surprising to me. He hated the place. His chapter on Bell Labs is called "In the Penal Colony," a reference to the Kafkaesque petty bureaucracy and enforced regimentation that he found. The reason that I was surprised was that in the period that we are discussing—the early 1980s—I spent a good deal of time at the labs, although I did not meet Derman there.

Indeed, I was not going there to do physics but rather to write about the place. I wrote a series of linked profiles of people at the laboratory. In 1984, it was published as a small book, *Three Degrees Above Zero*.[2] This is a reference to the temperature of the background radiation left over from the Big Bang, which was discovered in 1964 serendipitously, by two Bell Labs physicists, Arno Penzias and Robert Wilson, who received the Nobel Prize in 1978 for this work. I interviewed them as well as the third Nobelist at the labs, Philip Anderson. These three were part of a long tradition of Bell Labs scientists who had won the Nobel Prize, including William Shockley, Walter Brattain, and John Bardeen, all of whom shared the 1956 Nobel Prize for their discovery of the transistor. The people that I interviewed were at the top of their fields—people whom any university would have been delighted to have. At the labs they could do pretty much what they wanted. For them, Bell Labs was the real ivory tower, a first-rate research facility where in those pre-divestiture days—AT&T was broken up in January of 1984, and some of the Labs eventually became Lucent and the rest went with the Baby Bells—there was no concern about scrounging for money for research projects. Of all the people that I interviewed there was only one overlap with the people that Derman knew. This was Ken Thompson, a computer genius who, along with another Bell Labs computer genius, Dennis Ritchie, created the UNIX operating system. This is the multi-user, multi-task system that is used to run the computer complexes in most centers around the world. To run the system they created a language called "C," which, with its variants, became the language of choice for programmers. Thompson, along with a Bell Labs physicist named Joe Condon, had, at the time I met them, constructed a dedicated chess-playing machine they called "Belle." This was not a program but a machine that was hard-wired to play chess. It played just below the Grand Master level and was, at the time, the champion chess machine. As part of my interview, I was given the chance to play it—losing gracefully. Derman would have liked to join a research group with people like Thompson and Ritchie, but all his requests were refused. By the early 1980s he had had enough.

As it happened, this coincided with the time in which the major brokerage firms on Wall Street and elsewhere were building up their financial engineering departments. They were headhunting in places such as Bell Labs in search of potential quants. This had to do with a change in the brokerage business from merely selling stocks and bonds to dealing in all sorts of derivatives. For example, Salomon had put together a very powerful group of such analysts under the aegis of one John Meriwether (More about him later). One of the consultants to this group was Robert Merton, the Harvard professor who later shared the Nobel Prize with Scholes. Incidentally, it was Merton's father (also Robert but with a different middle initial), a noted sociologist of science at Columbia University, who coined the phrase "self-fulfilling prophecy," something that might well have characterized the later activities of his son and his collaborators. Even the staid brokerage firm of Goldman, Sachs was adding quants. In December of 1985, Derman took a job at Goldman in the Financial Strategies Group, where he had his first encounter with Black-Scholes.

---

[2] *Three Degrees Above Zero* by Jeremy Bernstein, Mentor Books, New American Library, New York, 1986.

Chapter 2
Black-Scholes

Through my parents and relatives I became interested in economics and, in particular, finance. My mother loved business and wanted me to work with her brother in his book publishing and promotion business. During my teenage years, I was always treasurer of my various clubs; I traded extensively among my friends; I gambled to understand probabilities and risks; and worked with my uncles to understand their business activities. I invested in the stock market while in high school and university. I was fascinated with the determinants of the level of stock prices. I spent long hours reading reports and books to glean the secrets of successful investing, but, alas, to no avail.

—Myron Scholes, Nobel autobiography, 1997

At this point let us interrupt the narrative in order to explain the Black-Scholes-Merton revolution. Otherwise you will not be able to make much sense out of what follows. Derman gives a good qualitative discussion of this, but, as the great nineteenth-century Scottish physicist James Clerk Maxwell used to say, "I didn't see the 'go' of it." If you put "Black-Scholes" into Google you will find something like 1.46 million entries. Most of them are technical, proposing solutions to the equations or trying to generalize or derive them. Some of these sites have clearly been posted by ex-physicists, who note, for example, that the Black-Scholes equation can be morphed into the equation that describes the flow of heat. (We will explain this, too.) There are offers to tutor you for a considerable fee. While wandering through this jungle I came across the perfect site for my purposes. It is called "Black-Scholes the Easy Way." You can find it at http://homepage.mac.com/j.norstad. The person who put it up, John Norstad, is a computer scientist who was learning this stuff as a hobby. His posted notes represent his own learning process and are very clear. We will use his examples in case someone wants to consult the site for more details. But what is the basic problem?

At this time, as mentioned, financial institutions were doing a substantial business in the sale of derivatives. A typical example is a stock option. This is a contract between two parties that allows the buyer of the option to purchase a particular stock at a future time from the seller of the option at a contractually specified price called the "strike price," which is often but not always the price of the stock when you buy the option. Until that future time you do not own the stock. You own an option *to buy* the stock at the fixed price. If the stock has gone up by the time you buy it in the future you are, using the term of choice, “in the money. ” If the stock goes down, you don’t buy it but are out the cost of the option— “out of the money. ” The question is, what should the price of the option be when you buy it? This is what the Black-Scholes equation purports to allow you to compute.

To see what is involved we will, following Norstad, consider a “toy” model. This illustrates many of the general features of the problem without the mathematical complexity.

In the toy model there is a stock whose current value is \$100, which, in this example, will be the strike price. What makes the model a toy is that at the time the option is to be exercised there are only two possible prices; \$120 and \$80. In the real world there will be a continuum of prices, which is what Black-Scholes must deal with. The kind of option we are considering here is called a “European call option. ” It can only be exercised at one definite time in the future. An “American call option” can be exercised at any time. We will further assume that the probability of the stock’s rising to \$120 is three quarters, while the probability of its falling to \$80 is one quarter. What then should you be willing to pay for the option? At first sight this seems like a simple question to answer. With these probabilities the expected outcome is $\frac{3}{4} \times \$20 + \frac{1}{4} \times \$0$, which is \$15. Thus one might assume that the option should be worth \$15 to you and that you can then, with a high probability, expect to earn \$5 if you buy the option. This would be true if we were not able to engage in financial engineering—an activity that goes under the rubric of “arbitrage. ” With arbitrage one can gain a certain profit with no risk at all. The cost of this arbitrage is what determines the cost of the option. This changes everything and explains why the financial institutions were hiring quants by the carload. Here is how the arbitrage works in this case.

Let’s assume there is a friendly bank that is willing to lend money interest free. This is a simplification in the analysis that can readily be corrected. If you want to see how including interest modifies the results in the toy model, you can look at Norstad’s website. Only a little high school algebra is required. We do not lose any matters of principle if we make this assumption. Likewise we assume that we can buy fractional shares of this stock from a friendly broker commission-free. To use another physics/economics term, we make the problem “frictionless. ” In physics a friction force generates heat without doing any useful work. Here, the friction generates money loss without helping us with the bottom line.

Now suppose you have made the expectation calculation given above and are willing to give \$15 to buy the option. We will now see how we can, using arbitrage, always pocket \$5 no matter what the outcome is. To achieve this we take the \$15 from you and put \$5 in our pocket. You will never see the fiver again. We then borrow \$40 from the friendly bank-“leverage. ” We take your \$10 and the borrowed \$40 and buy a half share of the stock. This is called the “hedge. ” In this case it has cost us \$10 to replicate the option. This will turn out to be its true value.

Now we have the two cases. If the final price is a \$120, you will exercise your option to buy the stock at a \$100. We are obliged to deliver the stock to you at that price. What we will do is to sell our half-share for $60, repay the bank its $40, and add the remaining $20 to the $100 you gave us to buy the share so we can give it to you. We have, of course, pocketed the $5. If, on the other hand, the final price is $80 you will not exercise your option. If you really like the stock you can simply buy it outright for $80. We will sell our half-share for $40, which we will then return to the bank, still pocketing the $5. This means that if you have given us $15 for the option you have overpaid by $5. If you think about it, you will see that the $10 price is a kind of tipping point. If we can sell the option for more than $10 we will make money, and if someone wants to sell us the option for less than $10 we will buy it and again make money. The $10 is a kind of equilibrium price at which it is not profitable to either buy or sell. Black and Scholes approached the problem of option evaluation using arbitrage as an equilibrium problem—you need to find the price at which there is equilibrium between buying and selling. Merton had a different approach, and this is the one that is now more generally used.

To understand it, note that in finding the correct option price in the presence of the possibility of arbitrage, the probabilities of three-quarters and one quarter played no role. These probabilities only entered when, in the absence of arbitrage, we used them to compute the expected gain. We never had to use them when we found the cost of the hedge—which is the correct option price. This is important because in real life there is little likelihood that we would ever be given these probabilities in any reliable way. In fact, in an important sense, the presence of a buyer of the option is irrelevant. Suppose we just construct a portfolio that consists of $10, and a $40 loan from the bank, which we then invest in half a share of the stock which is selling for $100 a share. This is called a "synthetic option. " You can easily persuade yourself that if we sell this stock when its value is either $120 or $80, the amount that we gain or lose is the same as what the buyer of the option gains or loses in the preceding example if he or she pays $10 for the option. The essence of Merton's approach is to show that one can, in general, construct synthetic options that have the same outcome as the real options. The cost of the synthetic option is the same as the cost of the real option and, by what is called the Law of One Price, this *is* the cost of the option. This is what these brokerage firms do—they construct synthetic options.

In discussing the equivalence of these two methods of pricing options, Derman was reminded of what happened in quantum electrodynamics in the late 1940s. There were two approaches. Julian Schwinger started from first principles and by carrying out a series of horrendous calculations—which, as Oppenheimer said, only Schwinger could have done—produced numbers for physical quantities that could be compared to experiment. Richard Feynman, on the other hand, arrived at these numbers by using pictorial methods and intuitive arguments. A reasonably competent graduate student could learn them in a few days. I remember the disquiet I felt when I first studied Feynman's papers. Why did these tricks work? Indeed, did they always work? The matter was laid to rest when Freeman Dyson in a mathematical tour de force showed that Schwinger and Feynman had found two equivalent representations of the same theory. You could use Feynman's Mozartean calculus knowing that Schwinger's Bach-like logic made it legitimate.

What Black, Scholes, and Merton had to confront was the fact that in the real world we do not have a situation in which there are just two future prices but in fact a continuum. This gets one into the question of how you can predict the future of a stock price. To deal with this Black and Scholes adopted a model that supposed that stock prices follow what is known as a "random walk." We will explain this, but first remarkably the same model was used by a French mathematician named Louis Bachelier to derive the price of what is known as a "barrier option, " an option that is extinguished if the stock price crosses a certain barrier. (One feature of Bachelier's model that was not realistic was his use of negative as well as positive stock prices.)[3] This work was contained in his Ph.D. thesis, *Théorie de la Spéculation*, which he presented at the Sorbonne in 1900! Although Bachelier wrote both books and papers on this kind of probability theory, his work was not much appreciated, even in France. A great deal of it was rediscovered and made more rigorous by people such as Norbert Weiner. It is now embedded in what is known as the "stochastic calculus."

In 1905, Einstein, who had not heard of Bachelier, used these ideas to analyze what is known as "Brownian motion. " In 1827, the Scottish botanist Robert Brown made a very odd discovery. He noticed that if pollen grains, which could only be seen though a microscope, were suspended in water they executed a curious dancing motion. Brown made the natural assumption that these pollens were alive, but then he tried other microscopic particles made of things that were clearly not alive and found the same effect. By the end of the century the correct explanation had been guessed at; namely, that these very small particles were being bombarded by the still smaller—indeed invisible—water molecules and that the "dance" was in response to these collisions. In 1905, Einstein showed that this assumption had precise and measurable consequences. The same work was done independently at about the same time by the Polish physicist Marian Smoluchowski. The basic idea can be illustrated by what is known as the "drunkard's walk. "

A drunkard begins his walk at, say, a lamp post. At each step he can go, say, 2 feet, but in a totally random direction. The question then is how far on the average the drunkard will go, as the crow flies, away from the lamp post after a given number of these steps, say N. When first confronted with this question many people are tempted to say the drunk won't get anywhere, since he will simply retrace each step. But a moment's reflection convinces one that this is essentially impossible. After each step the drunkard's new direction is totally random.[4] The path will appear jagged, but the distance from the lamp post continues to increase. Indeed, the fundamental result of this analysis is that the average distance increases as the square root of the number of steps N.[5] This square root feature shows up in the

---

[3] Bachelier's random-walk model predicted that the spread in stock prices would increase as the square root of the time. But there was no limit in his model as to how low a stock could go. It could become negative, which means the company would be paying you to buy the stock. This flaw in Bachelier was first noted by the economist Paul Samuelson.

[4] A special case is a one-dimensional random walk. After each step the drunkard is equally likely to go forward or backward. Nonetheless he will move inexorably away from the lamp post. This example makes it clear why it takes longer to reach a certain average distance than it would if one just walked there.

[5] To be precise we are talking about the root mean square distance. This is the square root of the average of the square of the distance.