

Horst Beyer

The Reasoning of Quantum Mechanics

Operator Theory and the Harmonic Oscillator

Synthesis Lectures on Engineering, Science, and Technology

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Operator Theory and the Harmonic Oscillator

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