Jane Clark

Calculate the Orbit of Mars!

An Observing Challenge and Historical Adventure

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Preface

Twelve years ago, I wrote a book [1] in Springer's Patrick Moore's Practical Astronomy Series entitled *Measure Solar System Objects and their Movements for Yourself!*, in which I worked out some planetary orbits assuming that these orbits, and that of the Earth, were coplanar circles. I deliberately made that restriction to keep the mathematical difficulty to approximately that of college freshman mathematics for science and engineering students. I didn't use much calculus and used almost no vectors at all.

Of course, I was well aware that this simplifying assumption had its limits. In particular, I was not satisfied with my orbital parameters for Mars, because the Red Planet has quite a noticeably elliptical orbit with an eccentricity of almost 0.1. I now believe I got lucky: my simple method didn't work for data from later years.

I quickly discovered methods for deducing the orbits of comets and asteroids, which work so long as they have a decent inclination to the ecliptic, the plane of the orbit of the Earth. Every single one of these methods fails when the celestial body orbits in almost the same plane as the Earth, as the planets do.

So, I hit a brick wall.

Meanwhile I was slowly gathering photographic position data on planets

Life also got in the way. For me 2010 was one of those disastrous years when my business and my marriage both hit the rocks. I ended up moving From King's Lynn in Norfolk, England, to Bristol on the other side of the country to begin a new job. Difficulty selling a house in the recession, plus the need for some major surgery, meant that I was in a rented house in Bristol for 5 years. In late 2015, I bought a house 34 miles from my Bristol office in Risca, across the border in Wales. The house was what real estate agents politely call a “project”: it did not even have a kitchen. I gathered no data for the Mars opposition of 2016 because I had to get my house fixed up.

At this point John Watson of Springer asked me if I had any ideas for a book. “Funny you should ask,” I replied, “I have this idea for a project to analyse the orbit of Mars.” Thus, the project was born.

By the time of Mars’ apparition of 2018, I had built an observatory, but unfortunately my mother unexpectedly died that summer after a short illness, and I was too shocked to attempt any astronomy. So, I got no data then either.

In 2019 I began to get my observatory to work well. The money from my parents’ estate did not go amiss – I upgraded to an 11” SCT telescope and bought a Celestron CGX mount, which proved to be a massive improvement on its predecessor. Thus, by 2020, I was well positioned to collect another dataset for Mars’ apparition.

But what about that brick wall I had hit with the analysis? I made pretty good progress in 2019, but still didn’t have a method. The next bit of life that got in the way was the Covid pandemic. My pandemic fortunes were mixed. On the one hand I managed to pick up long Covid, and was plagued by fatigue. Sometimes I still am. But two pieces of good luck came my way. First, I now work from home, so I’m not losing 2 hours a day to commuting. Second, what else was there to do? Astronomy has been the perfect lockdown hobby.

While dealing with the data analysis problem, I coined a word, for an activity which produced a combination of fascination and exasperation: I called it exasprinating. Eventually my persistence paid off, and I got a method to work. Fortunately for you, dear reader, explaining the method is much, much easier than finding it and making it work with no help was.

It is my strict policy to spell out all my mathematical working and to try to leave nothing as an exercise for the reader. That way, I hope you will experience more fascination and less exasperation than I had to put up with.

It proved to be impossible to make all the chapters self-contained. You will need Chap. 4 for all the least-squares fits I do, unless this is already a very familiar subject to you. You will also need to have read the mathematical parts of Chaps. 2 and 3 to read Chap. 7. You also need to read a small section of Chap. 7 while reading Chap. 6 if you have forgotten, or never knew, about rotation matrices.

I have assumed that the reader is a scientist, engineer or mathematician, and knows something about calculus and vectors and trigonometry No doubt you may have forgotten much of it. Recalling half-forgotten material may cause some short-term pain. Please don't expect to pick this book up and read it like a novel. You will need a pencil and paper. Please expect to have to put it down and think from time to time. In emergencies, there are plenty of inexpensive mathematics textbooks from which to revise.

The history of the orbit of Mars is worth telling because in order to provide convincing solutions, the natural philosophers of the Renaissance had to invent both physics and calculus. An English newspaper editor once said that while comment is free, facts are sacred [2]. I have tried to stick to this principle. For example, as a teeneager, I was much impressed by Arthur Koestler's book [3] covering much the same history. Even at that tender eage, I had an uneasy feeling that, for this author, facts might be subordinate to the quality of the yarn he spun. I have tried to provide a reference for every factual claim I make.

The rate-determining step in astronomical progress has always been our ability to observe, not our cleverness at theorizing. A combination of Tycho's positional data and Galileo's telescopes did for the ancients' view of the universe. The same thing happened with Hubble's observations of the distances to galaxies and the redshift. Without those insights, the theorists could have debated until the cows come home.

I would like to thank my editors at Springer, whose chief service was to keep chasing me. This includes Maury Solomon and John Watson, both retired, who encouraged me to start, and Hannah Kaufman and Clement Wilson Kamalesh, whose encouragement kept me going. I would also like to thank my many astronomer friends, who have taught me more than they know. Finally, I would like to thank my best friend Pauline Thomas for her encouragement and friendship.

No doubt there are better ways to do what I did. If anyone finds them, finds a historical nugget or two, or manages to do some other interesting piece of celestial mechanics, please share them with me at jane.clark@finerandd.com. Where next for me? One of my ambitions is to devise a method for measuring the astronomical unit in miles, based on inverting Rømer's discovery [4], while obsering the Moons of Jupiter, that light has a finite speed. Stay tuned.

Risca, Wales, UK Jane Clark
April 2021

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I also express my thanks to Dr. Jeannette M. Fine for permission to use a piece of software and Pauline Thomas for permission to use a photograph.

Finally, I would like to thank my best friend Pauline Thomas for her encouragement and friendship.

Contents

About the Author

Jane Clark is a British amateur astronomer who earns her living as an engineer. She has a Ph.D. in physics and an MBA from Warwick University. She completed 2 years of postdoctoral training at Case Western Reserve University in Ohio before returning to England to begin an industrial career. She became interested in both astronomy and photography as a teenager in the 1970s, photography much more seriously, although as her career progressed and family commitments increased, both interests lapsed. She acquired a telescope in 2006, shortly after completing her MBA, and quickly became hooked on observing. This experience made her realize that astronomy is a lot more fun than business administration. In 2017 she achieved her ambition of having an observatory in her back yard. She is a member of Bristol, Cardiff and Newtown Astronomical Societies; and was a founder member of West Norfolk Astronomy Society. Jane gives talks on the Solar System to astronomy clubs, and other societies as diverse as the cub scouts, the University of the Third Age, and church wives' groups.

Chapter 1

In the Beginning

Most, if not all, of the early civilizations developed their astronomy. If nothing else, it gave them a way to measure and predict the seasons. Evidence from Australia, where Europeans discovered a native culture whose land management was there, but could hardly have been called agricultural in a Western sense, is that the first Australians had good naked-eye astronomical knowledge and were well aware of the planets. Europeans found that the Maori in New Zealand used astronomy to predict seasons [5], even though they had no writing before they encountered Europeans. In other words, humans must have been astronomers for a very long time.

Not all the early civilizations had writing as we know it. This invention does not seem to have made it to Central America, for example. Yet we now know that, when they wanted to, the Mesoamerican civilizations could orient their buildings to indicate astronomical events. Therefore, something must have been recorded. The Ancient Egyptians used the disappearance of the star Sirius into the sunrise to know when the Nile would flood. They were exceptionally dependent on this, as rainfall is very rare indeed in much of Egypt. My father served as a draftee in the British Army along the Suez Canal. After it rained, he told that the locals debated whether or not the previous rainfall had been twenty years before. The first indication that the floods were coming was astronomical: there simply was no rainy season.

Civilizations on the Indus, and the Tigris and Euphrates rivers, were among those that kept records. One remarkably successful Macedonian king and general, Alexander the Great (356–323 BCE), conquered lands

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from Greece to the Indus River. Although he died young and his empire did not last, one side effect was the diffusion of astronomical knowledge from the Indian and Babylonian civilizations to the Middle East.

(CE, or the Common Era, is a way of dating. It is exactly the same as the system BC and AD and is simply a more neutral name that does not imply either recognition or rejection of a particular religious event. BCE is "Before the Common Era". One of the more curious characteristics of this dating system is that it has no year zero. It went from 1 BC/BCE straight to 1 AD/CE.)

In the city of Alexandria, Egypt, one of many founded by Alexander the Great as he conquered his way across the Middle East, there grew up a great library and a research institution called the Musaeum, said to have housed over a thousand scholars at any time. The heyday of the Musaeum was from roughly 300–145 BCE.

In that time, it hosted Archimedes, Euclid, Eratosthenes and Aristarchus of Samos.

Eratosthenes measured the circumference of the Earth. It's hard to know whether his answer was a good or brilliant first effort, because that depends on how we interpret the distance unit he used. He used the stadion, which might be anywhere from 159 m to 209 m [6]. If he used the lower end of the range, his Earth radius is almost smack on the modern value. He is popularly supposed to have done this by comparing the angles of the Sun at a mineshaft at Syene, modern Aswan, and in Alexandria. In fact, he was more careful than that, but the details are lost [7]. His own book has not been preserved.

We know of Aristarchus of Samos directly and indirectly. Directly we know of his book "On the sizes and distances of the Sun and Moon" [8]. Indirectly we know from a book by Archimedes, the Sand Reckoner, that

"Aristarchus has brought out a book consisting of certain hypotheses, wherein it appears, as a consequence of the assumptions made, that the universe is many times greater than the 'universe' just mentioned. His hypotheses are that the fixed stars and the sun remain unmoved, that the earth revolves about the sun on the circumference of a circle, the sun lying in the middle of the orbit, and that the sphere of the fixed stars, situated about the same center as the sun, is so great that the circle in which he supposes the earth to revolve bears such a proportion to the distance of the fixed stars as the center of the sphere bears to its surface" [9]. There is no evidence that Aristarchus ever made his hypotheses quantitative [10], although I find it hard to believe that his idea was purely qualitative.

Archimedes goes on to dismiss Aristarchus' idea. Unfortunately for posterity, so did just about everyone else.

What the world got instead was a textbook by Claudius Ptolemy, known as Ptolemy. Ptolemy's view was firmly geocentric. Why was this? It was partly down to the baleful influence of Aristotle.

Aristotle lived from 384 to 322 BCE. I am clearly not qualified to judge how good a philosopher Aristotle was. But I am suitably qualified and experienced to judge his physics. I cannot say it loudly enough: he was a lousy physicist. "In reality [Aristotle] does little but analyze the meanings of every-day experience and words in order thereby to solve the problems of nature." So says J. L. E. Dreyer in his "History of Astronomy from Thales to Kepler" [11]. Because he had such a following for his other endeavours, people unfortunately assumed that he know what the heck he was talking about. Well, he didn't. His idea that everything was made of four elements, Earth, Air, Fire and Water, was hogwash. His ideas on motion misled people for centuries. The likes of Huyghens, Galileo and Newton took a careful, experimental and quantitative view of motion, unlike Aristotle, and got much nearer to what we now believe to be the truth. Aristotle, on the other hand, tried to argue his physics from philosophical reasoning. He also postulated an idea totally untainted by evidence either for or against: that the heavens above the earth were made of a fifth element, which was "perfect". This frankly bizarre idea took a lot of dislodging by means of evidence.

One of Aristotle's fetishes was "prefect circular motion", that is, uniform circular motion about a point. There *is* a useful characteristic of circular motion: the centripetal force required to keep the motion of a point particle circular is always perpendicular to the distance moved. In an infinitesimal time *dt*, the work done is

$$dT = \mathbf{F}_{\text{centripetal}} \bullet \mathbf{dx} = \mathbf{0} \tag{1.1}$$

because **F** and **dx** are always perpendicular. In other words, no work is done to maintain circular motion. The extension to finite bodies is obvious. You simply integrate over the volume. This no energy input is required to keep a planet moving in a circular orbit in a vacuum. It also requires no energy input to keep a solid planet rotating about its axis. Of course, Aristotle did not have the modern vocabulary to describe the phenomenon of circular motion not changing kinetic energy. Nevertheless, he overdid it. As we shall see, it took the world millennia to move on from the fetishization of perfect circular motion.

He acquired over the centuries a tremendous following to the point where it became dangerous to question his claims. We shall see this later in our story.

Ptolemy was strongly influenced by Aristotle, although it would be somewhat unfair to accuse him of being a slavish follower. He modified Aristotle's idea that uniform circular motion was some kind of perfect state.

Ptolemy's Model: Salient Features

Ptolemy lived in Alexandria, rather later than the heyday of the Musaeum, from about 100 to 170 CE.

His was one of the first mathematical models of the universe and one of the earliest mathematical models of a natural phenomenon of any kind. To that extent it was a remarkable achievement. He based it on an earlier, but simpler model, due to Hipparchus. The model was published in a textbook, which we now know by its Arabic name, "the Almagest" [12]. This was partly because some parts survived in the original Greek, while other parts only survived in Arabic translation.

A discussion of the Almagest appears in the self-published online book by Fitzpatrick [13].

It has been superseded by better models. Nowadays, we think that that's how scientific theory advances. New observations and measurements come along which no longer fit the old paradigm, so a new paradigm is required.

That concept came along much later than Ptolemy's model. Even on its own terms, it only approximately matched the then known data. Nowadays we have statistical tools to handle approximation, but that was not true until well after the time of Newton, let alone the time of Ptolemy.

Without tools to help you choose which data to accept, and which to reject, and without much prior experience to go on, Ptolemy was on his own and had to do the best he could. Many scholars have tried to take him down by claiming that he made "convenient" choices of data. Owen Gingerich [14], in an essay entitled "Was Ptolemy a fraud?", argues that that's being a little harsh and judging him by modern standards.

The model he developed had the Earth "not moving", with everything else moving around it. Ancient Greek concepts of space and of motion and stationariness were, of course, pre-Newtonian and very different from ours.

As we now know, the problem the planetary astronomers had to solve was how to model and predict the motion of planets, including the one upon which we live, as they follow elliptical paths. Not only did Ptolemy not know that, but the data available to him were not accurate enough to distinguish elliptical paths from plausible alternatives.

The path of a superior planet (Mars, Jupiter or Saturn) in Ptolemy's model is shown in Fig. 1.1. The Earth is at a fixed point. The sold circle is

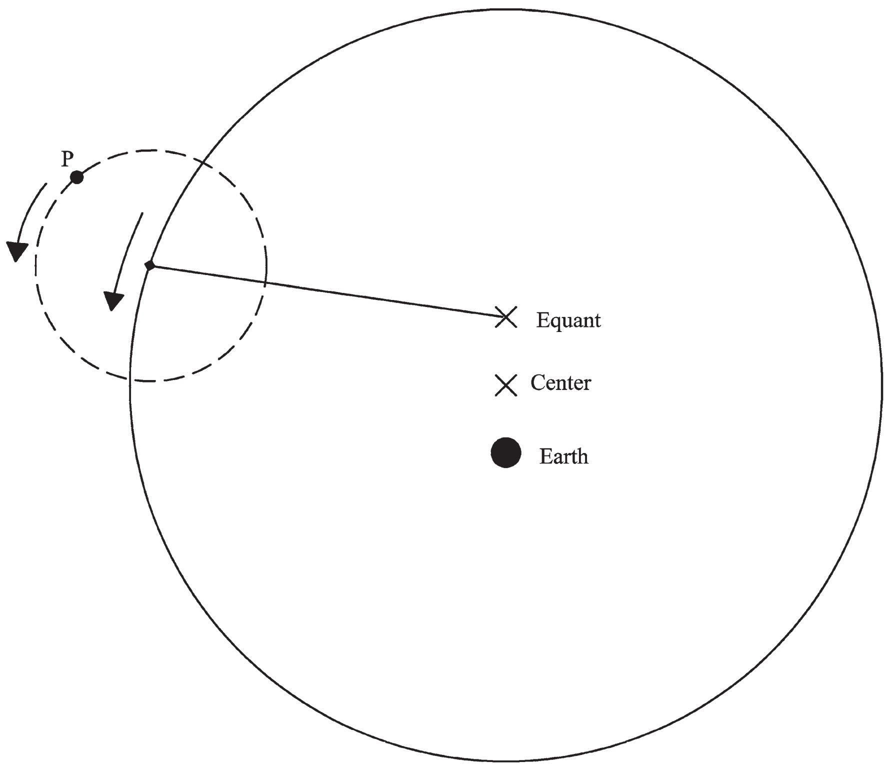

Fig. 1.1 Ptolemy's model of the path of a superior planet, that is, Mars, Jupiter or Saturn. The Earth is at a fixed point. The sold circle is called the "deferent" of the planet. Note that its centre is not at the Earth. Another circle, the "epicycle", shown dashed, rotates around the centre at a variable rate such that an observer at the Equant would see a uniform rate of rotation. The planet itself rotates around the epicycle. The plane of this arrangement is that of the Ecliptic, the apparent orbit of the Sun. (Image: Author)

called the "deferent" of the planet. Note that its centre is not at the Earth. Another circle, the "epicycle", shown dashed, rotates around the centre at a variable rate such that an observer at the Equant would see a uniform rate of rotation. The planet itself rotates around the epicycle.

The time to go around the deferent is what we would now call the orbital period of the superior planet. The time to go around the epicycle is roughly an Earth year.

The plane of this arrangement is that of the Ecliptic, the apparent orbit of the Sun.

The sun in Ptolemy's model also rotates about the Earth in a similar manner, except that the Equant is at the centre of the deferent, and there is no epicycle. The epicycle is not needed because the Sun does not undergo retrograde motion when observed from the Earth. This model is credited to Hipparchus of Nicaea (c. 190–120 BCE) although this author's writings do not survive. Hipparchus is also credited with being the "father" of trigonometry and with discovering the precession of the equinoxes by comparing his data against that from Babylon [15].

Through modern eyes, we might say that the deferent represents the planet's orbit and the epicycle represents the effect of the orbit of the Earth. Of course, we cannot expect Ptolemy to have thought like that.

A quantitative assessment of the calculating capability of this model will be made a little later in our story.

The Copernican Revolution: The Pun That Keeps oving

No doubt this pun has elevated Copernicus' role beyond the admittedly very high level it deserves (Fig. 1.2). It is a myth that he put the Sun at the centre of his system. He almost, but not quite, did this. But he did have the Earth moving around the Sun.

In fact, he was a dedicated Aristotelian, who was offended by the way that Ptolemy's system did not employ uniform motion about the centers of circles. He embarked upon his study with a view to rectifying this situation. In the process, he noticed that some of his epicycles would cancel out if he had the Earth go round the Sun, rather than vice versa.

His model for a heliocentric planet looks like Fig. 1.3.

Part of the conceptual leap that Copernicus had to make was to consider whether the Earth is a planet. He certainly discussed this question [16]. He also worried about the implications of his theory for gravity. If the Earth was but a planet, could it be the sole source of gravity? Did the Sun also exert gravitational pull? What about other bodies? [16]

This model actually made less accurate predictions of planetary motions than Ptolemy's model.

Before Galileo and Newton, particularly Newton, the concepts of moving and stationary were not what they are now. Nowadays, we believe that the universe has no centre and that a body, if it obeys classical mechanics to a good approximation, experiences force but is not affected by translational movement. Rotation is another matter: the General Theory of Relativity is required to explain this. Such explanation is outside the scope of this book. Any interested reader is referred to the section on Mach's principle in

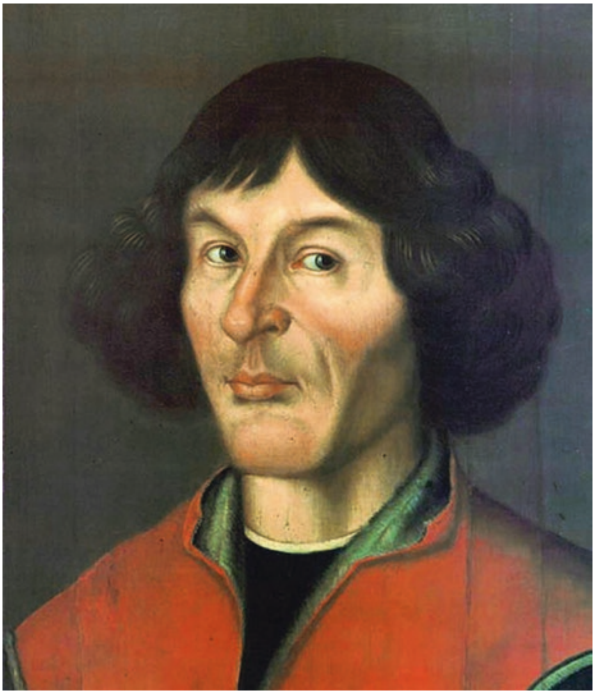

Fig. 1.2 Nicolaus Copernicus portrait from Town Hall in Thorn/Toruń – 1580. (Image courtesy of http://en.wikipedia.org/wiki/Copernicus#mediaviewer/File:Nikolaus_Kopernikus.jpg). Public domain

Misner, Thorne and Wheeler [17], but is warned that General Relativity is not for the faint-hearted. To understand what little I myself know, I had to sign up for a distance-learning undergraduate course, over 40 years after I graduated, having totally failed to teach myself the subject.

The next significant character in this story, Tycho Brahe, got rather hung up on this question, as we will see.

Tycho Brahe, the Greatest Pre-telescope Observer

Tycho (Fig. 1.4), who lived from 1546 to 1601 [18], did not quite live to see the telescope applied to astronomy. In this, he was a little unlucky: he died of an illness so sudden that rumours abounded that he was poisoned. These were eventually laid to rest in 2010 after his body was exhumed and tested [19]. It seems as though he died of an advanced bladder infection. This post-mortem did indicate that the best known legend about Tycho, that he had a

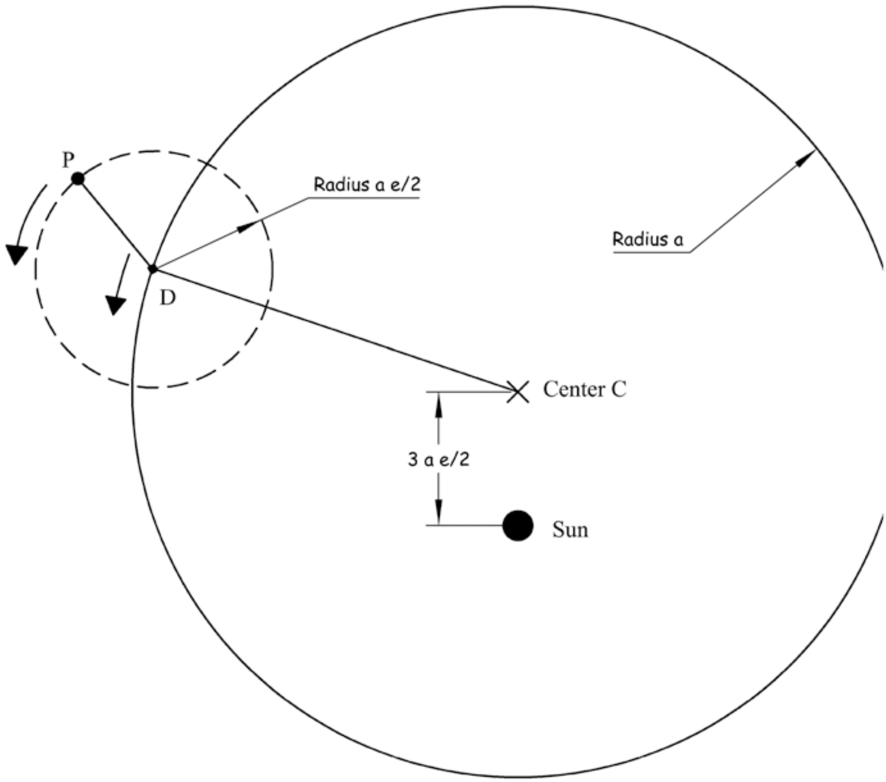

Fig. 1.3 A heliocentric orbit according to Copernicus. The lines CD and DP rotate uniformly about C and D, respectively, such that DP rotates twice as fast as CD. The perihelion is at the top and the aphelion is at the bottom. (Image: Author)

metal prosthetic nose because part of his was cut off in a duel, was true. The post-mortem showed traces of brass around the nose wound.

This was simply another example of what an extreme character Tycho was. Everything about him was outsized. He was a high-ranking, very wealthy Danish aristocrat. Confusingly, the land where he was born and mostly lived was in what is now south-western Sweden, but back then it was Danish. Unusually for an aristocrat, he married for love and produced thirteen children. He was a prodigiously determined observational astronomer. He had his own magnificent observatory on an Island, together with a paper mill and printing press. Over the years, he employed perhaps sixty people and drove them hard [20]. He even had his own court jester [21]. He was a great correspondent and author [22]. This

Fig. 1.4 Tycho Brahe. (Image courtesy of http://cache.eb.com/eb/image?id=83677&rendTypeId=4). Public domain

enabled him to disseminate his results and, in effect, to shout louder than any of his contemporaries. He was certainly competitive and keen to establish his own legend. But the thing is this: he had the talent to back this up. His measurements were an order of magnitude better than anything that went before.

Well educated in the Universities of Germany and Scandinavia [21], he discovered early in his adulthood that he could measure astronomical phenomena better than the ancient Greeks could. He would not have known about the astronomers of Islam. They were not widely known in Europe during the Renaissance [23].

Tycho and Mathematics

Geometry was an essential tool to convert Tycho's measurements into celestial positions. Living in the computer age as I do, I have to admit that I never gave this a second thought until I realized that he worked before logarithms were invented. Multiplication and division were therefore much more severe and labour-intensive problems than they were once logarithms became available.

The key art was spherical trigonometry, which is a shorthand name for the analysis of triangles on the surfaces of spheres. Of course, the sphere the early scientists thought of was a faraway sphere on whose inside surface were the "fixed" stars.

The parameters governing such a triangle are built up in Figs. 1.5, 1.6, 1.7, and 1.8.

Thus, our triangle on the surface of a sphere can be uniquely defined entirely in terms of angles, provided that its sides all lie on great circles.

Now let me define some planar triangles to help me analyse the spherical triangle (Fig. 1.9). I do this by drawing lines tangential to the two great circles that meet at A.

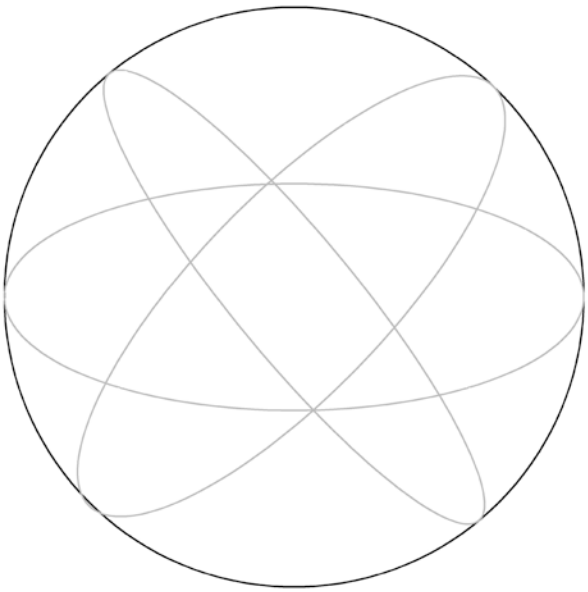

Fig. 1.5 Imagine a sphere onto whose surface three great circles are drawn. A great circle is a circle with the same radius as the sphere, whose centre is the centre of the sphere. (Image: Author)

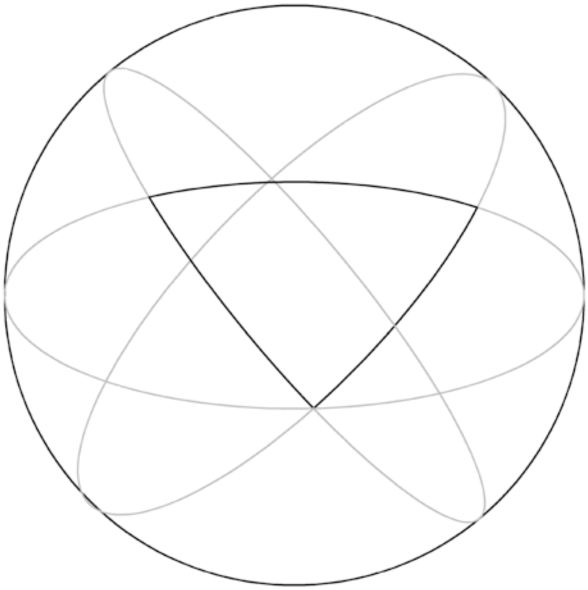

Fig. 1.6 These three great circles form the sides of a triangle on the sphere. They actually host another triangle on the other side of the sphere, but we will not consider that for now. (Image: Author)

Then by the cosine rule, these two equations are true.

$$\begin{aligned} DE^2 &= AD^2 + AE^2 - 2(AD)(AE)\cos A; \\ DE^2 &= OD^2 + OE^2 - 2(OD)(OE)\cos a. \end{aligned} \tag{1.2}$$

Because we chose tangents, the angles $\angle OAD$ and $\angle OAE$ are right angles. Thus,

$$\begin{aligned} OD^2 &= OA^2 + AD^2; \\ OE^2 &= OA^2 + AE^2. \end{aligned} \tag{1.3}$$

Substitute the equations of Eq. (1.3) into Eq. (1.2):

$$\begin{aligned} DE^2 &= AD^2 + AE^2 - 2(AD)(AE)\cos A; \\ DE^2 &= OA^2 + AD^2 + OA^2 + AE^2 - 2(OD)(OE)\cos a. \end{aligned} \tag{1.4}$$

Now subtract the upper from the lower of Eq. (1.4).

$$0 = 2(OA^2) + 2(AD)(AE)\cos A - 2(OD)(OE)\cos a. \tag{1.5}$$

Rearranging Eq. (1.5) gives

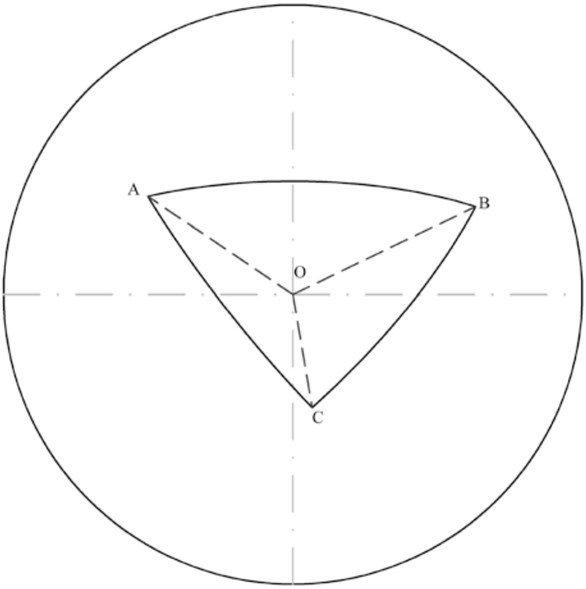

Fig. 1.7 Let this triangle have vertices A, B and C, all in upper case. Let the centre of the sphere be at O. (Image: Author)

$$\begin{aligned} \cos a &= \frac{\mathrm{OA}}{\mathrm{OE}}\frac{\mathrm{OA}}{\mathrm{OD}} + \frac{\mathrm{AE}}{\mathrm{OE}}\frac{\mathrm{AD}}{\mathrm{OD}}\cos A \\ \text{so } \cos a &= \cos b \cos c + \sin b \sin c \cos A \end{aligned} \tag{1.6}$$

using the definitions of sine as opposite/hypotenuse and cosine as adjacent/hypotenuse. Rearranging Eq. (1.6) gives

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}\ . \tag{1.7}$$

In the pre-logarithmic age, the very useful Eq. (1.7) would have been messy to manipulate. It involves two multiplications and a division.

It is therefore worth some effort to see if we can turn the multiplications into additions or subtractions.

We will only deal with the cases where angles are right angles or less because Tycho's measurements did not use angles >90°. By following the logic in Fig. 1.10, it can be seen that

$$\sin(g+h) = \sin g \, cos\, h \; + \; \cos g \, sin\, h \tag{1.8}$$

and

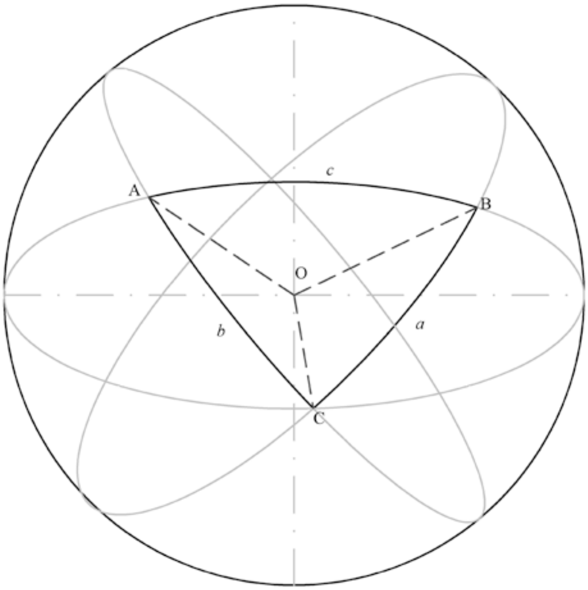

Fig. 1.8 Let the side opposite A has length *a*, let that opposite B has length *b* and let that opposite C has length *c*. A better way to analyse the triangle is to regard *a*, *b* and *c* as angles. *a* is the angle BOC; *b* is the angle COA and *c* is the angle BOA. (Image: Author)

$$\cos(g+h) = \cos g \cos h \; - \; \sin g \sin h \tag{1.9}$$

Proofs of these formulae for all angles, whether greater than 90° or not, are given in Appendix 3.

Eq. (1.8) can be used like this:

$$\sin g \sin \; h = \frac{1}{2}\left[\cos(f-g) - \cos(f+g)\right], \tag{1.10}$$

and Eq. (1.9) can be used like this:

$$\cos g \cos \; h = \frac{1}{2}\left[\cos(f-g) + \cos(f+g)\right], \tag{1.11}$$

These relations can be used in Eq. (1.7) to eliminate the multiplications but not the division:

$$\cos A = \frac{2\cos a - \left[\cos(b-c) + \cos(b+c)\right]}{\cos(b-c) - \cos(b+c)}. \tag{1.12}$$

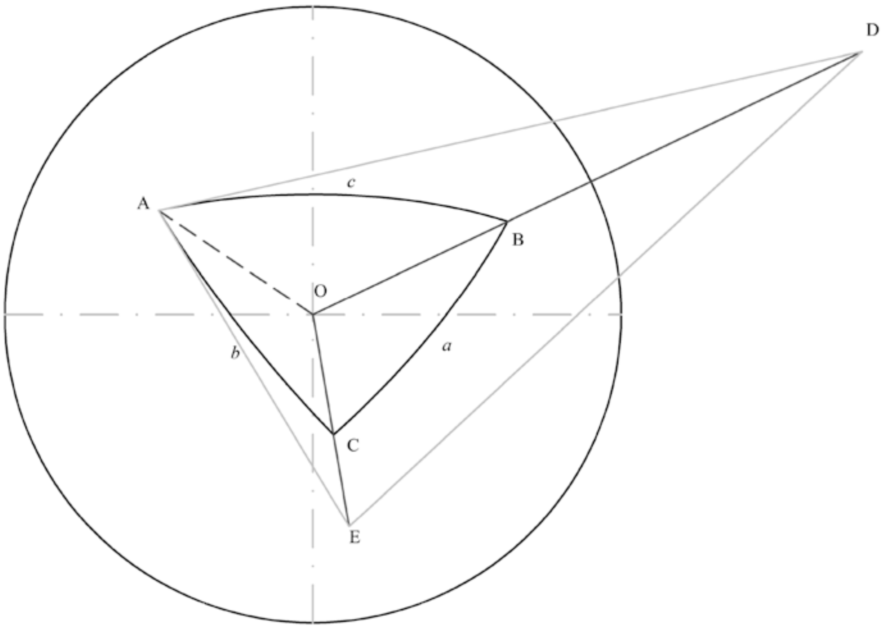

Fig. 1.9 Two lines are drawn tangent to the two great circles that cross at A. The lines OB and OC are projected until they meet these tangent lines at D and E, respectively. (Image: Author)

This elimination of multiplication was well worth the bother when you could not use logarithms. Indeed, the process was given a name: *prosthaphaeresis*. This compound word was built from the Greek words *prosthesis* and *aphaeresis* (subtract).

In those days, Eqs. (1.2)–(1.12) were not written out as formulae. They were written in words [24].

Tycho claimed priority for this technique. For this he has been in hot water with historians ever since. It's the perfect storm in a teacup. There does not appear to be enough evidence to settle the matter decisively [24], but it seems likely that Tycho was introduced to the formula by an assistant, Johannes Wittich [25]. Thoren [25] reports that Wittich brought the method to Tycho's observatory, having developed it beforehand. Tycho, who was nothing if not competitive, seems to have viewed prosthaphaeresis as something of a trade secret. He was not best pleased when he found out that, after leaving his employment, Wittich published the technique [24]. Such a driven man as Tycho venting his anger may well have been an impressive sight.

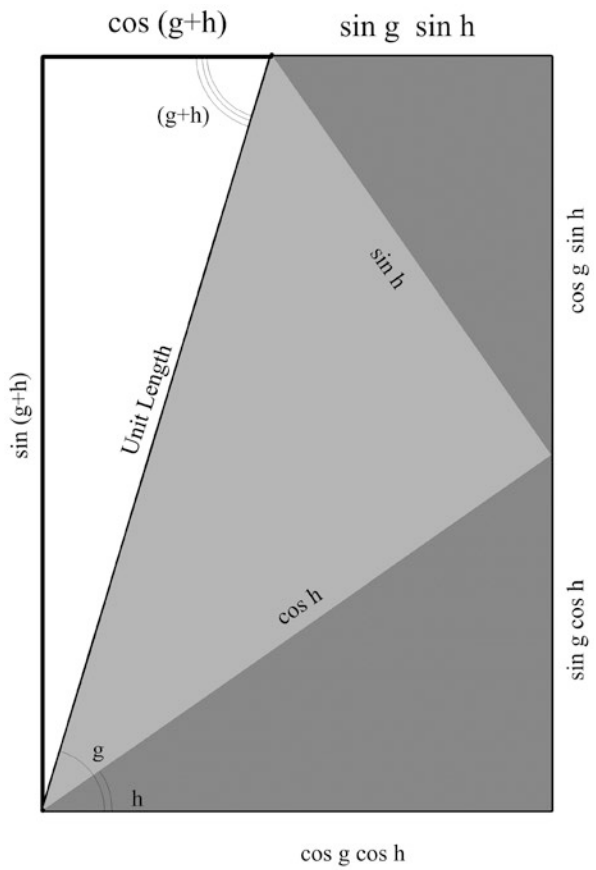

Fig. 1.10 Diagram showing how to manipulate sines and cosines of sums of angles

Tycho's Instruments

Tycho was no lone genius. In fact, you could even make a case for saying that his operation at his observatory in Uraniborg, Hveen, the Danish island now part of Sweden and known as Ven, was the first example of "big science". By the standards of today's big science, where the biggest projects employ thousands, Tycho's operation was small, yet his grant from the King of Denmark was 1% of the total royal expenditure. That was not to be sneezed at.

His first big challenge was to observe a supernova in 1572. He was careful to measure the position of the supernova relative to those of the other stars in Cassiopeia, within which constellation it had appeared. He looked very hard for evidence of parallax. His method was to compare what happened when the supernova first appeared in November and Cassiopeia was very high in the sky with what happened later when the circumpolar Cassiopeia was low in the sky. Cassiopeia is circumpolar for latitudes above 34°N: his latitude was around 56°N. He was looking for parallaxes of a few degrees for a nearby object. He found no parallax, and even checked that over its 18-month apparition, it kept pace with the precession of the equinoxes. He was in fact unable to detect whether it did or not, the effect in question being about 20 arcseconds. He reckoned that the supernova was more distant than the planets and was probably on the sphere on which people then believed the stars to exist [26]. The appearance of this very bright star cast doubt on the Aristotelian idea that the heavens were unchanging. That, however, was not what Tycho wrote about. He wrote about the astrological implications of the new star [26].

His instrument was a large wooden sextant on a stand, basically a huge protractor. When Cassiopeia was almost overhead, he was unable to make measurements. A later version of Tycho's instrument is shown in Fig. 1.11.

His next famous observation was of the great comet of 1577. This was by no means the only comet that he observed over the years. He again took an interest in parallax measurement and demonstrated that the comet was not in the Earth's atmosphere at all, but was beyond the Moon. If the supernova made him doubt the Greek belief in heavenly spheres to carry the planets, the comet blew his faith in this idea out of the water. He observed that the angular velocity of the comet decreased and deduced from this that it was travelling *through* these spheres. This made him doubt their very existence. This in turn drove him, over a period of years, to grope towards a cosmological model of his own [28, 29]. He had, however, another fixation to deal with. He could never shake off his belief that the Earth could not be moving. So he worked his way towards a system, without epicycles, in which the Earth was fixed, the Moon and Sun go around the Earth and the planets go around the Sun. The resultant system is shown in Fig. 1.12. It was published in 1588 [30].

For our story, the significance is that Fig. 1.12 appears to be the first diagram showing the Solar System without planets travelling in epicycles. The system became less elegant the more quantitatively it was analysed [28], and never caught on, despite the best efforts of Tycho, and later his heirs, to foist it on the world.

Over thirty years, he built better and better instruments as he sought the elusive prize of accuracy. He began with conventional enough instruments, but developed them. Over time they got bigger because it was then possible to make the scales finer. He had to contend with typical problems as you

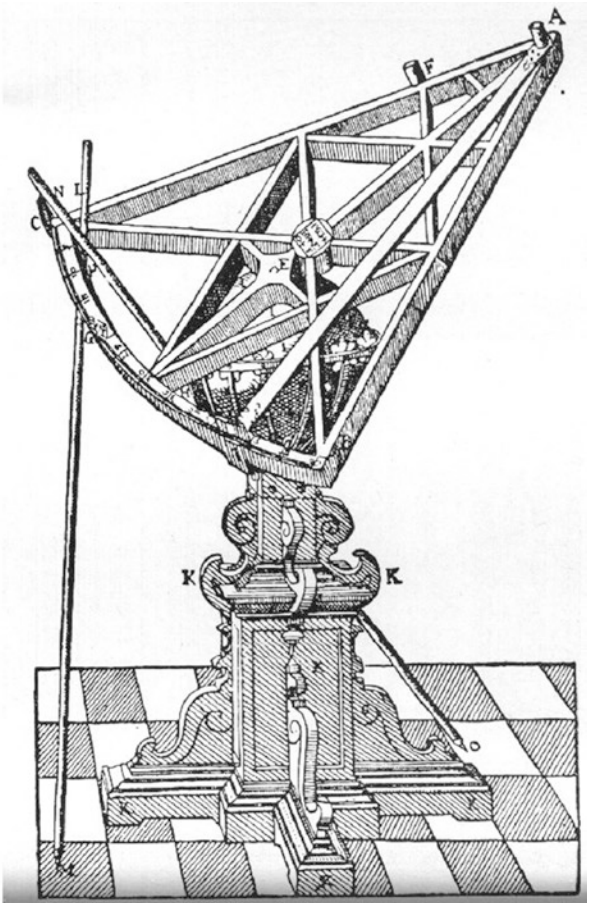

Fig. 1.11 A sextant built by Tycho in 1582, ten years after the supernova. He says in the text of his book on instruments that he built three of these instruments over the years. (Source: *Astronomiæ Instauratæ Mecanica* by Tcho Brahe, 1598 [27]). Public domain

make things bigger and bigger. Maintaining the rigidity of the equipment was a challenge. So was operating them in the wind. In the end, he built a separate observatory next door, in which the instruments were basically below ground.

Another of Tycho's instruments was the great equatorial armillary, a device capable of measuring both right ascension and declination. Despite owning the best clocks then available, Tycho was not able to measure time reliably. But he could use the equatorial armillary to get the declination of a star and then compare its right ascension with that of a known and trusted

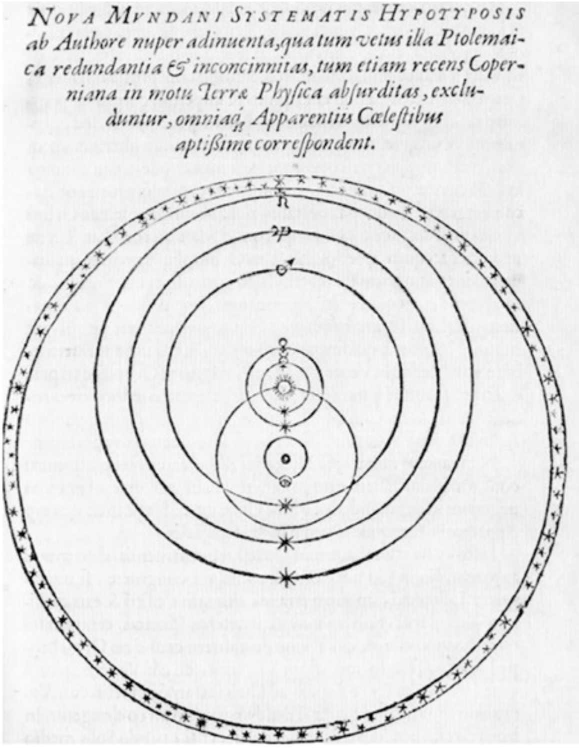

Fig. 1.12 The Tycho model of the world. The Earth is fixed, the Moon and Sun go around the Earth and the planets go around the Sun. My translation of the Latin inscription reads "New Hypotheses of the system of the World proposed recently by the author, by which the redundancy & inelegance of the Ptolemaic, and the physical absurdity of the Copernican, are both excluded, in very apt agreement with the appearance of the heavens". (Source: Brahe's *De Mundi Aetherei Recentioribus Phaenomenis*, 1588). Public domain

reference star. Armillaries were not new, but they were either too small to be accurate or very big and unwieldy: in particular they would flex. The word "armillary" is derived from the Latin word "armilla" meaning "bracelet". Tycho reduced this device to the bare minimum. Originally armillaries were much more complex than Tycho's eventual design. The one in Fig. 1.13 shows that they would typically have great circle rings for each of the equator, both tropics, the Arctic and Antarctic circles and the ecliptic.

The principle of Tycho's armillary is illustrated in Fig. 1.14.

His actual Great Equatorial Armillary is illustrated in Fig. 1.15. It was graduated to ¼-arcminute intervals [31].

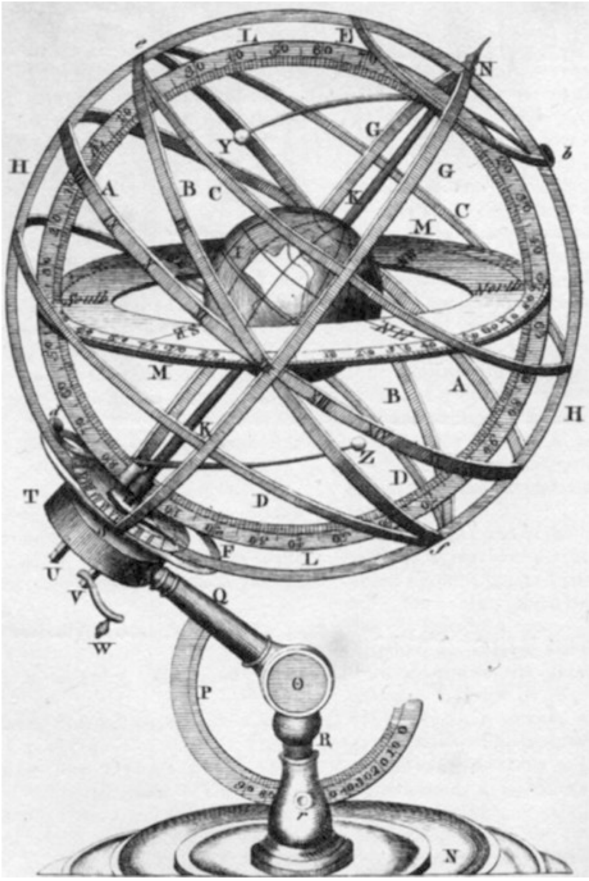

Fig. 1.13 An armillary from the 1771 edition of the Encyclopedia Britannica. Public domain

One of the uses to which he put this armillary was to measure the phenomenon of refraction by the atmosphere. He and his team would simultaneously use this instrument and a quadrant to measure the right ascension and declination of a star and measure its altitude, thereby mapping the phenomenon of atmospheric refraction as a star travelled across the sky.

The other big instrument of Tycho's was the Mural Quadrant, an enormous 90-degree protractor, shown in Fig. 1.16. This instrument was graduated to $^1/_6$-arcminute intervals [31].

Tycho put immense effort into calibrating the Mural Quadrant and the Great Equatorial Armillary.

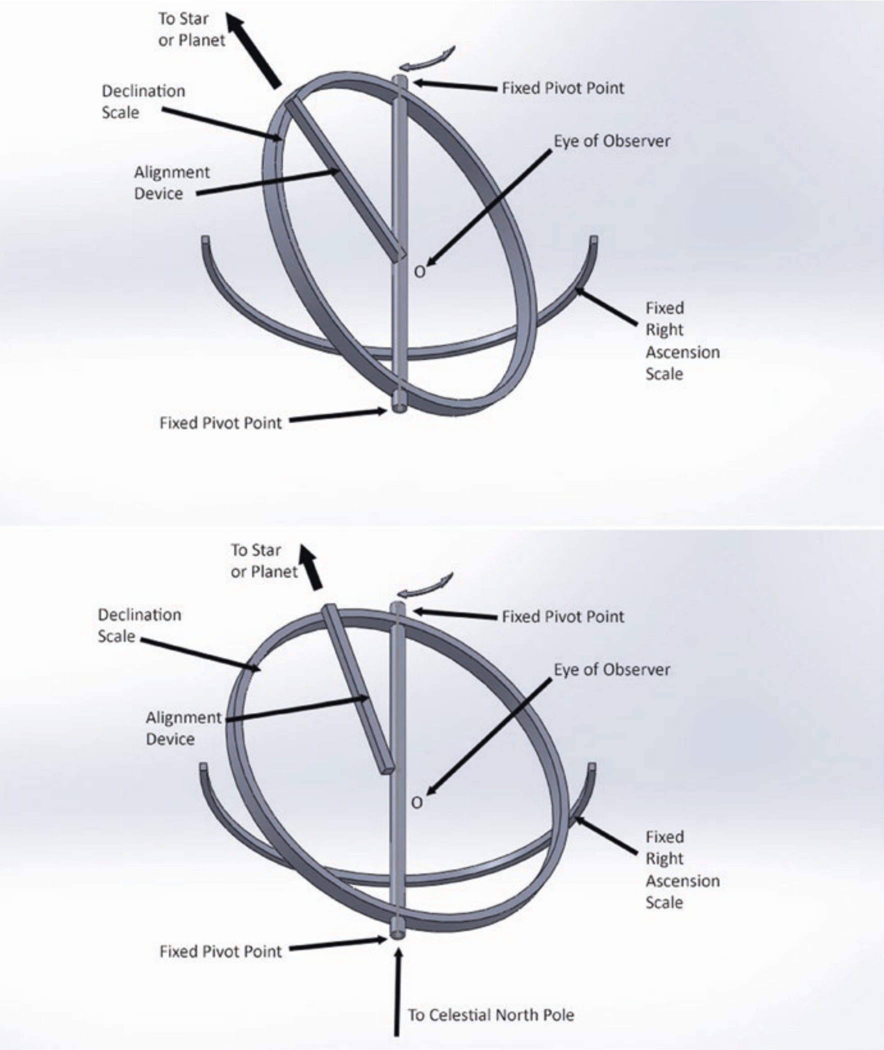

Fig. 1.14 The principle of Tycho's great Equatorial Armillary. The diameter of the ring was around nine feet [31]. The axis of the ring is allowed to rotate but fixed against translation at both ends and points towards the North Celestial Pole. Using the alignment device, the observer rotates the ring until they are looking at the star and reads off the declension on the ring. The fixed semi-circular scale enables the right ascension to be read. The observer would then quickly move to a reference star and do the same. The difference in the two right ascensions is then read directly without the need for trigonometric calculation