Practical Quantum Computing for Developers

Programming Quantum Rigs in the Cloud using Python, Quantum Assembly Language and IBM QExperience

Vladimir Silva

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About the Author

Vladimir Silva holds a Master's degree in Computer science from Middle TN State University. He worked for 5 years for IBM as a Research Engineer where he acquired extensive experience in distributed and Grid computing.

He holds numerous IT certifications, including OCP, MCSD, and MCP, and has written many technical articles for IBM developerWorks. His previous books include *Grid Computing for Developers* (Charles River Media), *Practical Eclipse Rich Client Platform* (Apress), *Pro Android Games* (Apress), and *Advanced Android 4 Games* (Apress).

An avid marathon runner, with over 16 races completed all over the state of NC (by the time of this writing), when not coding, writing or running he enjoys playing his classic guitar and pondering about awesome things like Quantum Mechanics.

About the Technical Reviewer

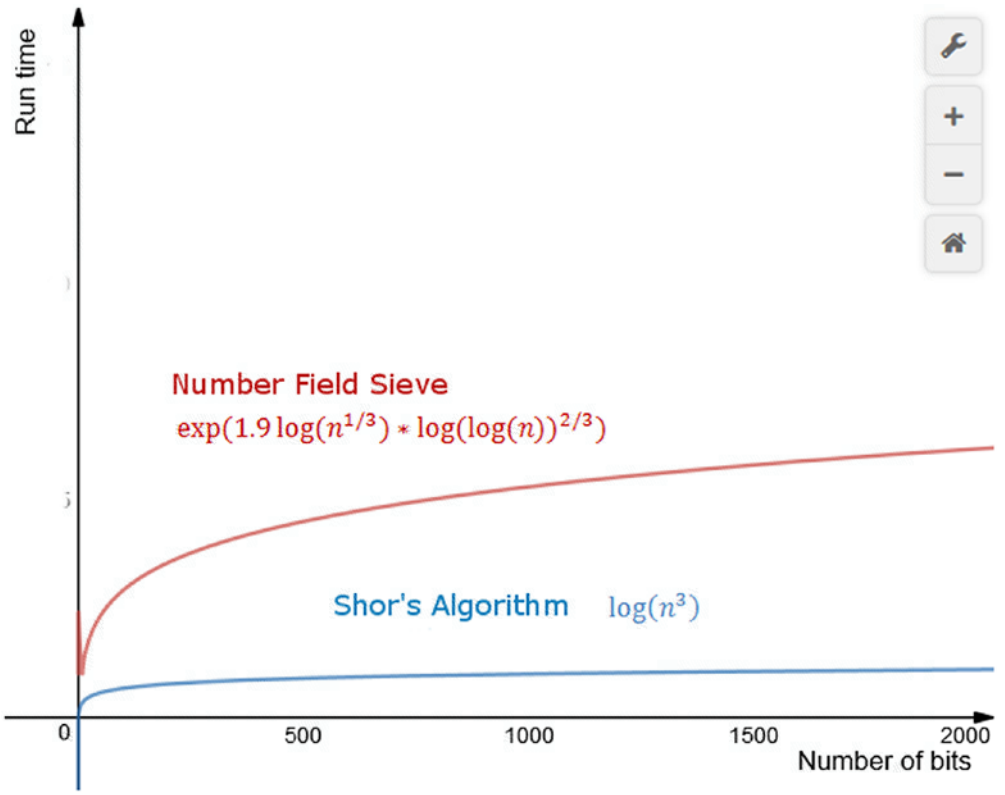
Jason Whitehorn is an experienced entrepreneur and software developer and has helped many oil and gas companies automate and enhance their oilfield solutions through field data capture, SCADA, and machine learning. Jason obtained his Bachelor of Science in Computer Science from Arkansas State University, but he traces his passion for development back many years before then, having first taught himself to program BASIC on his family's computer while still in middle school.

When he's not mentoring and helping his team at work, writing, or pursuing one of his many side projects, Jason enjoys spending time with his wife and four children and living in the Tulsa, Oklahoma region. More information about Jason can be found on his website: https://jason.whitehorn.us

Introduction

I wrote this book to be the ultimate guide for programming a quantum computer in the cloud. Thanks to the good folks at IBM Research, this is now possible. IBM has made their prototype quantum rig (known as the IBM Q Experience) available not only for research but for individuals in general interested in this field of computing.

Quantum computing is gaining traction, and now is the time to learn to program these machines. In years to come, the first commercial quantum computers should be available, and they promise significant computational speedups compared to classical computers. Consider the following graph showing the time complexities for two large integer factorization algorithms: the best classical algorithm, the Number Field Sieve, vs. the quantum factorization algorithm developed by Peter Shor.



Shor's algorithm provides a significant speedup over the Number Field Sieve for a problem, that is, the foundation of current cryptography. A practical implementation of this algorithm will render current asymmetric encryption useless!

All in all, this book is a journey of understanding. If you find the concepts explained throughout the chapters difficult to grasp, then you are not alone. The great physicist Richard Feynman once said: *If somebody tells you he understands quantum mechanics, it means he doesn't understand quantum mechanics*. Even the titans of this bizarre theory have struggled to understand what it all means.

I have tried to explore quantum computation to the best of my abilities by using real-world algorithms, circuits, code, and graphical results. Some of the algorithms included in this manuscript defy logic and seem more voodoo magic than a computational description of a physical system. This is the main reason I decided to tackle this subject. Even though I find it hard to understand the mind-bending principles of quantum mechanics, I've always been fascinated by this awesome theory. Thus when IBM came up with its one-of-a-kind quantum computing platform for the cloud, and opened it up for the rest of us, I jumped to the opportunity to learn and create this manuscript.

Ultimately, this is my take on quantum computing in the cloud, and I hope you find as much enjoyment reading it as I got writing it. My humble advice: Learn to program quantum computers; soon they will be ever present in the data center, doing everything from search and simulations to medicine and artificial intelligence. You name it. In general terms, the manuscript is divided into the following chapters:

Chapter 1: The Bizarre and Awesome World of Quantum Mechanics

It all began in the 1930s with Max Planck, the reluctant genius. He came up with a new interpretation for the energy distribution of the light spectrum. He started it all by unwillingly postulating that the energy of the photon was not described by a continuous function, as believed by classical physicists, but by tiny chucks he called *quanta*. He was about to start the greatest revolution in science in this century: *quantum mechanics*. This chapter is an appetizer to the main course and explores the clash of two titans of physics: Albert Einstein and Niels Bohr. Quantum mechanics was a revolutionary theory in the 1930s, and most of the scientific establishment was reluctant to accept it, including the colossus of the century: Albert Einstein. Fresh from winning the Nobel Prize, Einstein never accepted the probabilistic nature of quantum mechanics. This caused a rift with

its biggest champion: Niels Bohr. The two greats debated it out for decades and never resolved their differences. Ultimately, quantum mechanics has withstood 70 years of theoretical and experimental challenges, to emerge always triumphant. Read this chapter and explore the theory, experiments, and results, all under the cover of the incredible story of these two extraordinary individuals.

Chapter 2: Quantum Computing: Bending the Fabric of Reality Itself

In the 1980s, another great physicist – Richard Feynman – proposes a quantum computer, that is, a computer that can take advantage of the principles of quantum mechanics to solve problems faster. The race is on to construct such a machine. This chapter explores, in general terms, the basic architecture of a quantum computer: qubits, the basic blocks of quantum computation. They may not seem like much, but they have almost magical properties: superposition; believe it or not, a qubit can be in two states at the same time: 0 and 1. This concept is hard to grasp at the macroscale where we live. Nevertheless, at the atomic scale, all bets are off. This fact has been proven experimentally for over 70 years. Thus superposition allows a quantum computer to outmuscle a classical computer by performing large amounts of computation with relatively small numbers of qubits. Another mind bender is qubit entanglement: something that, when explored, seems more like voodoo magic than a physical principle. Entangled qubits transfer states, when observed, faster than the speed of light across time or space! Wrap your head around that. All in all, this chapter explores all the physical components of a quantum computer: quantum gates, types of qubits such as superconducting loops, ion traps, topological braids, and more. Furthermore, the current efforts of all major technology players in the subject are described, as well as other types of quantum computation such as quantum annealing.

Chapter 3: Enter the IBM Q Experience: A One-of-a-Kind Platform for Quantum Computing in the Cloud

In this chapter, you will get your feet wet with the IBM Q Experience. This is the first quantum computing platform in the cloud that provides real or simulated quantum devices for the rest of us. Traditionally, a real quantum device will be available only for

research purposes. Not anymore, thanks to the folks at IBM who have been building this stuff for decades and graciously decided to open it up for public use.

Learn how to create a quantum circuit using the visual Composer or write it down using the excellent Python SDK for the programmer within you. Then execute your circuit in the real thing, explore the results, and take the first step in your new career as a quantum programmer. IBM may have created the first quantum computing platform in the cloud, but its competitors are close behind. Expect to see new cloud platforms in the near future from other IT giants. Now is the time to learn.

Chapter 4: QISKit, Awesome SDK for Quantum Programming in Python

QISKit stands for Quantum Information Software Kit. It is a Python SDK to write quantum programs in the cloud or a local simulator. In this chapter, you will learn how to set up the Python SDK in your PC. Next, you will learn how the quantum gates are described using linear algebra to gain a deeper understanding of what goes on behind the scenes. This is the appetizer to your first quantum program, a very simple thing to familiarize yourself with the syntax of the Python SDK. Finally you will run it in a real quantum device. Of course, quantum programs can also be created visually in the Composer. Gain a deeper understanding of quantum gates, the basic building blocks of a quantum program. All this and more is covered in this chapter.

Chapter 5: Start Your Engines: From Quantum Random Numbers to Teleportation, Pit Stop at Super Dense Coding

This chapter is a journey through three remarkable information processing capabilities of quantum systems. Quantum random number generation explores the nature of quantum mechanics as a source for true randomness. You will learn how this can be achieved using very simple logic gates and the Python SDK. Next, this chapter explores two related information processing protocols: super dense coding and quantum teleportation. They have exuberant names and almost magical properties. Discover their secrets, write circuits for the Composer, execute remotely using Python, and finally interpret and verify their results.

Chapter 6: Fun with Quantum Games

In this chapter, you will learn how to implement a basic game in a quantum computer. For this purpose, we use the quintessential Quantum Battleship distributed with the QISKit Python tutorial. The first part looks at the mechanics of the game, yet we don't stop there. The second part of this chapter takes things to the next level by giving it a major face-lift. In this part, you will put Quantum Battleship in the cloud by giving it a browser-based user interface, an Apache CGI interface to consume events and dispatch them to the quantum simulator, and more. Impress your friends and family by playing Quantum Battleship with your web browsers in the cloud.

Chapter 7: Game Theory: With Quantum Mechanics, Odds Are Always in Your Favor

This is a weird one, even for quantum mechanics standards. This chapter explores two game puzzles that show the remarkable power of quantum algorithms over their classical counterparts: the counterfeit coin puzzle and the Mermin-Peres Magic Square. In the counterfeit coin puzzle, a quantum algorithm is used to reach quartic speedup over the classical solution for finding a fake coin using balance scale a limited number of times. The Mermin-Peres Magic Square is an example of quantum pseudo-telepathy or the ability of players to almost read each other's minds achieving outcomes only possible if they communicate during the game.

Chapter 8: Faster Search plus Threatening the Foundation of Asymmetric Cryptography with Grover and Shor

This chapter brings proceedings to a close with two algorithms that have generated excitement about the possibilities of practical quantum computation: Grover's search, an unstructured quantum search algorithm capable of finding inputs at an average of square root of N steps. This is much faster than the best classical solution at N/2 steps. It may not seem that much, but, when talking about very large databases, this algorithm can crush it in the data center. Expect all web searches to be performed by Grover's in the future. Shor's integer factorization: The notorious quantum factorization that experts say could bring current asymmetric cryptography to its knees. This is the best example of the power of quantum computation by providing exponential speedups over the best classical solution.

CHAPTER 1

The Bizarre and Awesome World of Quantum Mechanics

The story of quantum mechanics is a fable of wonder and bewilderment. It has elements of science, philosophy, religion, and dare I say magic. It'll turn your mind upside down, and sometimes it'll make you question the existence of an all-powerful creator out there. Even though I find its concepts difficult to grasp, I've always been fascinated by it. Some of the concepts presented in this chapter are hard to understand; however don't be troubled. Nobody has been able to fully describe what this all means, not even the titans of physics fully understand quantum mechanics. However that doesn't mean we can't be fascinated by it. The great physicist Richard Feynman once said: If somebody tells you he understands quantum mechanics, it means he doesn't understand quantum mechanics. This chapter is my take on this fascinating fable and how the struggle of two titans of science shaped its past, present, and future.

It all began in the 1930s, after Albert Einstein rose to world fame with the theory of special relativity which built upon Newtonian physics to unify the heavens and the earth. While Einstein was looking to the heavens, a new breed of scientists were looking at the very small. Spearheaded by giants of physics such as Max Planck, Ernest Rutherford, and Niels Bohr, it started a clash of titans and one of the greatest debates of physics in the twentieth century – on one side, Albert Einstein, fresh from winning the Nobel Prize for his groundbreaking discoveries on the nature of light and special relativity and, on the other side, Niels Bohr, whose contributions to the field of quantum mechanics would earn him a Nobel Prize in 1922 and the prestigious *Order of the Elephant*, a Danish distinction normally reserved for royalty. Let's take a look how the struggle between these two greats shaped the science masterpiece, that is, quantum mechanics.

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The Golden Age of Physics in the Twentieth Century

At the beginning of the twentieth century, British scientist Ernest Rutherford made a startling discovery about the nature of the atom. He postulated that atoms look like tiny solar systems, made of a tiny nucleus with positive charge and electrons negatively charged rotating like tiny planes around it. This was a remarkable insight as it was previously believed that the atom was a simple spherical blob of mass with positive and negative charges.

Bohr arrived at Rutherford's lab in Cambridge in 1920 and fell in love with Rutherford's model of the atom, but there was a problem, and a big one. If classical Newtonian physics are applied to Rutherford's model where negatively charged electrons rotate around a positively charged nucleus, the electron will eventually fall inside and crash against the nucleus creating a catastrophic paradox. Nothing should exist, as electrons will crash in a matter of seconds. Bohr saw this, and with undeterred excitement, he delayed his marriage and canceled his honeymoon in an effort to save Rutherford's model. Bohr postulates in a paper that electrons move in fixed orbits that cannot change. This goes against the basis of Newtonian physics but draws upon new ideas from the father of quantum mechanics, Max Planck.

Max Planck and the Ultraviolet Catastrophe Started It All

Planck suggested that heat and light come in units that cannot be divided, which he called "energy quanta." Planck's idea came from his efforts to solve the black-body radiation experiments where a body that completely absorbs all radiation (heat) inside it has a cavity that allows some radiation to escape (see Figure 1-1). As the heat increases inside the box, the frequency of the radiation reaches ranges visible to the human eye, glowing at different colors. It was well known by porcelain makers at the time that all bodies produce fixed colors at given temperatures (see Table 1-1).

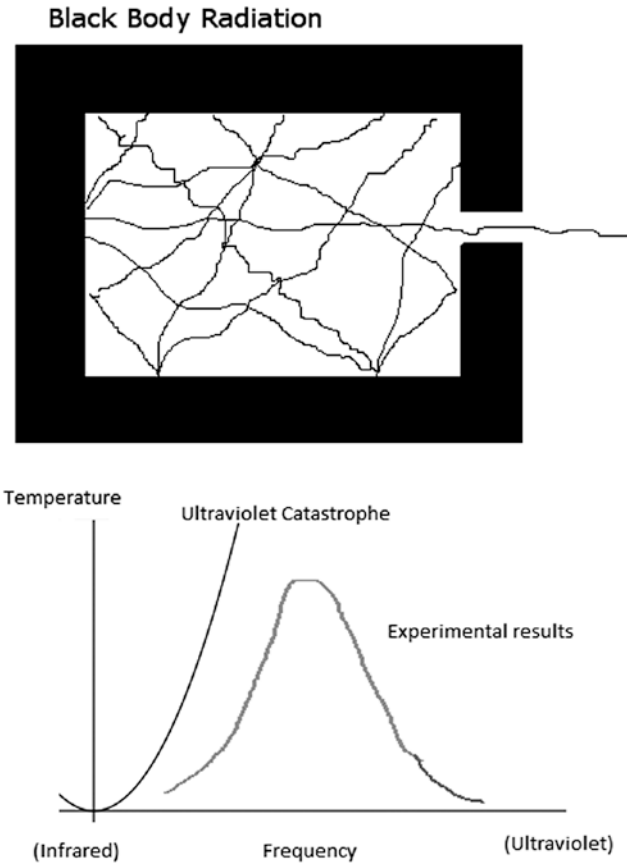

Figure 1-1. *Black-body radiation experiment results*

Table 1-1. *Light Colorization at Different Temperatures*

Temperature (°C)	Color
500	Dark red
800	Cherry red
900	Orange
1000	Yellow
1200	White

Figure 1-1 shows the black-body radiation experiment along with the results provided by the classical theory of radiation curves collected from experiments in the 1890s. Classical physicist's experiments predicted infinite intensities for the ultraviolet spectrum. This became known as the ultraviolet catastrophe and was the product of dubious theoretical arguments and experimental results. If true, this would mean, for example, that it will be dangerous to seat anywhere close to a fireplace! Planck sought to find a solution to the ultraviolet catastrophe.

Planck used the second law of thermodynamics also known as entropy to derive a formula for the experimental results derived from the black-body radiation problem.

$$S = k \log W$$

This is Boltzmann's entropy (S), where k is known as the Boltzmann's constant and W is the probability that a particular arrangement of atoms will occur for an element be that a solid, liquid, or gas.

Using Boltzmann's statistical method to calculate entropy, Planck sought a formula to match the results of the black-body experiment. By dividing the total energy (e) in chunks proportional to the frequency (f), he came up with the equation:

$$e = hf$$

where **e** is a chunk of energy, **h** is known as the Planck constant, and **f** is the frequency. Yet, he faced an obstacle; Boltzmann's statistical method demanded the chunks decrease to zero over time. This will nullify his equation and thus defeat its validity. After much struggle, Planck was forced reluctantly to postulate that the energy quantity must be finite. And here comes Planck's incredible insight; if this is correct, it meant that is not possible for an oscillator to absorb or emit energy in a continuous range. It must absorb or emit energy in small indivisible chunks of **e = hf** which he called "energy quanta," hence the term quantum mechanics.

Bohr's Quantum Jump

Bohr applied Plank's groundbreaking idea of energy quanta to the atom, the smallest unit of matter. He provided a bold description of the relationship of the atom and light where the electron which rotates around the nucleus will emit or absorb light causing a quantum jump. A quantum jump was therefore a transition between two states; however Bohr was incapable of fully describing it.

This idea was met with skepticism by other scientists who labelled his theory as nonsense, a cheap excuse for not knowing, or too bold, too fantastic to be true. The result was a rift in the physics community with one camp around Bohr believing in the quantum nature of matter and those supporting the classical view. Einstein will soon join the fight in the classical side of the struggle.

Clash of Titans: Quantum Cats and the Uncertainty Principle

By the mid-1920s the new theory about the quantum nature of matter is in shaky ground facing the real prospect of an early demise. It will take two new groundbreaking discoveries to solidify its foundation.

The first came around 1926 when German physicist Werner Heisenberg sought to legitimize Bohr's view by creating a mathematical description of the atom for what is now known as matrix mechanics. This idea was considered too complex to imagine even for the seasoned physicist. Nevertheless, Heisenberg's greatest contribution to the field is his famous uncertainty principle, which we will explore next. A second discovery came from Austrian physicist Erwin Schrödinger who came up with a new description of the atom not as a particle but as a wave. This idea built upon arguments of Louis de Broglie, a French prince who postulated that particles may exhibit wave properties and that duality may be necessary to understand the nature of light (see Figure 1-2).

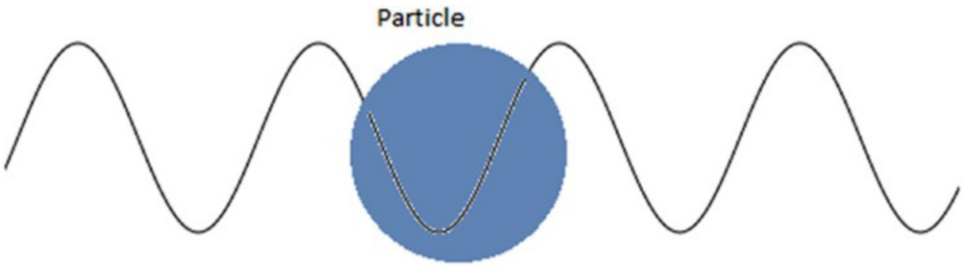

Figure 1-2. *Duality of the nature of the photon. It behaves as both a particle and wave.*

de Broglie used both Einstein's famous equation for energy $E = mc^2$ and Planck's energy quanta $e = hf$ to find a relation between the wavelength (λ) and the momentum (P) of a photon:

$$E = mc^2 = (m\ c) * c$$

Given that (mc) is the momentum (P) of the photon and c (speed) $= f$ (frequency) $* \lambda$ (wavelength), the equation becomes:

$$E = (P)\ (f\lambda)$$

But wait, Planck's relation states that energy $E = (h)(f)$; thus using basic algebra, de Broglie concluded:

$$h * f = P * (f\lambda)$$
$$h = P * \lambda$$
$$\lambda = h / P$$

de Broglie showed that the wavelength of a photon decreases as the momentum increases (see Figure 1-3). By analogy, he proposed that this relation was true not only for photons but for all particles. Given that at the time, the momentum of the electron $P = (mass)*(velocity)$ could be easily determined via experiment; this meant that the wavelength could be calculated from de Broglie's equation! The idea seemed preposterous at the time, as classical physicists knew that the electron was a particle, a discovery made long ago by J. J. Thomson in 1897.

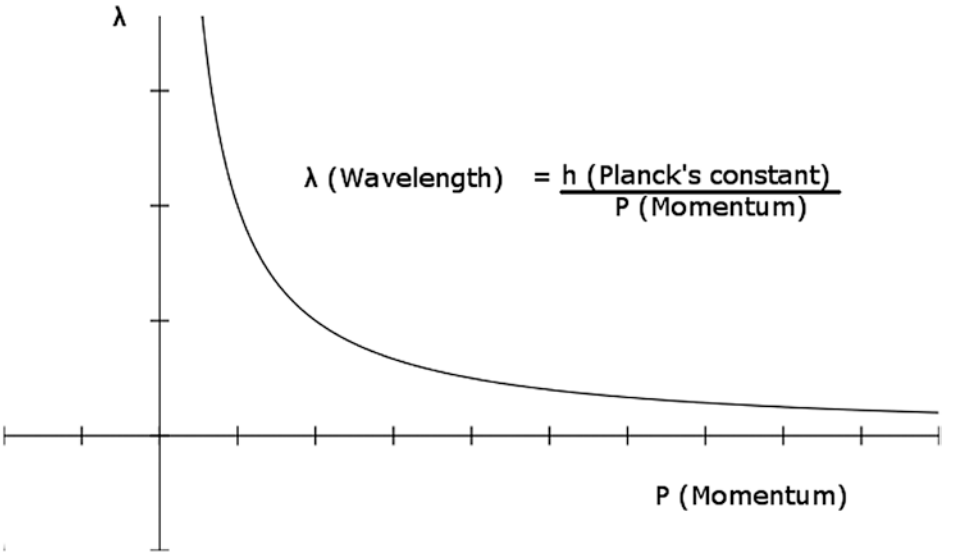

Figure 1-3. *de Broglie relation between the wavelength and the momentum of a photon*

Schrödinger used de Broglie's ideas to find an approach that was more acceptable to the status quo, marking a return to the continuous, visualizable world of classical physics. He was right about his wave function but dead wrong about appeasing the status quo.

Enter the Almighty Wave Function

Schrödinger sought to find a function that could be applied to any physical system for which a mathematical form of energy is known, thus creating his notorious wave function denoted by the Greek symbol ψ (pronounced Psi - see Figure 1-4). The wave function uses Fourier's method of solving equations by expressing any mathematical function as the sum of an infinite series of other periodic functions. This technique is called the method of **eigenvalues** (eigen being the German word for "certain" – a term that is commonplace in quantum physics). Schrödinger wave function was immediately accepted as a mathematical tool of exceptional power for solving problems related to the atomic structure of matter and is considered to be one of the greatest achievements of the twentieth century.

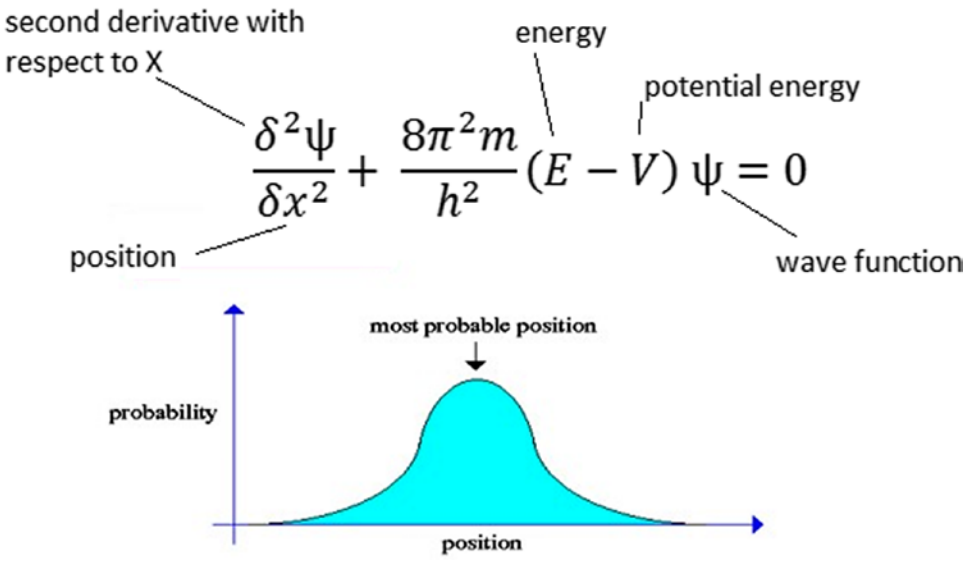

Figure 1-4. *Schrödinger famous wave function sought to describe any physical system with known energy*

Bohr and Heisenberg joined forces with Schrödinger given the incredible power of his wave function, but they needed to work out their differences first. It all took place in 1926 at a newly formed institute in Copenhagen where the three giants met to discuss.

Schrödinger rejected the Bohr/Heisenberg concept of discontinuous quantum jumps in the atom structure. He wanted to use his new discovery as a pathway back to the continuous process of physics undisturbed by sudden transitions. He was in fact proposing a classical theory of matter based entirely on waves, even to the point of doubting the existence of particles. Schrödinger proposed that particles are in fact a superposition of waves, a claim that was later proved wrong by Hendrik Lorentz who brought him to his senses, proving that you can't win them all after all. Schrödinger will later waver in his conviction on the importance of wave motion as the source of all physical reality.

Bohr, Heisenberg, and Schrödinger argued relentlessly until the point of exhaustion. Bohr demanded absolute clarity in all arguments, trying to force Schrödinger to admit that his interpretation was incomplete, Schrödinger clinging to his classical view, sometimes bemoaning his work on atomic theory and quantum jumps (something that he probably didn't mean).

Schrödinger loathed Bohr interpretation of the atomic structure. A final piece was required before these two could come to terms on a solid quantum theory.

Probabilistic Interpretation of ψ: The Wave Function Was Meant to Defeat Quantum Mechanics Not Become Its Foundation

Just like when the great rock guitarist Jimi Hendrix heard the tune *Hey Joe*, released a cover, and made it his own, thus creating arguably one of the greatest tune covers, so did the fathers of quantum mechanics. They realized the tremendous power of the wave function and made it their own. A little factoid about this story is that Schrödinger detested Planck's noncontinuous interpretation of energy and heat. He wanted to use his smooth and continuous wave function to defeat Planck's energy quanta. It is hard to believe, but in the 1930s, Planck's discovery was so revolutionary that most physicists thought he was nuts. Nevertheless, just as Hendrix did with that tune, the founders of quantum mechanics will make the wave function theirs.

A breakthrough came from German physicist Max Born, who developed the idea of the wave function as the probability of an electron for a given state to scatter in some direction. Born stated that the probability (P) of the existence of a state is given by the square of the normalized amplitude of the wave function, that is, $P = |\psi|^2$. This was groundbreaking at the time as Born claimed no more exact answers; all we get in atomic theory are probabilities. This brand new idea took Bohr interpretation of the atom in an entirely new direction (see Figure 1-5).

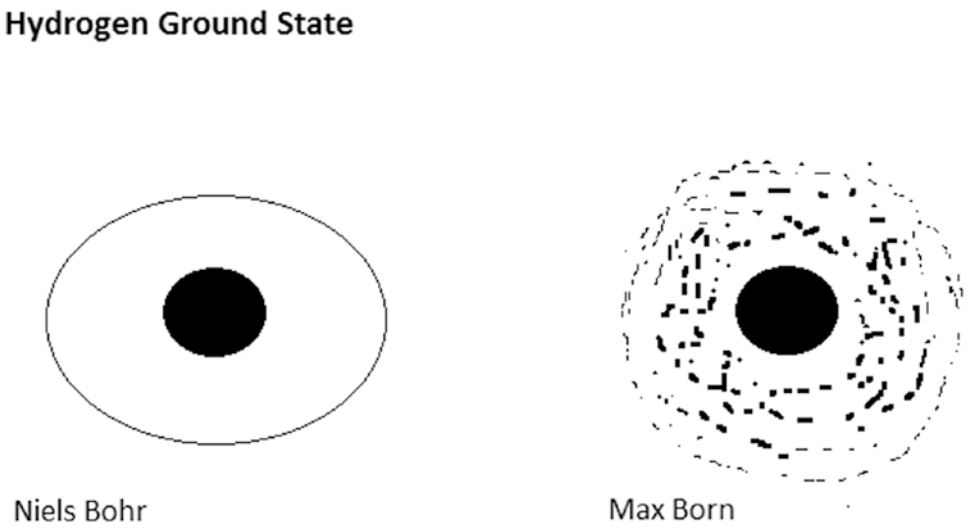

Figure 1-5. *Bohr vs. Max Born probabilistic view of the wave function*

The Quantum Cat Attempts to Crash Born's Probabilistic Party

As Born's idea about the probabilistic nature of ψ gained traction, Schrödinger through his wave function was being misused, and that originated the famous thought experiment that will be later known as the quantum cat, a story that you probably heard of. In the experiment, Schrödinger sought to rebuff Born's probabilistic interpretation of ψ. It goes like this: a live cat is placed in a box with a radioactive source that triggers the release of a hammer that breaks a flask with poison that will kill the cat. Assuming a 50% probability of radioactive decay per hour, after one hour the mechanism will be triggered, thus killing the cat. Schrödinger claimed that according to Born's interpretation, quantum theory will predict that after one hour, the box would contain

a cat that is neither dead nor alive but a mixture of both states, a superposition of both wave functions. Schrödinger thought this was ridiculous and would create a paradox. Yet today, this so-called paradox is used to teach about quantum probabilities and superposition of states.

This is the genius of superposition; as soon as the box is opened, the superimposed wave functions collapse into a single one making the cat dead or alive – thus the act of observation resolves the impasse. Yet another incredible insight will come from Heisenberg pondering about a certain amount of uncertainty about the position of a particle in the atomic structure championed by Bohr.

Uncertainty Principle

Heisenberg pondered about how the position of a particle cannot be known in Bohr's atom. After much reflection, in a moment of clarity, he realized that to know where a particle is, you have to look at it, and to look at it, you have to shine a photon of light on it. However, when you do this, it disturbs the particle position; thus the act of observing a particle changes its location. Heisenberg called this idea the uncertainty principle.

To study the problem, Heisenberg devised a hypothetical experiment using a microscope firing gamma rays, which carry high momentum and low frequency, toward a passing electron to be observed. With Bohr's help, the goal was to describe a quantitative relationship by estimating the imprecision on a simultaneous measurement of the position and momentum. The imprecision of the position was found to be close to the wavelength of the radiation being used, $\Delta X \sim \lambda$.

Similarly, the imprecision of the momentum of the electron is close to the momentum of the photon used to illuminate the particle, $\Delta P \sim h/\lambda$. Note that from the de Broglie equation it is known that the momentum of the photon (P) = h (Planck constant)/ λ (wavelength). Heisenberg showed that multiplying both inequalities, the product will always be greater or equal to h.

$$\Delta X * \Delta P \geq \lambda * h/\lambda$$
$$\Delta X * \Delta P \geq h$$

This is Heisenberg uncertainty principle (HUP) which formally states: "The uncertainty of a simultaneous measurement of the momentum and position is always greater than a fixed amount and close to Planck's constant h."

There is a simple experiment physicists commonly use to show the uncertainty principle in action. It's called the *single slit experiment*, and it goes as follows: A laser beam is fired through a single vertical wide slit and is reflected in a projection screen. What we see with the wide slit is exactly what we suspect, a dot projected on the screen. Now, if we make the width of the slit narrower and narrower, the sides of the dot start to get narrower too. Nevertheless, at around 1/100 of an inch, the uncertainty principle kicks in, and the direction of the beam becomes uncertain, according to Heisenberg. Thus now we observe the light to spread becoming wider and wider! Sounds crazy, how can the light become wider if we are making the slit narrower! It is extremely nonintuitive, but that's how things work.

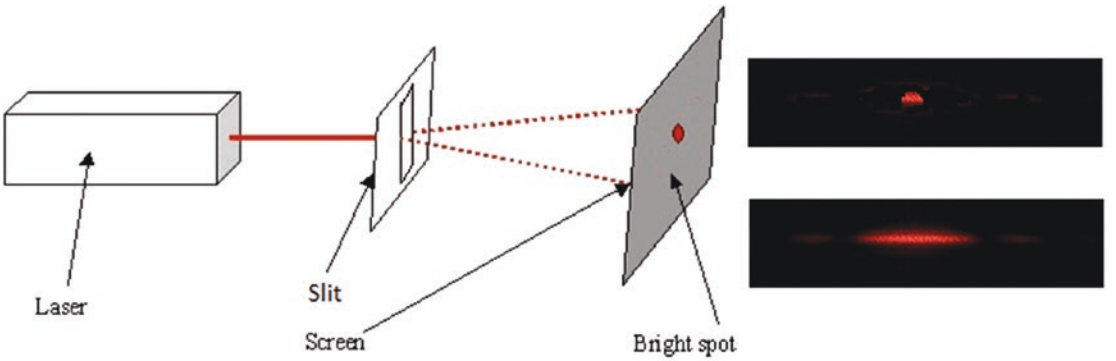

Figure 1-6. *Single slit experiment used to show the uncertainty principle in action*

The uncertainty principle is extremely important because it unifies the rift between Schrödinger and Bohr laying down the foundation of the modern quantum theory. That is, the electron is a particle, as Bohr postulated, but we don't know exactly where it is, as the uncertainty principle states (Heisenberg). Lastly, the probability of finding it is given by the wave function (Schrödinger/Born). Thus there is a duality in the nature of the electron, both as a particle and wave. With all this, a rock-solid view of quantum mechanics emerges that will later be known as the Copenhagen interpretation.

Interference and the Double Slit Experiment

Interference is another incredible property of quantum mechanics, one that makes you think what in the world is going on behind the scenes of our reality. The great physicist Richard Feynman once said about interference: The essentials of quantum mechanics could be grasped from an exploration of interference and the double slit experiment.

It is well known that at the beginning of the nineteenth century, there was a debate raging about the nature of light. Some like Newton claimed it was made of particles; others postulated that it behaved like waves. Thus in 1801, Thomas Young came up with the double slit experiment in an attempt to settle things up: In the experiment, a beam of light is aimed at a barrier with two vertical slits. After the light passes through the slits, the resulting pattern is recorded on a photographic plate. When one slit is covered, a single line of light is displayed, aligned with whichever slit is open. Common sense and intuition tells us that when both slits are open, the resulting pattern would display as two lines of light, aligned with the slits. Incredibly this is not the case. What occurs in practice is that light passing through the slits and displayed on the photographic plate is separated into multiple lines of brightness and darkness in varying degrees (see Figure 1-7).

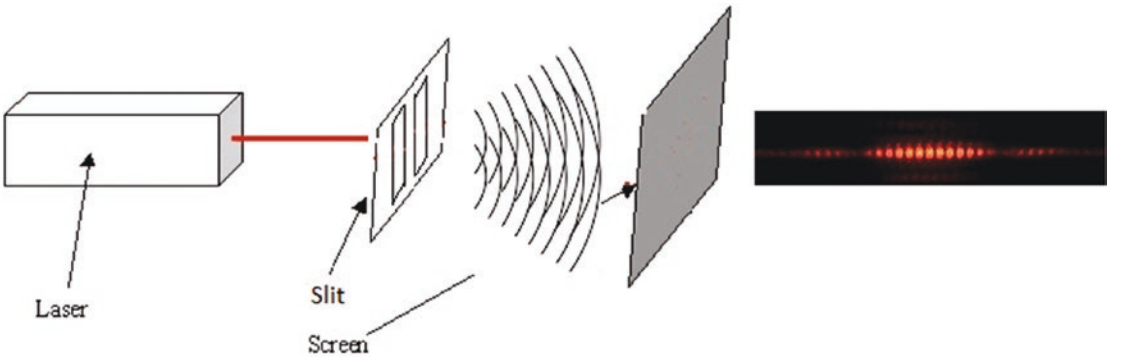

Figure 1-7. *Double slit experiment by Thomas Young*

This mind-bending result perplexed physicists who hypothesized that interference is taking place between the waves and particles going through the slits. If the beam of photons is slowed enough to ensure that individual photons are hitting the plate, one might expect there to see two lines of light (a single photon going through one slit or the other and ending up in one of two possible light lines). However that is not true. What happens is that somehow the light is doing the impossible: each photon not only goes through both slits but also simultaneously traverses every possible trajectory en route to the target (a principle called interference).

The fact that events like interference, which seem impossible, can occur at the atomic scale baffled the greatest minds at the time. Yet soon, this new theory will face its biggest challenge from the titan of physics, Albert Einstein.

Einstein to Bohr: God Does Not Throw Dice

If you are involved in science, or even if you aren't, you probably heard the famous phrase by Einstein "God does not throw dice." It was coined during a series of letters exchanged with Bohr about the nature of quantum mechanics. Bohr believed the concepts of space-time do not apply at the atomic level. Einstein, on the other hand, was a firm believer in the fabric of space-time and thought this idea could be extended to the atomic scale. This was essentially the root of the disagreement between the two.

Einstein postulated that the properties of an atomic particle could be measured without disturbing it, an idea that goes against the Bohr/Heisenberg interpretation. The two giants faced in a gathering of the greatest physicists of the time in Brussels in 1927 where Einstein sought to prove once and for all that uncertainty does not rule reality.

Einstein challenged Bohr to a series of thought experiments to disprove the uncertainty principle. In round one, Einstein devised a box that he thought will be able to register the precise moment a particle of light was emitted from a small opening in the side of the box and at the same time measure its weight (see Figure 1-7).

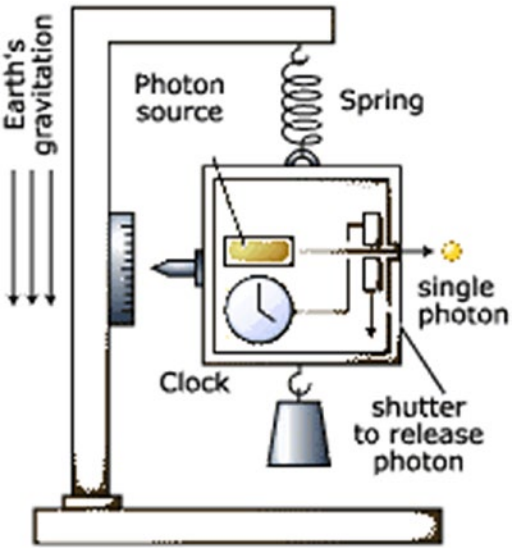

Figure 1-8. *Einstein's experimental box to disprove the uncertainty principle*

In the thought experiment in Figure 1-7, the box has a light source with a clock designed to measure the precise time a photon is emitted. At the same time, the box hangs from a spring with a weight at the bottom and corresponding measuring device. The idea was simple: weight the box before and after the photon is emitted and at the same time register the precise time using the clock. The energy levels could be easily calculated using Einstein's own equation $E = mc^2$. Things didn't look good for the uncertainty principle at that point. If the experiment was correct, the uncertainty principle will be disproven and quantum theory defeated.

Bohr got to work immediately trying to persuade Einstein that if his box works it would mean the end of physics. Bohr prevailed at the end by stating that Einstein forgot to take his own theory into account, as clocks are affected by gravity yielding uncertainty at the time of measurement. He proved the following uncertainty calculation $\Delta E\ \Delta t \geq h$ using Einstein's equation and the red shift formula. Given (Δp) uncertainty of the momentum and (Δq) uncertainty of the position:

$$\Delta p\ \Delta q \geq h \tag{1-1}$$

The uncertainty of the momentum (Δp) is given by $\Delta p \leq t\ g\ \Delta m$; then we have:

$$t\ g\ \Delta m\ \Delta q \geq h \tag{1-2}$$

From the redshift formula and principle of time dilation:

$$\Delta t = c^{-2}\ g\ t\ \Delta q \tag{1-3}$$

$$\Delta E = c^2\ \Delta m \tag{1-4}$$

Now, multiply (1-3) and (1-4) to obtain (1-5):

$$\Delta E\ \Delta t = g\ t\ \Delta m\ \Delta q \tag{1-5}$$

Finally, comparing (1-5) and (1-2), we obtain an inequality for the uncertainty principle $\Delta E\ \Delta t \geq h$. With this result, round one goes to Bohr; however this will not be the end of it. Einstein believed in a complete picture of physical reality, and the uncertainty principle stood in his way. He will come back with a bigger challenge.

Bohr to Einstein: You Should Not Tell God What to Do

God does not throw dice was Einstein unshakable principle. The firm belief that reality exists independent of one's self. When Einstein wrote to Bohr that god does not throw dice, he replied that he should not tell god what to do. This set the stage