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Stabilization for Some Fractional-Evolution Systems

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Chapter 1
Introduction

In recent years, fractional calculus has been increasingly applied in different fields of science [40, 55, 60]. Physical phenomena related to electromagnetism, propagation of energy in dissipative systems, thermal stresses, models of porous electrodes, relaxation vibrations, viscoelasticity, and thermoelasticity are successfully described by fractional differential equations [32, 39, 42, 43]. Fractional calculus allows for the investigation of the nonlocal response of mechanical systems, and this is the main advantage when compared to the classical calculus.

Fractional derivatives provide an excellent instrument for the description of memory and hereditary properties of various materials and processes. This is the main advantage of fractional derivatives in comparison with classical integer-order models, in which such effects are in fact neglected. The advantages of fractional derivatives become apparent in modeling mechanical and electrical properties of real materials, as well as in the description of rheological properties of rocks, and in many other fields.

Fractional integrals and derivatives also appear in the theory of control of dynamical systems, when the controlled system or/and the controller is described by a fractional differential equation [3, 28, 52]. Integer-order derivatives and integrals have clear physical interpretation and are used for describing different concepts in classical physics. For example, the position of a moving object can be represented as a function of time, the object velocity is then the first derivative of the function, and the acceleration is the second derivative. Fractional derivatives and integrals, being generalization of the classical derivative and integrals, are expected to have an even broader meaning. Unfortunately, there is no such result in the literature until now.

Fractional calculus includes various extensions of the usual definition of derivative from integer to real order [31], including the Riemann–Liouville derivative, the Caputo derivative, the Riesz derivative, the Weyl derivative, etc. In this book, we only consider the Caputo derivative by Michele Caputo and Mauro Fabrizio in [25] (which is most widely used [38] and has the same Laplace transform as the integer-order one, so it is widely used in control theory) that leads to an initial condition

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