RANDOM HOUSE eBOOKS

100 Essential Things You Didn't Know You Didn't Know About Sport

John D. Barrow

Contents

About the Book

What can maths tell us about sports?

100 Essential Things You Didn’t Know You Didn’t Know About Sport sheds new light on the mysteries of running, jumping, swimming and points-scoring across the whole sporting spectrum. Whether you are a competitor striving to go faster or higher, or an armchair enthusiast wanting to understand more about your favourite sport, this is a fascinating read. Find out why high-jumpers use the Fosbury Flop; how Usain Bolt could break his records without running any faster; what is the best strategy for taking football penalties; what are the effects of those banned skin-tight swimsuits; and last but not least, why the bounce of a Superball seems to defy Newton’s laws of motion.

Written for anyone interested in sport or simple mathematics, this book will enrich your understanding of sport and enliven your appreciation of maths.

About the Author

John D. Barrow is Professor of Mathematical Sciences and Director of the Millennium Mathematics Project at Cambridge University, Fellow of Clare Hall, Cambridge, a Fellow of the Royal Society, and the current Gresham Professor of Geometry at Gresham College, London. He is the bestselling author of *100 Essential Things You Didn't Know You Didn't Know*.

ALSO BY JOHN D. BARROW

Theories of Everything

The Left Hand of Creation
(with Joseph Silk)

L'Homme et le Cosmos
(with Frank J. Tipler)

The Anthropic Cosmological Principle
(with Frank J. Tipler)

The World within the World

The Artful Universe

Pi in the Sky

Perché il mondo è matematico?

Impossibility

The Origin of the Universe

Between Inner Space and Outer Space

The Universe that Discovered Itself

The Book of Nothing

The Constants of Nature: From Alpha to Omega

The Infinite Book: A Short Guide to the Boundless, Timeless and Endless

New Theories of Everything

Cosmic Imagery: Key Images in the History of Science

100 Essential Things You Didn't Know You Didn't Know

The Book of Universes

TO MAHLER

who can already run
and soon will count

100 Essential Things You Didn't Know You Didn't Know About Sport

JOHN D. BARROW

THE BODLEY HEAD
LONDON

‘Heck, gold medals, what can you do with them’
Eric Heiden

Preface

In this Olympic year I have taken the opportunity to demonstrate some of the unexpected ways in which simple mathematics and science can shed light on what is going on in a wide range of sporting activities. The following chapters will look into the science behind aspects of human movement, systems of scoring, record breaking, paralympic competition, strength events, drug testing, diving, riding, running, jumping and throwing. If you are a coach or a competitor you may get a glimpse of how a mathematical perspective can enrich your understanding of your event. If you are a spectator or commentator then I hope that you will develop a deeper understanding of what is going on in the pool, gymnasium or stadium, on the track or on the road. If you are an educator you will find examples to enliven the teaching of many aspects of science and mathematics, and to broaden the horizons of those who thought that mathematics and sport were no more than a timetable clash. And if you are a mathematician you will be pleased to discover how essential your expertise is to yet another area of human activity. The collection of examples you are about to read covers a great many sports and tries to pick topics that have not been discussed extensively before. Occasionally, there is a little bit of Olympic history for perspective, but it is balanced by chapters about several non-Olympic sports as well, and if you wish to delve deeper with your reading or push a calculation further there are notes to show you where to begin.

I would like to thank Katherine Ailes, David Alciatore, Philip Aston, Bill Atkinson, Henry Baker, Melissa Bray, James Cranch, Marianne Freiberger, Franz Fuss, John Haigh, Jörg Hensgen, Steve Hewson, Sean Lip, Justin Mullins, Kay Peddle, Stephen Ryan, Jeffrey Shallit, Owen Smith, David Spiegelhalter, Ian Stewart, Will Sulkin, Rachel Thomas, Roger Walker, Peter Weyand and Peng Zhao for the help, discussions and useful communications that helped this book come into being. A few of the topics covered here have been presented in lectures at Gresham College in London and as part of the Millennium Mathematics Project's activity for the London 2012 Olympics. I am most grateful to these audiences for their interest, questions and input. I must also thank family members, Elizabeth, David, Roger and Louise for their enthusiasm – although it turned to disbelief when they realised that this book wasn't going to help them get any Olympic tickets.

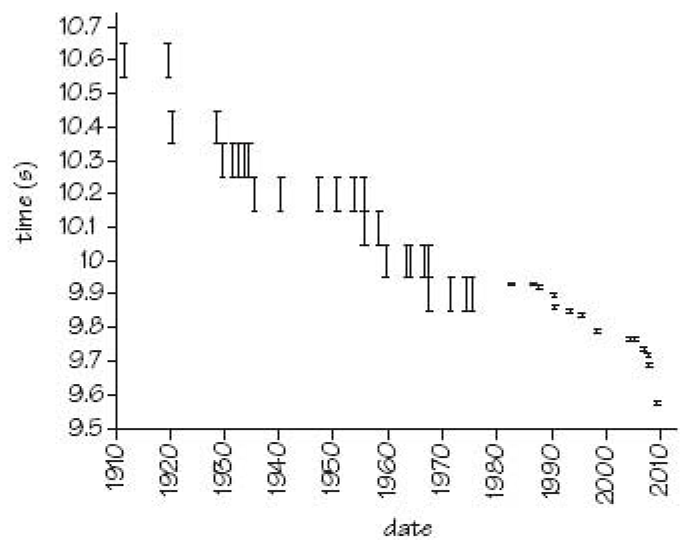
John D. Barrow, Cambridge 2012.

1

How Usain Bolt Could Break His World Record With No Extra Effort

USAIN BOLT IS THE best human sprinter there has ever been. Yet, few would have guessed that he would run so fast over 100m after he started out running 400m and 200m races when in his mid teens. His coach decided to shift him down to running 100m one season so as to improve his basic sprinting speed. No one expected him to shine there. Surely he is too big to be a 100m sprinter? How wrong they were. Instead of shaving the occasional hundredth of a second off the world record, he took big chunks out of it, first reducing Asafa Powell’s time of 9.74s down to 9.72 in New York in May 2008, and then down to 9.69 (actually 9.683) at the Beijing Olympics later that year, before dramatically reducing it again to 9.58 (actually 9.578) at the 2009 Berlin World Championships. His progression in the 200m was even more astounding: reducing Michael Johnson’s 1996 record of 19.32s to 19.30 (actually 19.296) in Beijing and then to 19.19 in Berlin. These jumps are so big that people have started to calculate what Bolt’s maximum possible speed might be. Unfortunately, all the commentators have missed the two key factors that would permit Bolt to run significantly faster without any extra effort or improvement in physical conditioning. ‘How could that be?’ I hear you ask.

The recorded time of a 100m sprinter is the sum of two parts: the reaction time to the starter’s gun and the subsequent running time over the 100m distance. An athlete is judged to have false-started if he reacts by applying foot pressure to the starting blocks within 0.10s of the start gun firing. Remarkably, Bolt has one of the longest reaction times of leading sprinters – he was the second slowest of all the finalists to react in Beijing and third slowest in Berlin when he ran 9.58. Allowing for all this, Bolt’s average running speed in Beijing was 10.50m/s and in Berlin (where he reacted faster) it was 10.60m/s. Bolt is already running faster than the ultimate maximum speed of 10.55m/s that a team of Stanford human biologists recently predicted for him.[1]



Bolt	0.146 + 9.434 = 9.58	Thompson	0.119 + 9.811 = 9.93
Gay	0.144 + 9.566 = 9.71	Chambers	0.123 + 9.877 = 10.00
Powell	0.134 + 9.706 = 9.84	Burns	0.165 + 9.835 = 10.00
Bailey	0.129 + 9.801 = 9.93	Patton	0.149 +10.191 = 10.34

In the Beijing Olympic final, where Bolt’s reaction time was 0.165s for his 9.69 run, the other seven finalists reacted in 0.133, 0.133, 0.134, 0.142, 0.145, 0.147, 0.165 and 0.169s.

From these stats it is clear what Bolt’s weakest point is: he has a very slow reaction to the gun. This is not quite the same as having a slow start. A very tall athlete, with longer limbs and larger inertia, has got more moving to do in order to rise upright from the starting blocks.[2] If Bolt could get his reaction time down to 0.13, which is very good but not exceptional, then he would reduce his 9.58 record run to 9.56. If he could get it down to an outstanding 0.12 he is looking at 9.55 and if he responded as quickly as the rules allow, with 0.1, then 9.53 is the result. And he hasn't had to run any faster!

This is the first key factor that has been missed in assessing Bolt's future potential. What are the others? Sprinters are allowed to receive the assistance of a following wind that must not exceed 2m/s in speed. Many world records have taken advantage of that and the most suspicious set of world records in sprints and jumps were those set at the Mexico Olympics in 1968 where the wind gauge often seemed to record 2m/s when a world record was broken. But this is certainly not the case in Bolt's record runs. In Berlin his 9.58s time benefited from only a modest 0.9m/s tailwind and in Beijing there was nil wind, so he has a lot more still to gain from advantageous wind conditions. Many years ago, I worked out how the best 100m times are changed by wind.[3] A 2m/s tailwind is worth about 0.11s compared to a nil-wind performance, and a 0.9m/s tailwind 0.06s, at a low-altitude site. So, with the best possible legal wind assistance and reaction time, Bolt's Berlin time is down from 9.53s to 9.47s and his Beijing time becomes 9.51s. And finally, if he were to run at a high-altitude site like Mexico City, then he could go faster still and effortlessly shave off another 0.07s.[4] So he could improve his 100m time to an amazing 9.4s without needing to run any faster.[5]

2

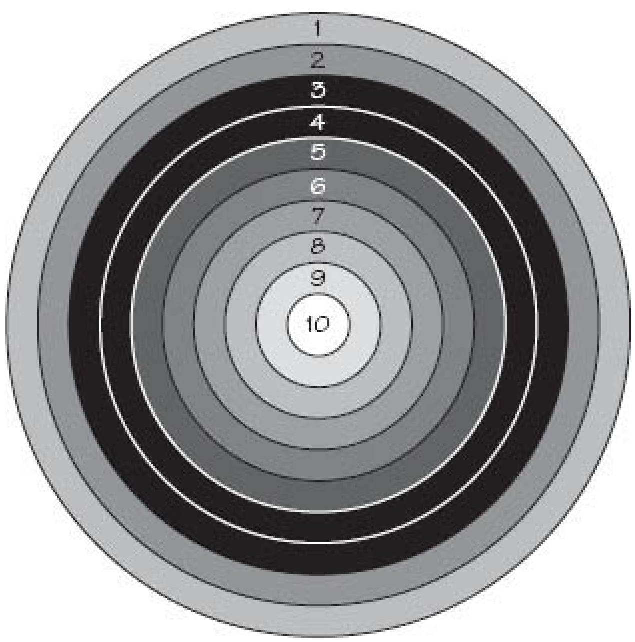
All-rounders

HUMANS ARE OFTEN compared rather unfavourably with the champions of the animal kingdom: cheetahs sprinting faster than the motorway speed limit, ants carrying many times their body weight, squirrels and monkeys performing fantastic feats of aerial gymnastics, seals that swim at superhuman speeds, and birds of prey that can pluck pigeons out of the air without the need for guns. It is easy to feel inadequate. But really we shouldn't. All these stars of the animal kingdom are really nowhere near as impressive athletes as humans. They are very good at very special things and evolution has honed their ability to dominate their competitors in a very particular niche. We are quite different. We can swim for miles, run a marathon, run 100m in less than ten seconds, turn a somersault, ride a bike or a horse, high jump over eight feet, shoot accurately with rifles and bows, throw small objects nearly a hundred metres, ride a bicycle for hundreds of kilometres, row a boat, and lift much more than our body weight over our heads. Our range of physical prowess is exceptional. It's easy to forget that no other living creature can match us for the diversity of our physical abilities. We are the greatest multi-eventers on earth.

3

The Archers

OLYMPIC ARCHERY IS a dramatic participation sport but it is not so easy to see what is happening without a good pair of binoculars or big video monitors to replay the shots. The archers shoot seventy-two arrows at a circular target 70m away. The target is 122cm in diameter and divided into ten concentric rings, each of which is 6.1cm wide.



The two inner rings are gold and arrows landing there score 10 and 9 points. Going outwards the next two are red and score 8 and 7 points; the next two are blue and score 6 and 5; the next two are black and score 4 and 3; the last two are white and score 2 and 1. If you hit the target further out than this (or miss it completely) you score zero. These coloured circles are printed on a 125cm × 125cm square of paper that is backed by a protective layer to stop the arrows from penetrating through it.

The world’s best archer is the South Korean woman Park Sung-Hyun. She scored a total of 682 points from seventy-two arrows to win individual and team gold medals at the 2004 Athens Olympics.[1] If she only scored 10s and 9s with all her arrows we can work out how she would have achieved that score. If T arrows scored 10 and the other 72–T arrows scored 9 then we know that 10T + *9(72–T)* = 682 and so T = 34 gives the number of 10s scored. The number of 9s would have been 72–34 = 38. If she only scored 10s, 9s and 8s you might like to show that she must have scored thirty-five 10s, thirty-six 9s and one 8.

The difficulty of getting a particular score with one arrow is determined by the area of the annular ring that you have to hit to obtain it. The outer radii (in centimetres) of each of the ten circular rings are 6.1, 12.2, 18.3, 24.4, 30.5, 36.6, 42.7, 48.8, 54.9 and 61. Since the area of a circle is just π (= 3.14) times the square of its radius we can work out the area of each annular ring by subtracting the area of its inner bounding circle from the area of its outer bounding circle. So, for example, the area of the ring in which arrows score 9 points is $\pi (12.2^2 - 6.1^2) = \pi \times 6.1 \times 18.3 = 350.7$. I won’t work out the areas of all the target rings but the same principle gives them very easily. Now, the likelihood of your arrow gaining a particular score is given by the fraction of the target area occupied by that part of the target. The area of the whole circular target is $\pi \times 61^2 = 11689.9$sq cm and so the probability of scoring a 9 with a randomly shot arrow that hits the target area is given by the ratio of the area in which you score 9 to the total area and this is 350.7/11689.9 = 0.03, or 3%. If I do these sums for the relative areas of all the scoring rings I get the probabilities that randomly shot arrows will hit any one of them. There is a simple pattern. The probabilities rise by 2% per ring as you move outwards through the rings. The hardest to hit is the centre ring with a 1% (i.e. 0.01) chance for a random shot; the easiest is the outer ring with a 19% (i.e. 0.19) chance of scoring 1 point.

If we add up all these average contributions we get 3.85 as the score we are likely to get from shooting a single arrow randomly at the target. If we shoot seventy-two arrows randomly then the average score we will get will be seventy-two times this, or 277, to the nearest round number. As you might expect this is far, far less than the world record score of 682. A score of 277 is what you would achieve with a purely random shooting strategy with no skill at all (except to hit some part of the target).

In calculating this we assumed that a random archer always hits the circular target. Suppose that they are not even that accurate and end up hitting anywhere at random inside the 125cm × 125cm square on which the target is printed. Its area is 15,625sq cm and you score zero if you hit this square beyond the outer circle of radius 61cm. In this case, all the overall probabilities and scores are reduced by a factor equal to the ratio of the area of the outer circle divided by the square, which is 11689.9 ÷ 15625 = 0.75. Therefore the average score obtained by shooting seventy-two arrows at random within the bounding square falls to 207.4.

If you want to test your arithmetic then you can apply exactly the same principles to calculate what score would be obtained by a random darts player. You should find that the average score is 13 points per dart, giving a score of 39 for three darts.[2]

4

The Flaw of Averages

AVERAGES ARE FUNNY things. Ask the statistician who drowned in a lake of average depth equal to 3cm. Yet, they are so familiar and seemingly so straightforward that we trust them completely. But should we? Let’s imagine two cricketers. We’ll call them, purely hypothetically, Anderson and Warne. They are playing in a crucial Test match which will decide the outcome of the series. The sponsors have put up big cash prizes for the best bowling and batting performances in the match. Anderson and Warne don’t care about batting performances – except in the sense that they want to make sure there aren’t any good ones at all on the opposing side – and are going all out to win the big bowling prize.

In the first innings Anderson gets some early wickets but is taken off after a long spell of very economical bowling and ends up with figures of 3 wickets for 17 runs, an average of 5.67. Anderson’s side then have to bat and Warne is on top form, taking a succession of wickets for final figures of 7 for 40, an average of 5.71 runs per wicket taken. Anderson therefore has the better (i.e. lower) bowling average in the first innings, 5.67 to 5.71.

In the second innings Anderson is expensive at first, but then proves to be unplayable for the lower-order batsmen, taking 7 wickets for 110 runs, an average of 15.71. Warne then bowls at Anderson’s team during the last innings of the match. He is not as successful as in the first innings but still takes 3 wickets for 48 runs, for an average of 16. So, Anderson has the better average bowling performance in the second innings as well, this time by 15.71 to 16.

Bowler	First Innings figures	First Innings average	Second Innings figures	Second Innings average	Combined figures	Overall average
Anderson	3 for 17	5.67	7 for 110	15.71	10 for 127	12.7
Warne	7 for 40	5.71	3 for 48	16	10 for 88	8.8

Who should win the bowling man-of-the-match prize for the best figures? Anderson had the better average in the first innings and the better average in the second innings. Surely, there is only one winner. But the sponsor takes a different view and looks at the overall match figures. Over the two innings Anderson took 10 wickets for 127 runs for an average of 12.7 runs per wicket. Warne, on the other hand, took 10 wickets for 88 runs and an average of 8.8. Warne clearly has the better average and wins the bowling award despite Anderson having a superior average in the first innings and in the second innings!

5

Going Round the Bend

HAVE YOU EVER wondered whether it's best to have an inside or an outside lane in track races like the 200m where you have to sprint around the bend? Athletes have strong preferences. Tall runners find it harder to negotiate the tighter curve of the inside lane than that of the gentle outer lanes. The situation is even more extreme when sprinters race indoors where the track is only 200m around, so the bends are far tighter and the lanes are reduced in width from 1.22m to 1m. This was such a severe restriction that it became common for the athlete who drew the inside lane for the final (by being the slowest qualifier on times) to scratch from the final in indoor championships because there was so little chance of winning from the inside and a considerable risk of injury. As a result, this event has largely disappeared from the indoor championship roster.

But what about the outdoor situation where the curve is not so extreme? Most athletes don't like to be right on the outside because you can't see anyone (unless they pass you) for the first half of the race and you can't run 'off' their pace. On the inside you have a metal kerb marking the inside of your lane and you tend not to get as close to it as you would to the simple white painted line that marks the inside of the other lanes. Generally, the fastest qualifiers from the previous round are placed in the centre two or three lanes – a clear signal that they might be advantageous. A runner's physique is a factor too. If you are tall and long-legged you will have a harder time in the inner lanes and may have to chop your stride or run towards the outside of your lane to run freely. Potentially even more significant is the wind. If the wind is blowing at right angles to the finishing straight, into the faces of the runners when they run around the bend, then you will want to be in the outside lane so that you will be starting some way around the bend and will not have to run directly into the wind for so long – unlike those runners on the inside.

Finally, it is easy to show that you need to work harder if you run in the inside lanes. The two bends of an athletics track are semicircular. The radius of the circle traced by the inner line of the inside lane is 36.5m and each lane is 1.22m wide. So, the radius of the circle that you run in gets larger and the extra force that you have to exert to run in a circular path gets smaller and you actually run a smaller part of a circle as well. The radius of the circle traced by lane eight is 36.5 + (7 × 1.22) = 45.04m. The force needed for a runner of mass m to run in a circular path of radius r at speed v is mv2/r, so as r gets larger,[1] and the bend is less tight, the force needed to maintain a given speed v decreases. If two identical runners, one in lane one and the other in lane eight, exert the same force over the first 100m of a 200m race, then the runner in lane one will have achieved a speed that is about 0.9 of that achieved in lane eight and the runner in lane eight will take 0.9 of the time. This is a very large factor – worth a whole second off the time for the first half of the race if you are running a 20s time for 200m. In practice there isn’t such a large systematic advantage to running in the outside lanes and the runner only has to supply a fraction of the full circular motion force to sprint around the curve.[2]

If this simple model were complete then all 200m runners would run their best times from the outside lane. In practice most records are set from lanes three and four. Even this fact is slightly biased because the fastest qualifiers for the finals of big championships will have been put in those lanes. Presumably, the psychological and tactical advantages of being able to see your opponents and judge your speed against them from an inside lane helps outweigh the mechanical advantage of running around a gentler curve.

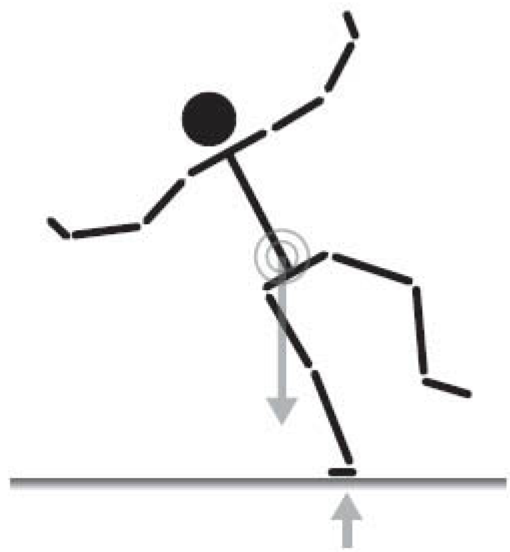
A good final comparison to make which illustrates the effect of the curve on 200m runs is to compare the world records run on a straight track with those around a curve. Straight 200m tracks are very rare now. There used to be one at the old Oxford University track at Iffley Road (where Roger Bannister ran the first sub-four-minute mile in 1954) that was still there when I began as a student in 1974 but had been removed by the time I graduated in 1977. When Tommie Smith set his world 200m record of 19.83s around a curve at altitude in the 1968 Mexico Olympics, he had already run a remarkable[3] 19.5s on a straight cinder track in San Jose in 1966. This latter record was only beaten by Tyson Gay, who ran 19.41s at the Birmingham City Games in 2010, watched by a 65-year-old Smith. Gay's fastest time around a curve is 19.58s. These time differences show the considerable slowing that is created by negotiating the curve. You might be lucky and have the wind behind you all the way in a straight 200m, but nonetheless runners find it strange to sprint such a long way without the reference points of the curve and other runners to dictate where they are and how they should apportion their effort.

6

A Question of Balance

IF THERE IS one attribute that is invaluable in just about any sport, it is balance. Whether you are a gymnast on the beam, a high-board diver, a spinning hammer thrower, a rugby forward snaking through the opposition's defence, a wrestler, a judoka trying to throw an opponent or a fencer lunging forwards, it is all about balance. Try a little experiment to see how well balanced you are and get a feeling for the muscle control behind it. Just stand completely still with one foot immediately in front of the other, so that the heel of your left foot touches the toe of your right foot. You can shift your weight so that it is mainly over the front foot or the back foot but keep your hands by your sides. You will probably find that standing completely still in a relaxed way is surprisingly difficult and your calf muscles are being tensed this way and that all the time. If you spread your arms out sideways you will find it much easier to balance. But now try leaning to one side. You won't lean very far before you lose your balance completely. Now, if you move your feet apart, in a normal standing position so they are not one behind the other in a straight line, then you will find it easier still, even with your arms by your sides – this, after all, is probably your usual stance. Lastly, go back to that difficult position with one foot directly in front of the other, but slowly crouch down low. You will find that balancing gets easier as you get nearer to the ground.

These little exercises reveal some simple principles for maintaining a good balance:



Make sure that the vertical line through your body’s centre of gravity doesn’t fall outside the base of support created by your feet. Once it does, you will fall away from equilibrium. You can experiment for yourself to see how far you can lean sideways, while keeping the body straight, before you start falling. The high-board diver will often use this instability in order to initiate his dive, leaning forward until his movement is taken over by gravity.

Broaden your base of support as much as possible. This makes it harder for your centre of gravity to fall outside your base. If you can stand on two feet, rather than on one, this will always help.

Keep your centre of gravity as low as possible. This is why you often see female gymnasts on the beam going into a low crouch position during a swing, perhaps with only one foot on the beam and one leg dangling below the beam – this lowers the centre of gravity even more. Sit astride the beam and you will see that balance is easy – your centre of gravity can’t get much lower.

Spread your weight as far from your centre as possible. This is what was happening when you spread your arms out sideways. This is changing the distribution of your mass. By moving more of it far from your centre you are increasing your inertia, or your tendency not to move. Increasing your inertia in this way won't stop you wobbling but, crucially, it will make you wobble more slowly.[1] This gives you more time to take corrective action, shift your centre of gravity sideways or downwards, as required. This is why tightrope walkers carry long poles: they are ensuring that they wobble more slowly and have more time to correct a dangerous imbalance. Without that helpful pole, the man walking between skyscrapers on a high wire would surely fall to his death once he started to wobble in the breeze.

Watch wrestling and judo, where competitors are constantly trying to make their opponents lose their balance in subtle ways, or by using their strength to force them to violate one of the principles we have highlighted.

7

Anyone for Baseball, Tennis or Cricket?

A LOT OF people spend a lot of time hitting or chasing small spherical projectiles while dressed in unusual items of clothing. Games like baseball, tennis and cricket involve someone receiving one of these projectiles at very high speed. They have a split second to respond, either by getting out of the way, or hitting the projectile back as skilfully as they can. Which of these three sports requires the quickest reactions?

In each case the ball is different in size and can be launched by the pitcher, server or bowler at different speeds. Baseball has the simplifying feature that the ball only flies through the air, whereas in cricket and tennis it will hit the ground and rebound unpredictably because of its spin. In all three cases, the ball can swerve in the air to deceive the receiver in many ways. Let's ignore these extra degrees of difficulty and just focus on how quickly the receiver has to react to the incoming ball in each of these three games.

First, take cricket: a cricket pitch is 22 yards (= 20.12m) long.[1] The fastest bowlers achieve speeds exceeding 100mph, which is about 45m/s. The bowler will generally take a lengthy approach run in order to build up speed but the ball must be released with a straight arm or a 'no-ball' will be called for 'throwing'. If the batsman stands 1m in