The Number Concept

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Preface.

In the selection of authorities which have been consulted in the preparation of this work, and to which reference is made in the following pages, great care has been taken. Original sources have been drawn upon in the majority of cases, and nearly all of these are the most recent attainable. Whenever it has not been possible to cite original and recent works, the author has quoted only such as are most standard and trustworthy. In the choice of orthography of proper names and numeral words, the forms have, in almost all cases, been written as they were found, with no attempt to reduce them to a systematic English basis. In many instances this would have been quite impossible; and, even if possible, it would have been altogether unimportant. Hence the forms, whether German, French, Italian, Spanish, or Danish in their transcription, are left unchanged. Diacritical marks are omitted, however, since the proper key could hardly be furnished in a work of this kind.

With the above exceptions, this study will, it is hoped, be found to be quite complete; and as the subject here investigated has never before been treated in any thorough and comprehensive manner, it is hoped that this book may be found helpful. The collections of numeral systems illustrating the use of the binary, the quinary, and other number systems, are, taken together, believed to be the most extensive now existing in any language. Only the cardinal numerals have been considered. The ordinals present no marked peculiarities which would, in a work of this kind, render a separate discussion necessary. Accordingly they have, though with some reluctance, been omitted entirely.

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Chapter I.

Counting.

Among the speculative questions which arise in connection with the study of arithmetic from a historical standpoint, the origin of number is one that has provoked much lively discussion, and has led to a great amount of learned research among the primitive and savage languages of the human race. A few simple considerations will, however, show that such research must necessarily leave this question entirely unsettled, and will indicate clearly that it is, from the very nature of things, a question to which no definite and final answer can be given.

Among the barbarous tribes whose languages have been studied, even in a most cursory manner, none have ever been discovered which did not show some familiarity with the number concept. The knowledge thus indicated has often proved to be most limited; not extending beyond the numbers 1 and 2, or 1, 2, and 3. Examples of this poverty of number knowledge are found among the forest tribes of Brazil, the native races of Australia and elsewhere, and they are considered in some detail in the next chapter. At first thought it seems quite inconceivable that any human being should be destitute of the power of counting beyond 2. But such is the case; and in a few instances languages have been found to be absolutely destitute of pure numeral words. The Chiquitos of Bolivia had no real numerals whatever, [1] but expressed their idea for "one" by the word *etama* , meaning alone. The Tacanas of the same country have no numerals except those borrowed from Spanish, or from Aymara or Peno, languages with which they have long been in contact. [2] A few other South American languages are almost equally destitute of numeral words. But even here, rudimentary as the number sense undoubtedly is, it is not wholly lacking; and some indirect expression, or some form of circumlocution, shows a conception of the difference between *one* and *two* , or at least, between *one* and *many* .

These facts must of necessity deter the mathematician from seeking to push his investigation too far back toward the very origin of number. Philosophers have endeavoured to establish certain propositions concerning this subject, but, as might have been expected, have failed to reach any common ground of agreement. Whewell has maintained that “such propositions as that two and three make five are necessary truths, containing in them an element of certainty beyond that which mere experience can give.” Mill, on the other hand, argues that any such statement merely expresses a truth derived from early and constant experience; and in this view he is heartily supported by Tylor. [3] But why this question should provoke controversy, it is difficult for the mathematician to understand. Either view would seem to be correct, according to the standpoint from which the question is approached. We know of no language in which the suggestion of number does not appear, and we must admit that the words which give expression to the number sense would be among the early words to be formed in any language. They express ideas which are, at first, wholly concrete, which are of the greatest possible simplicity, and which seem in many ways to be clearly understood, even by the higher orders of the brute creation. The origin of number would in itself, then, appear to lie beyond the proper limits of inquiry; and the primitive conception of number to be fundamental with human thought.

In connection with the assertion that the idea of number seems to be understood by the higher orders of animals, the following brief quotation from a paper by Sir John Lubbock may not be out of place: “Leroy … mentions a case in which a man was anxious to shoot a crow. ‘To deceive this suspicious bird, the plan was hit upon of sending two men to the watch house, one of whom passed on, while the other remained; but the crow counted and kept her distance. The next day three went, and again she perceived that only two retired. In fine, it was found necessary to send five or six men to the watch house to put her out in her calculation. The crow, thinking that this number of men had passed by, lost no time in returning.’ From this he inferred that crows could count up to four. Lichtenberg mentions a nightingale which was said to count up to three. Every day he gave it three mealworms, one at a time. When it had finished one it returned for another, but after the third it knew that the feast was over.... There is an amusing and suggestive remark in Mr. Galton's interesting *Narrative of an Explorer in Tropical South Africa* . After describing the Demara's weakness in calculations, he says: ‘Once while I watched a Demara floundering hopelessly in a calculation on one side of me, I observed, “Dinah,” my spaniel, equally embarrassed on the other; she was overlooking half a dozen of her new-born puppies, which had been removed two or three times from her, and her anxiety was excessive, as she tried to find out if they were all present, or if any were still missing. She kept puzzling and running her eyes over them backwards and forwards, but could not satisfy herself. She evidently had a vague notion of counting, but the figure was too large for her brain. Taking the two as they stood, dog and Demara, the comparison reflected no great honour on the man....’ According to my bird-nesting recollections, which I have refreshed by more recent experience, if a nest contains four eggs, one may safely be taken; but if two are removed, the bird generally deserts. Here, then, it would seem as if we had some reason for supposing that there is sufficient intelligence to distinguish three from four. An interesting consideration arises with reference to the number of the victims allotted to each cell by the solitary wasps. One species of Ammophila considers one large caterpillar of *Noctua segetum* enough; one species of Eumenes supplies its young with five victims; another 10, 15, and even up to 24. The number appears to be constant in each species. How does the insect know when her task is fulfilled? Not by the cell being filled, for if some be removed, she does not replace them. When she has brought her complement she considers her task accomplished, whether the victims are still there or not. How, then, does she know when she has made up the number 24? Perhaps it will be said that each species feels some mysterious and innate tendency to provide a certain number of victims. This would, under no circumstances, be any explanation; but it is not in accordance with the facts. In the genus Eumenes the males are much smaller than the females.... If the egg is male, she supplies five; if female, 10 victims. Does she count? Certainly this seems very like a commencement of arithmetic." [4]

Many writers do not agree with the conclusions which Lubbock reaches; maintaining that there is, in all such instances, a perception of greater or less quantity rather than any idea of number. But a careful consideration of the objections offered fails entirely to weaken the argument. Example after example of a nature similar to those just quoted might be given, indicating on the part of animals a perception of the difference between 1 and 2, or between 2 and 3 and 4; and any reasoning which tends to show that it is quantity rather than number which the animal perceives, will apply with equal force to the Demara, the Chiquito, and the Australian. Hence the actual origin of number may safely be excluded from the limits of investigation, and, for the present, be left in the field of pure speculation.

A most inviting field for research is, however, furnished by the primitive methods of counting and of giving visible expression to the idea of number. Our starting-point must, of course, be the sign language, which always precedes intelligible speech; and which is so convenient and so expressive a method of communication that the human family, even in its most highly developed branches, never wholly lays it aside. It may, indeed, be stated as a universal law, that some practical method of numeration has, in the childhood of every nation or tribe, preceded the formation of numeral words.

Practical methods of numeration are many in number and diverse in kind. But the one primitive method of counting which seems to have been almost universal throughout all time is the finger method. It is a matter of common experience and observation that every child, when he begins to count, turns instinctively to his fingers; and, with these convenient aids as counters, tallies off the little number he has in mind. This method is at once so natural and obvious that there can be no doubt that it has always been employed by savage tribes, since the first appearance of the human race in remote antiquity. All research among uncivilized peoples has tended to confirm this view, were confirmation needed of anything so patent. Occasionally some exception to this rule is found; or some variation, such as is presented by the forest tribes of Brazil, who, instead of counting on the fingers themselves, count on the joints of their fingers. [5] As the entire number system of these tribes appears to be limited to *three* , this variation is no cause for surprise.

The variety in practical methods of numeration observed among savage races, and among civilized peoples as well, is so great that any detailed account of them would be almost impossible. In one region we find sticks or splints used; in another, pebbles or shells; in another, simple scratches, or notches cut in a stick, Robinson Crusoe fashion; in another, kernels or little heaps of grain; in another, knots on a string; and so on, in diversity of method almost endless. Such are the devices which have been, and still are, to be found in the daily habit of great numbers of Indian, negro, Mongolian, and Malay tribes; while, to pass at a single step to the other extremity of intellectual development, the German student keeps his beer score by chalk marks on the table or on the wall. But back of all these devices, and forming a common origin to which all may be referred, is the universal finger method; the method with which all begin, and which all find too convenient ever to relinquish entirely, even though their civilization be of the highest type. Any such mode of counting, whether involving the use of the fingers or not, is to be regarded simply as an extraneous aid in the expression or comprehension of an idea which the mind cannot grasp, or cannot retain, without assistance. The German student scores his reckoning with chalk marks because he might otherwise forget; while the Andaman Islander counts on his fingers because he has no other method of counting,—or, in other words, of grasping the idea of number. A single illustration may be given which typifies all practical methods of numeration. More than a century ago travellers in Madagascar observed a curious but simple mode of ascertaining the number of soldiers in an army. [6] Each soldier was made to go through a passage in the presence of the principal chiefs; and as he went through, a pebble was dropped on the ground. This continued until a heap of 10 was obtained, when one was set aside and a new heap begun. Upon the completion of 10 heaps, a pebble was set aside to indicate 100; and so on until the entire army had been numbered. Another illustration, taken from the very antipodes of Madagascar, recently found its way into print in an incidental manner, [7] and is so good that it deserves a place beside de Flacourt's time-honoured example. Mom Cely, a Southern negro of unknown age, finds herself in debt to the storekeeper; and, unwilling to believe that the amount is as great as he represents, she proceeds to investigate the matter in her own peculiar way. She had "kept a tally of these purchases by means of a string, in which she tied commemorative knots." When her creditor "undertook to make the matter clear to Cely's comprehension, he had to proceed upon a system of her own devising. A small notch was cut in a smooth white stick for every dime she owed, and a large notch when the dimes amounted to a dollar; for every five dollars a string was tied in the fifth big notch, Cely keeping tally by the knots in her bit of twine; thus, when two strings were tied about the stick, the ten dollars were seen to be an indisputable fact." This interesting method of computing the amount of her debt, whether an invention of her own or a survival of the African life of her parents, served the old negro woman's purpose perfectly; and it illustrates, as well as a score of examples could, the methods of numeration to which the children of barbarism resort when any number is to be expressed which exceeds the number of counters with which nature has provided them. The fingers are, however, often employed in counting numbers far above the first decade. After giving the Il-Oigob numerals up to 60, Müller adds: [8] "Above 60 all numbers, indicated by the proper figure pantomime, are expressed by means of the word *ipi* ." We know, moreover, that many of the American Indian tribes count one ten after another on their fingers; so that, whatever number they are endeavouring to indicate, we need feel no surprise if the savage continues to use his fingers throughout the entire extent of his counts. In rare instances we find tribes which, like the Mairassis of the interior of New Guinea, appear to use nothing but finger pantomime. [9] This tribe, though by no means destitute of the number sense, is said to have no numerals whatever, but to use the single word *awari* with each show of fingers, no matter how few or how many are displayed.

In the methods of finger counting employed by savages a considerable degree of uniformity has been observed. Not only does he use his fingers to assist him in his tally, but he almost always begins with the little finger of his left hand, thence proceeding towards the thumb, which is 5. From this point onward the method varies. Sometimes the second 5 also is told off on the left hand, the same order being observed as in the first 5; but oftener the fingers of the right hand are used, with a reversal of the order previously employed; *i.e.* the thumb denotes 6, the index finger 7, and so on to the little finger, which completes the count to 10.

At first thought there would seem to be no good reason for any marked uniformity of method in finger counting. Observation among children fails to detect any such thing; the child beginning, with almost entire indifference, on the thumb or on the little finger of the left hand. My own observation leads to the conclusion that very young children have a slight, though not decided preference for beginning with the thumb. Experiments in five different primary rooms in the public schools of Worcester, Mass., showed that out of a total of 206 children, 57 began with the little finger and 149 with the thumb. But the fact that nearly three-fourths of the children began with the thumb, and but one-fourth with the little finger, is really far less significant than would appear at first thought. Children of this age, four to eight years, will count in either way, and sometimes seem at a loss themselves to know where to begin. In one school room where this experiment was tried the teacher incautiously asked one child to count on his fingers, while all the other children in the room watched eagerly to see what he would do. He began with the little finger—and so did every child in the room after him. In another case the same error was made by the teacher, and the child first asked began with the thumb. Every other child in the room did the same, each following, consciously or unconsciously, the example of the leader. The results from these two schools were of course rejected from the totals which are given above; but they serve an excellent purpose in showing how slight is the preference which very young children have in this particular. So slight is it that no definite law can be postulated of this age; but the tendency seems to be to hold the palm of the hand downward, and then begin with the thumb. The writer once saw a boy about seven years old trying to multiply 3 by 6; and his method of procedure was as follows: holding his left hand with its palm down, he touched with the forefinger of his right hand the thumb, forefinger, and middle finger successively of his left hand. Then returning to his starting-point, he told off a second three in the same manner. This process he continued until he had obtained 6 threes, and then he announced his result correctly. If he had been a few years older, he might not have turned so readily to his thumb as a starting-point for any digital count. The indifference manifested by very young children gradually disappears, and at the age of twelve or thirteen the tendency is decidedly in the direction of beginning with the little finger. Fully three-fourths of all persons above that age will be found to count from the little finger toward the thumb, thus reversing the proportion that was found to obtain in the primary school rooms examined.

With respect to finger counting among civilized peoples, we fail, then, to find any universal law; the most that can be said is that more begin with the little finger than with the thumb. But when we proceed to the study of this slight but important particular among savages, we find them employing a certain order of succession with such substantial uniformity that the conclusion is inevitable that there must lie back of this some well-defined reason, or perhaps instinct, which guides them in their choice. This instinct is undoubtedly the outgrowth of the almost universal right-handedness of the human race. In finger counting, whether among children or adults, the beginning is made on the left hand, except in the case of left-handed individuals; and even then the start is almost as likely to be on the left hand as on the right. Savage tribes, as might be expected, begin with the left hand. Not only is this custom almost invariable, when tribes as a whole are considered, but the little finger is nearly always called into requisition first. To account for this uniformity, Lieutenant Gushing gives the following theory, [10] which is well considered, and is based on the results of careful study and observation among the Zuñi Indians of the Southwest: “Primitive man when abroad never lightly quit hold of his weapons. If he wanted to count, he did as the Zuñi afield does to-day; he tucked his instrument under his left arm, thus constraining the latter, but leaving the right hand free, that he might check off with it the fingers of the rigidly elevated left hand. From the nature of this position, however, the palm of the left hand was presented to the face of the counter, so that he had to begin his score on the little finger of it, and continue his counting from the right leftward. An inheritance of this may be detected to-day in the confirmed habit the Zuñi has of gesticulating from the right leftward, with the fingers of the right hand over those of the left, whether he be counting and summing up, or relating in any orderly manner.” Here, then, is the reason for this otherwise unaccountable phenomenon. If savage man is universally right-handed, he will almost inevitably use the index finger of his right hand to mark the fingers counted, and he will begin his count just where it is most convenient. In his case it is with the little finger of the left hand. In the case of the child trying to multiply 3 by 6, it was with the thumb of the same hand. He had nothing to tuck under his arm; so, in raising his left hand to a position where both eye and counting finger could readily run over its fingers, he held the palm turned away from his face. The same choice of starting-point then followed as with the savage—the finger nearest his right hand; only in this case the finger was a thumb. The deaf mute is sometimes taught in this manner, which is for him an entirely natural manner. A left-handed child might be expected to count in a left-to-right manner, beginning, probably, with the thumb of his right hand.

To the law just given, that savages begin to count on the little finger of the left hand, there have been a few exceptions noted; and it has been observed that the method of progression on the second hand is by no means as invariable as on the first. The Otomacs [11] of South America began their count with the thumb, and to express the number 3 would use the thumb, forefinger, and middle finger. The Maipures, [12] oddly enough, seem to have begun, in some cases at least, with the forefinger; for they are reported as expressing 3 by means of the fore, middle, and ring fingers. The Andamans [13] begin with the little finger of either hand, tapping the nose with each finger in succession. If they have but one to express, they use the forefinger of either hand, pronouncing at the same time the proper word. The Bahnars, [14] one of the native tribes of the interior of Cochin China, exhibit no particular order in the sequence of fingers used, though they employ their digits freely to assist them in counting. Among certain of the negro tribes of South Africa [15] the little finger of the right hand is used for 1, and their count proceeds from right to left. With them, 6 is the thumb of the left hand, 7 the forefinger, and so on. They hold the palm downward instead of upward, and thus form a complete and striking exception to the law which has been found to obtain with such substantial uniformity in other parts of the uncivilized world. In Melanesia a few examples of preference for beginning with the thumb may also be noticed. In the

Banks Islands the natives begin by turning down the thumb of the right hand, and then the fingers in succession to the little finger, which is 5. This is followed by the fingers of the left hand, both hands with closed fists being held up to show the completed 10. In Lepers' Island, they begin with the thumb, but, having reached 5 with the little finger, they do not pass to the other hand, but throw up the fingers they have turned down, beginning with the forefinger and keeping the thumb for 10. [16] In the use of the single hand this people is quite peculiar. The second 5 is almost invariably told off by savage tribes on the second hand, though in passing from the one to the other primitive man does not follow any invariable law. He marks 6 with either the thumb or the little finger. Probably the former is the more common practice, but the statement cannot be made with any degree of certainty. Among the Zulus the sequence is from thumb to thumb, as is the case among the other South African tribes just mentioned; while the Veis and numerous other African tribes pass from thumb to little finger. The Eskimo, and nearly all the American Indian tribes, use the correspondence between 6 and the thumb; but this habit is by no means universal. Respecting progression from right to left or left to right on the toes, there is no general law with which the author is familiar. Many tribes never use the toes in counting, but signify the close of the first 10 by clapping the hands together, by a wave of the right hand, or by designating some object; after which the fingers are again used as before.

One other detail in finger counting is worthy of a moment's notice. It seems to have been the opinion of earlier investigators that in his passage from one finger to the next, the savage would invariably bend down, or close, the last finger used; that is, that the count began with the fingers open and outspread. This opinion is, however, erroneous. Several of the Indian tribes of the West [17] begin with the hand clenched, and open the fingers one by one as they proceed. This method is much less common than the other, but that it exists is beyond question.

In the Muralug Island, in the western part of Torres Strait, a somewhat remarkable method of counting formerly existed, which grew out of, and is to be regarded as an extension of, the digital method. Beginning with the little finger of the left hand, the natives counted up to 5 in the usual manner, and then, instead of passing to the other hand, or repeating the count on the same fingers, they expressed the numbers from 6 to 10 by touching and naming successively the left wrist, left elbow, left shoulder, left breast, and sternum. Then the numbers from 11 to 19 were indicated by the use, in inverse order, of the corresponding portions of the right side, arm, and hand, the little finger of the right hand signifying 19. The words used were in each case the actual names of the parts touched; the same word, for example, standing for 6 and 14; but they were never used in the numerical sense unless accompanied by the proper gesture, and bear no resemblance to the common numerals, which are but few in number. This method of counting is rapidly dying out among the natives of the island, and is at the present time used only by old people. [18] Variations on this most unusual custom have been found to exist in others of the neighbouring islands, but none were exactly similar to it. One is also reminded by it of a custom [19] which has for centuries prevailed among bargainers in the East, of signifying numbers by touching the joints of each other's fingers under a cloth. Every joint has a special signification; and the entire system is undoubtedly a development from finger counting. The buyer or seller will by this method express 6 or 60 by stretching out the thumb and little finger and closing the rest of the fingers. The addition of the fourth finger to the two thus used signifies 7 or 70; and so on. "It is said that between two brokers settling a price by thus snipping with the fingers, cleverness in bargaining, offering a little more, hesitating, expressing an obstinate refusal to go further, etc., are as clearly indicated as though the bargaining were being carried on in words.

The place occupied, in the intellectual development of man, by finger counting and by the many other artificial methods of reckoning,—pebbles, shells, knots, the abacus, etc.,—seems to be this: The abstract processes of addition, subtraction, multiplication, division, and even counting itself, present to the mind a certain degree of difficulty. To assist in overcoming that difficulty, these artificial aids are called in; and, among savages of a low degree of development, like the Australians, they make counting possible. A little higher in the intellectual scale, among the American Indians, for example, they are employed merely as an artificial aid to what could be done by mental effort alone. Finally, among semi-civilized and civilized peoples, the same processes are retained, and form a part of the daily life of almost every person who has to do with counting, reckoning, or keeping tally in any manner whatever. They are no longer necessary, but they are so convenient and so useful that civilization can never dispense with them. The use of the abacus, in the form of the ordinary numeral frame, has increased greatly within the past few years; and the time may come when the abacus in its proper form will again find in civilized countries a use as common as that of five centuries ago.

In the elaborate calculating machines of the present, such as are used by life insurance actuaries and others having difficult computations to make, we have the extreme of development in the direction of artificial aid to reckoning. But instead of appearing merely as an extraneous aid to a defective intelligence, it now presents itself as a machine so complex that a high degree of intellectual power is required for the mere grasp of its construction and method of working.

Chapter II.

Number System Limits.

With respect to the limits to which the number systems of the various uncivilized races of the earth extend, recent anthropological research has developed many interesting facts. In the case of the Chiquitos and a few other native races of Bolivia we found no distinct number sense at all, as far as could be judged from the absence, in their language, of numerals in the proper sense of the word. How they indicated any number greater than one is a point still requiring investigation. In all other known instances we find actual number systems, or what may for the sake of uniformity be dignified by that name. In many cases, however, the numerals existing are so few, and the ability to count is so limited, that the term number system is really an entire misnomer.

Among the rudest tribes, those whose mode of living approaches most nearly to utter savagery, we find a certain uniformity of method. The entire number system may consist of but two words, one and many ; or of three words, one , two , many . Or, the count may proceed to 3, 4, 5, 10, 20, or 100; passing always, or almost always, from the distinct numeral limit to the indefinite many or several, which serves for the expression of any number not readily grasped by the mind. As a matter of fact, most races count as high as 10; but to this statement the exceptions are so numerous that they deserve examination in some detail. In certain parts of the world, notably among the native races of South America, Australia, and many of the islands of Polynesia and Melanesia, a surprising paucity of numeral words has been observed. The Encabellada of the Rio Napo have but two distinct numerals; tey , 1, and cayapa , 2. [20] The Chaco languages [21] of the Guaycuru stock are also notably poor in this respect. In the Mbocobi dialect of this language the only native numerals are yña tvak , 1, and yfioaca , 2. The Puris [22] count omi , 1, curiri , 2, prica , many; and the Botocudos [23] mokenam , 1, uruhu , many. The Fuegans, [24] supposed to have been able at one time to count to 10, have but three numerals,— kaoueli , 1, compaipi , 2, maten , 3. The Campas of Peru [25] possess only three separate words for the expression of number,— patrio , 1, pitteni , 2, mahuani , 3. Above 3 they proceed by combinations, as 1 and 3 for 4, 1 and 1 and 3 for 5. Counting above 10 is, however, entirely inconceivable to them, and any number beyond that limit they indicate by tohaine , many. The Conibos, [26] of the same region, had, before their contact with the Spanish, only atchoupre , 1, and rrabui , 2; though they made some slight progress above 2 by means of reduplication. The Orejones, one of the low, degraded tribes of the Upper Amazon, [27] have no names for number except nayhay , 1, nenacome , 2, feninichacome , 3, ononoeomere , 4. In the extensive vocabularies given by Von Martins, [28] many similar examples are found. For the Bororos he gives only couai , 1, maeouai , 2, ouai , 3. The last word, with the proper finger pantomime, serves also for any higher number which falls within the grasp of their comprehension. The Guachi manage to reach 5, but their numeration is of the rudest kind, as the following scale shows: tamak , 1, eu-echo, 2, eu-echo-kailau, 3, eu-echo-way, 4, localau , 5. The Carajas counted by a scale equally rude, and their conception of number seemed equally vague, until contact with the neighbouring tribes furnished them with the means of going beyond their original limit. Their scale shows clearly the uncertain, feeble number sense which is so marked in the interior of South America. It contains wadewo , 1, wadebothoa , 2, wadeboaheodo , 3, wadebojeodo , 4, wadewajouclay , 5, wadewasori , 6, or many.

Turning to the languages of the extinct, or fast vanishing, tribes of Australia, we find a still more noteworthy absence of numeral expressions. In the Gudang dialect [29] but two numerals are found— pirman , 1, and ilabiu , 2; in the Weedookarry, ekkamurda , 1, and kootera , 2; and in the Queanbeyan, midjemban , 1, and bollan , 2. In a score or more of instances the numerals stop at 3. The natives of Keppel Bay count webben , 1, booli , 2, koorel , 3; of the Boyne River, karroon , 1, boodla , 2, numma , 3; of the Flinders River, kooroin , 1, kurto , 2, kurto kooroin , 3; at the mouth of the Norman River, lum , 1, buggar , 2, orinch , 3; the Eaw tribe, koothea , 1, woother , 2, marronoo , 3; the Moree, mal , 1, boolar , 2, kooliba , 3; the Port Essington, [30] erad , 1, nargarick , 2, nargarickelerad , 3; the Darnly Islanders, [31] netat , 1, naes , 2, naesa netat , 3; and so on through a long list of tribes whose numeral scales are equally scanty. A still larger number of tribes show an ability to count one step further, to 4; but beyond this limit the majority of Australian and Tasmanian tribes do not go. It seems most remarkable that any human being should possess the ability to count to 4, and not to 5. The number of fingers on one hand furnishes so obvious a limit to any of these rudimentary systems, that positive evidence is needed before one can accept the statement. A careful examination of the numerals in upwards of a hundred Australian dialects leaves no doubt, however, that such is the fact. The Australians in almost all cases count by pairs; and so pronounced is this tendency that they pay but little attention to the fingers. Some tribes do not appear ever to count beyond 2—a single pair. Many more go one step further; but if they do, they are as likely as not to designate their next numeral as two-one, or possibly, one-two. If this step is taken, we may or may not find one more added to it, thus completing the second pair. Still, the Australian's capacity for understanding anything which pertains to number is so painfully limited that even here there is sometimes an indefinite expression formed, as many, heap, or plenty, instead of any distinct numeral; and it is probably true that no Australian language contains a pure, simple numeral for 4. Curr, the best authority on this subject, believes that, where a distinct word for 4 is given, investigators have been deceived in every case. [32] If counting is carried beyond 4, it is always by means of reduplication. A few tribes gave expressions for 5, fewer still for 6, and a very small number appeared able to reach 7. Possibly the ability to count extended still further; but if so, it consisted undoubtedly in reckoning one pair after another, without any consciousness whatever of the sum total save as a larger number.

The numerals of a few additional tribes will show clearly that all distinct perception of number is lost as soon as these races attempt to count above 3, or at most, 4. The Yuckaburra [33] natives can go no further than wigsin , 1, bullaroo , 2, goolbora , 3. Above here all is referred to as moorgha , many. The Marachowies [34] have but three distinct numerals,— cooma , 1, cootera , 2, murra , 3. For 4 they say minna , many. At Streaky Bay we find a similar list, with the same words, kooma and kootera , for 1 and 2, but entirely different terms, karboo and yalkata for 3 and many. The same method obtains in the Minnal Yungar tribe, where the only numerals are kain , 1, kujal , 2, moa , 3, and bulla , plenty. In the Pinjarra dialect we find doombart , 1, gugal , 2, murdine , 3, boola , plenty; and in the dialect described as belonging to “Eyre's Sand Patch,” three definite terms are given— kean , 1, koojal , 2, yalgatta , 3, while a fourth, murna , served to describe anything greater. In all these examples the fourth numeral is indefinite; and the same statement is true of many other Australian languages. But more commonly still we find 4, and perhaps 3 also, expressed by reduplication. In the Port Mackay dialect [35] the latter numeral is compound, the count being warpur , 1, boolera , 2, boolera warpur , 3. For 4 the term is not given. In the dialect which prevailed between the Albert and Tweed rivers [36] the scale appears as yaburu , 1, boolaroo , 2, boolaroo yaburu , 3, and gurul for 4 or anything beyond. The Wiraduroi [37] have numbai , 1, bula , 2, bula numbai , 3, bungu , 4, or many, and bungu galan or bian galan , 5, or very many. The Kamilaroi [38] scale is still more irregular, compounding above 4 with little apparent method. The numerals are mal , 1, bular , 2, guliba , 3, bular bular , 4, bular guliba , 5, guliba guliba , 6. The last two numerals show that 5 is to these natives simply 2-3, and 6 is 3-3. For additional examples of a similar nature the extended list of Australian scales given in Chapter V. may be consulted.

Taken as a whole, the Australian and Tasmanian tribes seem to have been distinctly inferior to those of South America in their ability to use and to comprehend numerals. In all but two or three cases the Tasmanians [39] were found to be unable to proceed beyond 2; and as the foregoing examples have indicated, their Australian neighbours were but little better off. In one or two instances we do find Australian numeral scales which reach 10, and perhaps we may safely say 20. One of these is given in full in a subsequent chapter, and its structure gives rise to the suspicion that it was originally as limited as those of kindred tribes, and that it underwent a considerable development after the natives had come in contact with the Europeans. There is good reason to believe that no Australian in his wild state could ever count intelligently to 7. [40]

In certain portions of Asia, Africa, Melanesia, Polynesia, and North America, are to be found races whose number systems are almost and sometimes quite as limited as are those of the South. American and Australian tribes already