

CHAOS IN TRANSIT SYSTEMS

A dissertation by
JORGE VILLALOBOS

Submitted to the
School of Engineering of
UNIVERSIDAD DE LOS ANDES
in partial fulfilment for the
requirements for the Degree of
DOCTOR IN ENGINEERING

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Departamento de Ingeniería Industrial



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ABSTRACT

We present two traffic models based on the same idea: that of a single vehicle interacting with traffic lights. The vehicle is capable of accelerating, traveling at constant speed, and braking. Braking takes place when the vehicle is interacting with a traffic light. The first model (Car model) will mimic the expected dynamics of a vehicle on a long road that contains several traffic lights. This car will interact with the lights depending on their color (either red or green) in the usual way drivers do, i.e., stop if a red light is present at a given point in space, and continue if green. The second model (Bus model) will obey the same dynamics as the Car model but with the possibility of a forced stop between traffic lights, this in order to copy the action of picking up or leaving passengers. Two variants of the Bus model will be developed and studied. All models are highly minimalistic and avoid the issue of human reactions (the vehicles will never run a red light or try to emulate other human behavior like traveling at different speeds or such).

The principal objective of this document is to characterize chaotic behavior in these simple traffic models. The characterization we propose is based on the following program: First, based on the dynamic equations of the system, write a discrete non linear map. Then use bifurcation diagrams to explore the parameters, and find symbolic solutions to period-1 and period-2 orbits, as well as some interesting values for the diagrams. Since there are many bi-

furcation parameters, represent combinations of parameter values that have resulted in chaotic behavior in 2D plots where positive Lyapunov exponents are marked; these 2D Lyapunov plots give a broad picture of the combinations of parameters for which chaos is present on the systems.

We will aim our efforts in order to achieve a goal and answer two simple questions:

1. To characterize the systems' asymptotic behavior.
2. Is non-trivial dynamics and chaos (understood as high sensitivity to initial conditions) a fundamental part of traffic systems?
3. Is this sensitivity a consequence of human behavior or not?

Presentamos dos modelos de tráfico basados en la misma idea: un vehículo interactuando con semáforos. El vehículo puede acelerar, viajar a velocidad constante y frenar. El frenado ocurre cuando se da la interacción con el semáforo. El primer modelo (el del Carro) imita la dinámica esperada de un móvil en una carretera larga que tiene varios semáforos. Este carro interactúa con las luces dependiendo del color de éstas (rojo o verde) de la forma usual como lo hacen los conductores: para si la luz está en rojo y continúa si está en verde. El segundo modelo (el del Bus) sigue la misma dinámica que el del Carro, pero incluye la posibilidad de hacer una parada obligada entre semáforos; esto para simular la carga y descarga de pasajeros. Se desarrollarán y estudiarán dos variantes del modelo del Bus. Todos los modelos mencionados son altamente minimalistas, evitando así comportamientos causados por el comportamiento típico de la gente (estos vehículos nunca se pasarán una luz en rojo, viajarán a velocidades distintas, o tendrán comportamientos no deterministas).

El objetivo principal del documento es caracterizar el comportamiento caótico y no trivial de estos modelos simples de tránsito. La propuesta de caracterización sigue el siguiente programa: primero, basados en las ecuaciones dinámicas del sistema, escribir un mapa no lineal. Utilizar diagramas de bifurcaciones para explorar la dependencia de los parámetros y encontrar soluciones analíticas para las órbitas de periodos 1 y 2, junto con algunos valores interesantes de los diagramas. Dado que existe más de un parámetro de bifurcación, se representan combinaciones de parámetros que resultan en comportamiento caótico, en gráficas en dos dimensiones en las que los exponentes de Lyapunov positivos se marcan; estas gráficas dan una idea global de las combinaciones de parámetros que presentan caos en estos sistemas.

Dirigimos nuestros esfuerzos a lograr un objetivo y responder dos preguntas:

1. Caracterizar el comportamiento asintótico de los sistemas
2. ¿Es parte fundamental de los sistemas de tránsito la dinámica no trivial y el caos (entendido como alta sensibilidad a las condiciones iniciales)?
3. ¿Es esta sensibilidad causada solamente por comportamientos humanos?

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