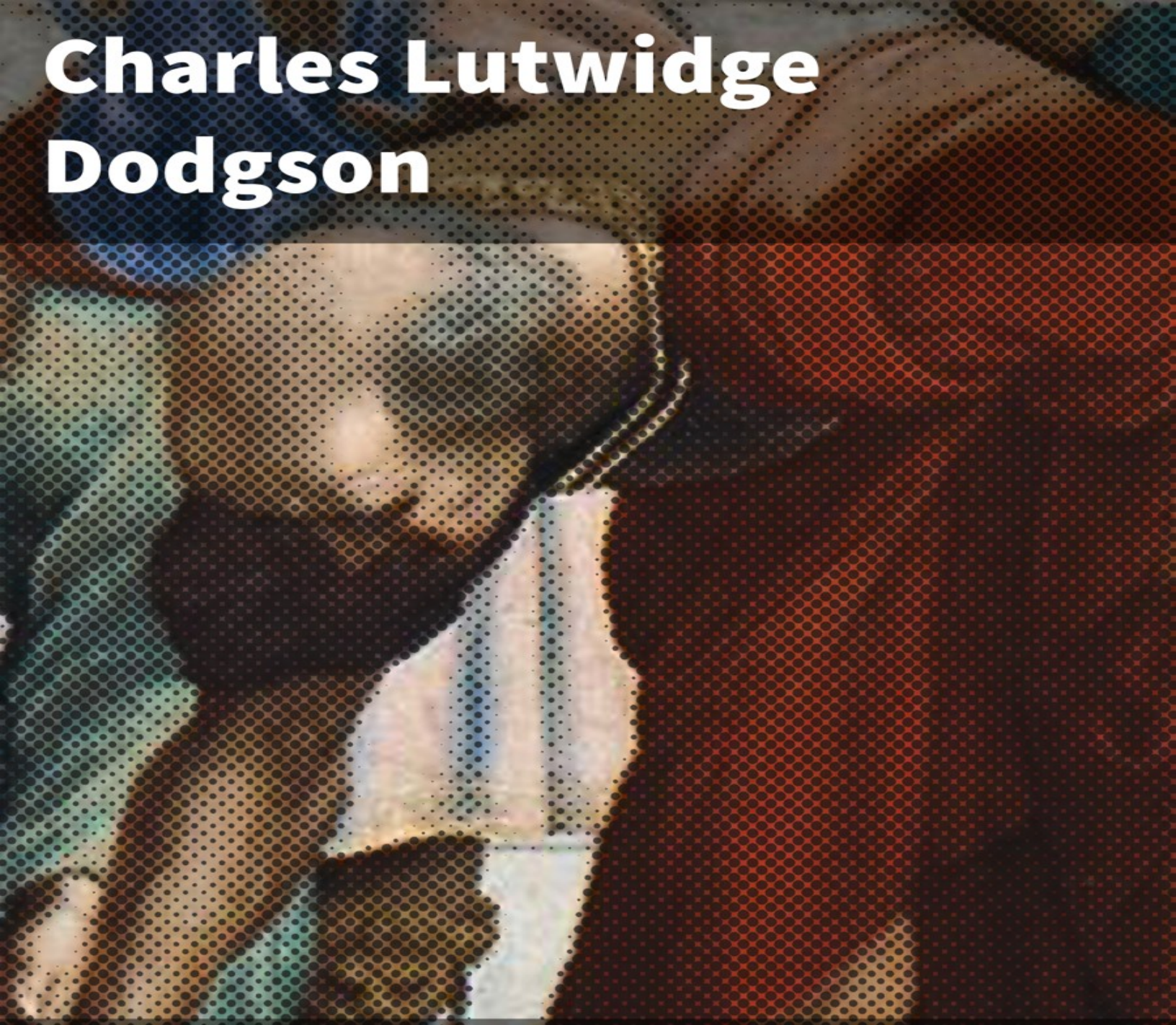
Charles Lutwidge Dodgson



Euclid and His Modern Rivals

Charles Lutwidge Dodgson

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'All for your delight
We are not here. *That you should here repent you*
The actors are at hand; and, by their show,
You shall know all, that you are like to know.'

SECOND EDITION

London

MACMILLAN AND CO.

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1885

Oxford

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PRINTED BY HORACE HART, PRINTER TO THE UNIVERSITY

Dedicated

to

the memory

of

Euclid

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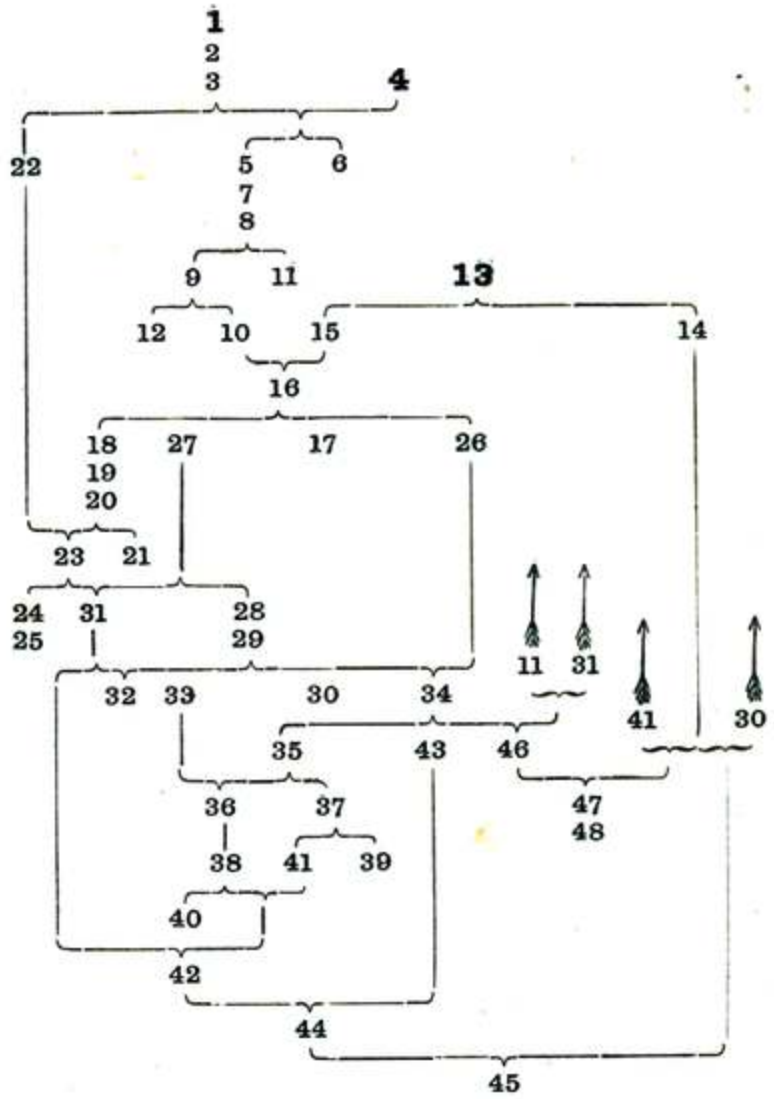
Content (not individually listed)

EUCLID, BOOK I.

Arranged in Logical Sequence.



PREFACE TO SECOND EDITION.

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THE only new features, worth mentioning, in the second edition, are the substitution of words for the symbols introduced in the first edition, and one additional review—of Mr. Henrici, to whom, if it should appear to him that I have at all exceeded the limits of fair criticism, I beg to tender my sincerest apologies.

C. L. D.

Ch. Ch. 1885.

PREFACE TO FIRST EDITION.

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'ridentem dicere verum
Quid vetat?'

THE object of this little book is to furnish evidence, first, that it is essential, for the purpose of teaching or examining in elementary Geometry, to employ one textbook only; secondly, that there are strong *a priori* reasons for retaining, in all its main features, and specially in its sequence and numbering of Propositions and in its treatment of Parallels, the Manual of Euclid; and thirdly, that no sufficient reasons have yet been shown for abandoning it in favour of any one

of the modern Manuals which have been offered as substitutes.

It is presented in a dramatic form, partly because it seemed a better way of exhibiting in alternation the arguments on the two sides of the question; partly that I might feel myself at liberty to treat it in a rather lighter style than would have suited an essay, and thus to make it a little less tedious and a little more acceptable to unscientific readers.

In one respect this book is an experiment, and may chance to prove a failure: I mean that I have not thought it necessary to maintain throughout the gravity of style which scientific writers usually affect, and which has somehow come to be regarded as an 'inseparable accident' of scientific teaching. I never could quite see the reasonableness of this immemorial law: subjects there are, no doubt, which are in their essence too serious to admit of any lightness of treatment—but I cannot recognise Geometry as one of them. Nevertheless it will, I trust, be found that I have permitted myself a glimpse of the comic side of things only at fitting seasons, when the tired reader might well crave a moment's breathing-space, and not on any occasion where it could endanger the continuity of a line of argument.

Pitying friends have warned me of the fate upon which I am rushing: they have predicted that, in thus abandoning the dignity of a scientific writer, I shall alienate the sympathies of all true scientific readers, who will regard the book as a mere *jeu d'esprit*, and will not trouble themselves to look for any serious argument in it. But it must be borne

in mind that, if there is a Scylla before me, there is also a Charybdis—and that, in my fear of being read as a jest, I may incur the darker destiny of not being read at all.

In furtherance of the great cause which I have at heart—the vindication of Euclid's masterpiece—I am content to run some risk; thinking it far better that the purchaser of this little book should *read* it, though it be with a smile, than that, with the deepest conviction of its seriousness of purpose, he should leave it unopened on the shelf.

To all the authors, who are here reviewed, I beg to tender my sincerest apologies, if I shall be found to have transgressed, in any instance, the limits of fair criticism, To Mr. Wilson especially such apology is due—partly because I have criticised his book at great length and with no sparing hand—partly because it may well be deemed an impertinence in one, whose line of study has been chiefly in the lower branches of Mathematics, to dare to pronounce any opinion at all on the work of a Senior Wrangler. Nor should I thus dare, if it entailed my following him up 'yonder mountain height' which *he* has scaled, but which *I* can only gaze at from a distance: it is only when he ceases 'to move so near the heavens,' and comes down into the lower regions of Elementary Geometry, which I have been teaching for nearly five-and-twenty years, that I feel sufficiently familiar with the matter in hand to venture to speak.

Let me take this opportunity of expressing my gratitude, first to Mr. Todbunter, for allowing me to quote *ad libitum* from the very interesting Essay on Elementary Geometry, which is included in his volume entitled 'The Conflict of

Studies, and other Essays on subjects connected with Education,' and also to reproduce some of the beautiful diagrams from his edition of Euclid; secondly, to the Editor of the Athenæum, for giving me a similar permission with regard to a review of Mr. Wilson's Geometry, written by the late Professor De Morgan, which appeared in that journal, July 18, 1868.

C. L. D.

Ch. Ch. 1879.

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(2) Different treatment of Parallels.

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(3) Superposition;

(4) Use of diagonals in Euc. II;

(5) Treatment of Lines;

(6) „ of Angles;

(7) Euclid's Propositions omitted;

(8) „ „ newly treated;

(9) New Propositions;

(10) Style, &c.

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Legendre, Cooley, Cuthbertson, Henrici, Wilson, Pierce, Willock, Chauvenet, Loomis, Morell, Reynolds, Wright, Syllabus of Association for Improvement of Geometrical Teaching, Wilson's 'Syllabus'-Manual.

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