



**Charles Lutwidge
Dodgson**

*Euclid and
His Modern
Rivals*

Charles Lutwidge Dodgson

Euclid and His Modern Rivals



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'All for your delight
We are not here. *That you should here repent you*
The actors are at hand; and, by their show,
You shall know all, that you are like to know.'

SECOND EDITION

London

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1885

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Dedicated

to

the memory

of

Euclid

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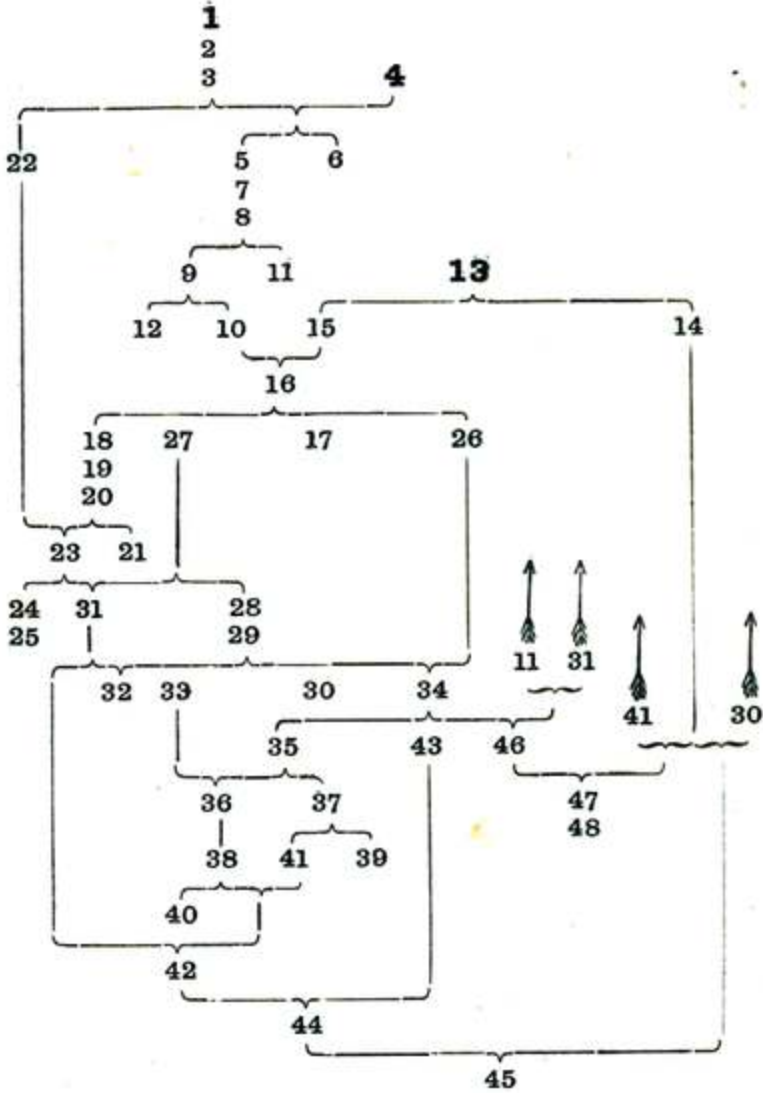
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PREFACE TO SECOND EDITION.

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THE only new features, worth mentioning, in the second edition, are the substitution of words for the symbols introduced in the first edition, and one additional review—of Mr. Henrici, to whom, if it should appear to him that I have at all exceeded the limits of fair criticism, I beg to tender my sincerest apologies.

C. L. D.

Ch. Ch. 1885.

PREFACE TO FIRST EDITION.

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'ridentem dicere verum

Quid vetat?'

THE object of this little book is to furnish evidence, first, that it is essential, for the purpose of teaching or examining in elementary Geometry, to employ one textbook only; secondly, that there are strong *a priori* reasons for retaining, in all its main features, and specially in its sequence and numbering of Propositions and in its treatment of Parallels, the Manual of Euclid; and thirdly, that no sufficient reasons have yet been shown for abandoning it in favour of any one

of the modern Manuals which have been offered as substitutes.

It is presented in a dramatic form, partly because it seemed a better way of exhibiting in alternation the arguments on the two sides of the question; partly that I might feel myself at liberty to treat it in a rather lighter style than would have suited an essay, and thus to make it a little less tedious and a little more acceptable to unscientific readers.

In one respect this book is an experiment, and may chance to prove a failure: I mean that I have not thought it necessary to maintain throughout the gravity of style which scientific writers usually affect, and which has somehow come to be regarded as an 'inseparable accident' of scientific teaching. I never could quite see the reasonableness of this immemorial law: subjects there are, no doubt, which are in their essence too serious to admit of any lightness of treatment—but I cannot recognise Geometry as one of them. Nevertheless it will, I trust, be found that I have permitted myself a glimpse of the comic side of things only at fitting seasons, when the tired reader might well crave a moment's breathing-space, and not on any occasion where it could endanger the continuity of a line of argument.

Pitying friends have warned me of the fate upon which I am rushing: they have predicted that, in thus abandoning the dignity of a scientific writer, I shall alienate the sympathies of all true scientific readers, who will regard the book as a mere *jeu d'esprit*, and will not trouble themselves to look for any serious argument in it. But it must be borne

in mind that, if there is a Scylla before me, there is also a Charybdis—and that, in my fear of being read as a jest, I may incur the darker destiny of not being read at all.

In furtherance of the great cause which I have at heart—the vindication of Euclid's masterpiece—I am content to run some risk; thinking it far better that the purchaser of this little book should *read* it, though it be with a smile, than that, with the deepest conviction of its seriousness of purpose, he should leave it unopened on the shelf.

To all the authors, who are here reviewed, I beg to tender my sincerest apologies, if I shall be found to have transgressed, in any instance, the limits of fair criticism, To Mr. Wilson especially such apology is due—partly because I have criticised his book at great length and with no sparing hand—partly because it may well be deemed an impertinence in one, whose line of study has been chiefly in the lower branches of Mathematics, to dare to pronounce any opinion at all on the work of a Senior Wrangler. Nor should I thus dare, if it entailed my following him up 'yonder mountain height' which *he* has scaled, but which *I* can only gaze at from a distance: it is only when he ceases 'to move so near the heavens,' and comes down into the lower regions of Elementary Geometry, which I have been teaching for nearly five-and-twenty years, that I feel sufficiently familiar with the matter in hand to venture to speak.

Let me take this opportunity of expressing my gratitude, first to Mr. Todhunter, for allowing me to quote *ad libitum* from the very interesting Essay on Elementary Geometry, which is included in his volume entitled 'The Conflict of

Studies, and other Essays on subjects connected with Education,' and also to reproduce some of the beautiful diagrams from his edition of Euclid; secondly, to the Editor of the Athenæum, for giving me a similar permission with regard to a review of Mr. Wilson's Geometry, written by the late Professor De Morgan, which appeared in that journal, July 18, 1868.

C. L. D.

Ch. Ch. 1879.

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[MINOS *and* RHADAMANTHUS.]

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[MINOS *and* EUCLID.]

§ I. A priori reasons for retaining Euclid's Manual.

We require, in a Manual, a selection rather than
. . . a complete repertory of Geometrical truths

Discussion limited to subject-matter of Euc. I, II.

One fixed logical sequence essential

One system of numbering desirable

A priori claims of Euclid's sequence and
 .. numeration to be retained

New Theorems might be interpolated without
 .. change of numeration

§ 2. Method of procedure in examining Modern Rivals.

Proposed changes which, even if proved to be
 essential, *would not* necessitate the
 .. abandonment of Euclid's Manual:—

- (1) Propositions to be omitted;
- (2) „ to be replaced by new proofs;
- (3) New Propositions to be added.

Proposed changes which, if proved to be
 essential, *would* necessitate such
 .. abandonment:—

- (1) Separation of Problems and Theorems;
- (2) Different treatment of Parallels.

Other subjects of enquiry:—

- (3) Superposition;
- (4) Use of diagonals in Euc. II;
- (5) Treatment of Lines;
- (6) „ of Angles;

- (7) Euclid's Propositions omitted;
- (8) „ „ newly treated;
- (9) New Propositions;
- (10) Style, &c.

List of authors to be examined, viz.:—

Legendre, Cooley, Cuthbertson, Henrici,
 Wilson, Pierce, Willock, Chauvenet, Loomis,
 Morell, Reynolds, Wright, Syllabus of
 Association for Improvement of Geometrical
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§ 3. The combination, or separation of Problems and Theorems.

Reasons assigned for separation

Reasons for combination:—

- (1) Problems are also Theorems;
- (2) Separation would necessitate a new numeration,
- (3) and hypothetical constructions.

§ 4. Syllabus of propositions relating to Pairs of Lines.

Three classes of Pairs of Lines:—

- (1) Having two common points;
- (2) Having a common point and a separate point;
- (3) „ no common point.

Four kinds of 'properties';

- (1) common or separate points;
- (2) equality, or otherwise, of angles made with transversals;
- (3) equidistance, or otherwise, of points on the one from the others;
- (4) direction.

Conventions as to language

Propositions divisible into two classes:—

- (1) Deducible from undisputed Axioms;
- (2) „ disputable „

Three classes of Pairs of Lines:—

- (1) Coincidental;
- (2) Intersectional;
- (3) Separational.

Subjects and predicates of Propositions concerning these three classes:—

Coincidental

Intersectional

Separational

TABLE I. *Containing twenty Propositions, of which some are undisputed Axioms, and the rest real and valid Theorems, deducible from .. undisputed Axioms*

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.. concerning Separational Lines

TABLE II. *Containing eighteen Propositions, of which no one is an undisputed Axiom, but all are real and valid Theorems, which, though not deducible from undisputed Axioms, are such that, if any one be admitted as an Axiom, the rest can be proved*

TABLE III. *Containing five Propositions, taken from Table II, which have been proposed as Axioms*

- (1) Euclid's Axiom;
- (2) T. Simpson's Axiom;
- (3) Clavius' „
- (4) Playfair's „
- (5) R. Simpson's Axiom.

It will be shown (in Appendix III) that *any* Theorem of Table II is sufficient logical basis for .. all the rest

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***Manuals which reject Euclid's treatment
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If angle be constant, the Axiom involves two assumptions: viz. that

- (1) there can be a Pair of different Lines that .. make equal angles with *any* transversal
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| | |
|--|--|
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| | |
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'Un enfant terrible'

Summary of results:—

Of 73 Propositions of Euclid, this Manual has

14 omitted;

43 done as in Euclid;

10 done by new but objectionable methods, viz.—

1 illogical;

1 'hypothetical construction';

2 needlessly using 'superposition';

2 algebraical;

4 omitting the diagonals of Euc. II.;

6 done by new and admissible methods.

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.. numeration

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[MINOS *and* EUCLID.]

Manual of Euclid.

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