100

essential THINGS YOU DIDN'T KNOW you didn't know

JOHN D. BARROW

Contents

Cover
About the Book
About the Author
Also by John Barrow
Dedication
Title Page
Epigraph
Preface

1. Pylon of the Month
2. A Sense of Balance
3. Monkey Business
4. Independence Day
5. Rugby and Relativity
6. Wagons Roll
7. A Sense of Proportion
8. Why Does the Other Queue Always Move Faster?
9. Two’s Company, Three’s a Crowd
10. It’s a Small World After All
11. Bridging That Gap
12. On the Cards
13. Tally Ho
14. Relationships
15. Racing Certainties
16. High Jumping
17. Superficiality
18. VAT in Eternity
19. Living in a Simulation
20. Emergence
21. How to Push a Car

22. Positive Feedback
23. The Drunkard's Walk
24. Faking It
25. The Flaw of Averages
26. The Origami of the Universe
27. Easy and Hard Problems
28. Is This a Record?
29. A Do-It-Yourself Lottery
30. I Do Not Believe It!
31. Flash Fires
32. The Secretary Problem
33. Fair Divorce Settlements: the Win–Win Solution
34. Many Happy Returns
35. Tilting at Windmills
36. Verbal Conjuring
37. Financial Investment with Time Travellers
38. A Thought for Your Pennies
39. Breaking the Law of Averages
40. How Long are Things Likely to Survive?
41. A President who Preferred the Triangle to the Pentagon
42. Secret Codes in Your Pocket
43. I've Got a Terrible Memory for Names
44. Calculus Makes You Live Longer
45. Getting in a Flap
46. Your Number's Up
47. Double Your Money
48. Some Reflections on Faces
49. The Most Infamous Mathematician
50. Roller Coasters and Motorway Junctions
51. A Taylor-made Explosion
52. Walk Please, Don't Run!
53. Mind-reading Tricks
54. The Planet of the Deceivers
55. How to Win the Lottery
56. A Truly Weird Football Match
57. An Arch Problem

58. Counting in Eights
59. Getting a Mandate
60. The Two-headed League
61. Creating Something out of Nothing
62. How to Rig An Election
63. The Swing of the Pendulum
64. A Bike with Square Wheels
65. How Many Guards Does an Art Gallery Need?
66. . . . and What About a Prison?
67. A Snooker Trick Shot
68. Brothers and Sisters
69. Playing Fair with a Biased Coin
70. The Wonders of Tautology
71. What a Racket
72. Packing Your Stuff
73. Sent Packing Again
74. Crouching Tiger
75. How the Leopard Got His Spots
76. The Madness of Crowds
77. Diamond Geezer
78. The Three Laws of Robotics
79. Thinking Outside the Box
80. Googling in the Caribbean – The Power of the Matrix
81. Loss Aversion
82. The Lead in Your Pencil
83. Testing Spaghetti to Destruction
84. The Gherkin
85. Being Mean with the Price Index
86. Omniscience can be a Liability
87. Why People aren't Cleverer
88. The Man from Underground
89. There are No Uninteresting Numbers
90. Incognito
91. The Ice Skating Paradox
92. The Rule of Two
93. Segregation and Micromotives

94. Not Going with the Flow
95. Venn Vill They Ever Learn
96. Some Benefits of Irrationality
97. Strange Formulae
98. Chaos
99. All Aboard
100. The Global Village

Notes
Copyright

About the Book

‘If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is.’
John von Neumann

Mathematics can tell you things about the world that can’t be learned in any other way. This hugely informative and wonderfully entertaining little book answers one hundred essential questions about existence. It unravels the knotty, clarifies the conundrums and sheds light into dark corners. From winning the lottery, placing bets at the races and escaping from bears to sports, Shakepeare, Google, game theory, drunks, divorce settlements and dodgy accounting; from chaos to infinity and everything in between, *100 Essential Things You Didn’t Know You Didn’t Know* has all the answers!

About the Author

John D. Barrow is Professor of Mathematical Sciences and Director of the Millenium Mathematics Project at Cambridge University, Fellow of Clare Hall, Cambridge, a Fellow of the Royal Society, and the current Gresham Professor of Geometry at Gresham College, London. His previous books include *The Origin of the Universe*; *The Universe that Discovered Itself*; *The Book of Nothing*; *The Constants of Nature: From Alpha to Omega*; *The Infinite Book: A Short Guide to the Boundless, Timeless and Endless*; *The Artful Universe Expanded*; *New Theories of Everything* and, most recently, *Cosmic Imagery: Key Images in the History of Science*. He is also the author of the award-winning play *Infinities*.

Also by John Barrow

Theories of Everything

The Left Hand of Creation
(with Joseph Silk)

L'Homme et le Cosmos
(with Frank J. Tipler)

The Anthropic Cosmological Principle
(with Frank J. Tipler)

The World within the World

The Artful Universe

Pi in the Sky

Perchè il mondo è matematico?

Impossibility

The Origin of the Universe

Between Inner Space and Outer Space

The Universe that Discovered Itself

The Book of Nothing

The Constants of Nature:
From Alpha to Omega

The Infinite Book:
A Short Guide to the Boundless,
Timeless and Endless

Cosmic Imagery:
Key Images in the History of Science

100 Essential Things You Didn't Know

You Didn’t Know About Sport

The Book of Universes

To David and Emma

100 Essential Things You Didn't Know You Didn't Know

JOHN D. BARROW

THE BODLEY HEAD
LONDON

I continued to do arithmetic with my father, passing proudly through fractions to decimals. I eventually arrived at the point where so many cows ate so much grass, and tanks filled with water in so many hours. I found it quite enthralling.

Agatha Christie

Preface

This is a little book of bits and pieces – bits about off-beat applications of mathematics to everyday life, and pieces about a few other things not so very far away from it. There are a hundred to choose from, in no particular order, with no hidden agenda and no invisible thread. Sometimes you will find only words, but sometimes you will find some numbers as well, and very occasionally a few further notes that show you some of the formulae behind the appearances. Maths is interesting and important because it can tell you things about the world that you can't learn in any other way. When it comes to the depths of fundamental physics or the breadth of the astronomical universe we have almost come to expect that. But I hope that here you will see how simple ideas can shed new light on all sorts of things that might otherwise seem boringly familiar or just pass by unnoticed.

Lots of the examples contained in the pages to follow were stimulated by the goals of the Millennium Mathematics Project[fn1], which I came to Cambridge to direct in 1999. The challenge of showing how mathematics has something to tell about most things in the world around us is one that, when met, can play an important part in motivating and informing people, young and old, to appreciate and understand the place of mathematics at the root of our understanding of the world.

I would like to thank Steve Brams, Marianne Freiberger, Jenny Gage, John Haigh, Jörg Hensgen, Helen Joyce, Tom Körner, Imre Leader, Drummond Moir, Robert Osserman, Jenny Piggott, David Spiegelhalter, Will Sulkin, Rachel Thomas, John H. Webb, Marc West, and Robin Wilson for helpful discussions, encouragement, and other practical inputs that contributed to the final collection of essential things you now find before you.

Finally, thanks to Elizabeth, David, Roger and Louise for their unnervingly close interest in this book. Several of these family members now often tell me why pylons are made of triangles and tightrope walkers carry long poles. Soon you will know too.

John D. Barrow
August 2008, Cambridge

fn1 www.mmp.maths.org

1

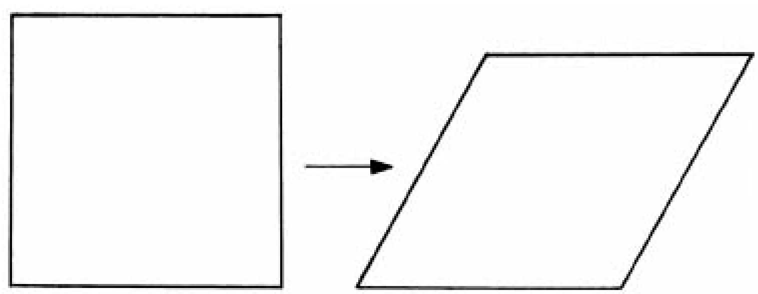
Pylon of the Month

> Like Moses parting the waves, National Grid Company PLC's 4YG8 leads his fellow pylons through this Oxfordshire housing estate towards the 'promised land' of Didcot Power Station.
>
> The December 1999 *Pylon of the Month*

There are some fascinating websites about, but none was more beguiling than the iconic *Pylon of the Month*,[fn1] once devoted to providing monthly pin-ups of the world's most exciting and seductive electricity pylons. The ones shown on the website below are from Scotland. Alas, *Pylon of the Month* now seems to have become a cobweb site, but there is still something to learn from it, since for the mathematician every pylon tells a story. It is about something so prominent and ubiquitous that, like gravity, it goes almost unnoticed.

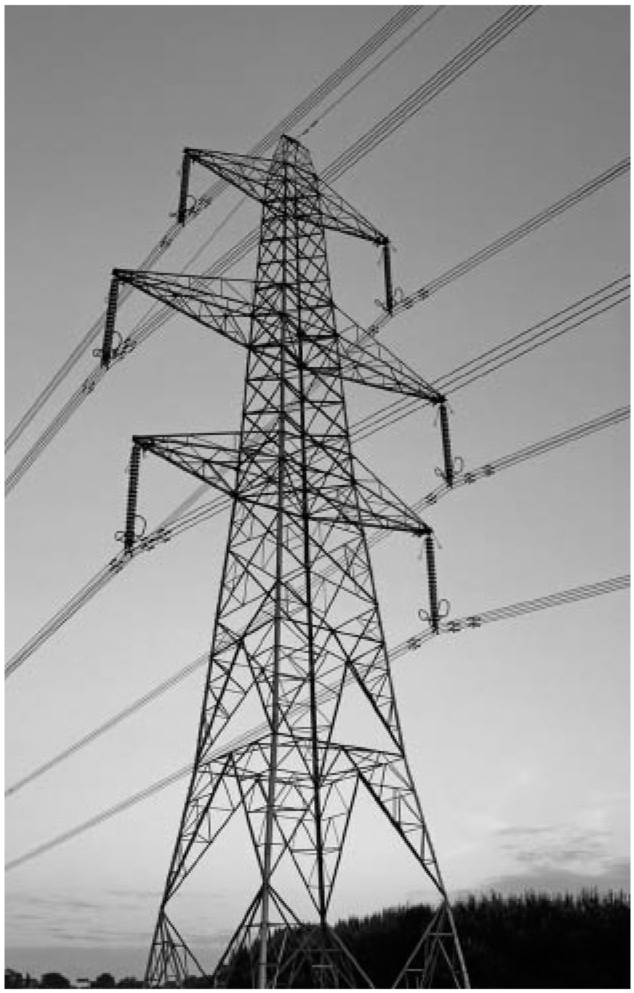
Next time you go on a train journey, look carefully at the pylons as they pass swiftly by the windows. Each is made of a network of metal struts that make use of a single recurring polygonal shape. That shape is the triangle. There are big triangles and smaller ones nested within them. Even apparent squares and rectangles are merely separate pairs of triangles. The reason forms a small part of an interesting mathematical story that began in the early nineteenth century with the work of the French mathematician Augustin-Louis Cauchy.

Of all the polygonal shapes that we could make by bolting together straight struts of metal, the triangle is special. It is the only one that is *rigid*. If they were hinged at their corners, all the others can be flexed gradually into a different shape without bending the metal. A square or a rectangular frame provides a simple example: we see that it can be deformed into a parallelogram without any buckling. This is an important consideration if you aim to maintain structural stability in the face of winds and temperature changes. It is why pylons seem to be great totems to the god of all triangles.



If we move on to three-dimensional shapes then the situation is quite different: Cauchy showed that *every* convex polyhedron (i.e. in which the faces all point outwards) with rigid faces, and hinged along its edges, is rigid. And, in fact, the same is true for convex polyhedra in spaces with four or more dimensions as well.

What about the non-convex polyhedra, where some of the faces can point inwards? They look much more squashable. Here, the question remained open until 1978 when Robert Connelly found an example with non-convex faces that is not rigid and then showed that in all such cases the possible flexible shifts keep the total volume of the polyhedron the same. However, the non-convex polyhedral examples that exist, or that may be found in the future, seem to be of no immediate practical interest to structural engineers because they are special in the sense that they require a perfectly accurate construction, like balancing a needle on its point. Any deviation from it at all just gives a rigid example, and so mathematicians say that 'almost every' polyhedron is rigid. This all seems to make structural stability easy to achieve – but pylons do buckle and fall down. I'm sure you can see why.



[fn1] http://www.drookitagain.co.uk/coppermine/thumbnails.php?album=34

2

A Sense of Balance

> Despite my privileged upbringing, I'm quite well-balanced. I have a chip on both shoulders.
>
> Russell Crowe in *A Beautiful Mind*

Whatever you do in life, there will be times when you feel you are walking a tightrope between success and failure, trying to balance one thing against another or to avoid one activity gobbling up every free moment of your time. But what about the people who really are walking a tightrope. The other day I was watching some old newsreel film of a now familiar sight: a crazy tightrope walker making a death-defying walk high above a ravine and a rushing river. One slip and he would have become just another victim of Newton's law of gravity.

We have all tried to balance on steps or planks of wood at times, and we know from experience that some things help to keep you balanced and upright: don't lean away from the centre, stand up straight, keep your centre of gravity low. All the things they teach you in circus school. But those tightrope walkers always seem to carry very long poles in their hands. Sometimes the poles flop down at the ends because of their weight, sometimes they even have heavy buckets attached. Why do you think the funambulists do that?

The key idea you need to understand why the tightrope walker carries a long pole to aid balance is inertia. The larger your inertia, the slower you move when a force is applied. It has nothing to do with centre of gravity. The farther away from the centre that mass is distributed, the higher a body's inertia is, and the harder it is to move it. Take two spheres of different materials that have the same diameter and mass, one solid and one hollow, and it will be the hollow one with all its mass far away at its surface that will be slower to move or to roll down a slope. Similarly, carrying the long pole increases the tightrope walker's inertia by placing mass far away from the body's centre line – inertia has units of mass times distance squared. As a result, any small wobbles about the equilibrium position happen more slowly. They have a longer time period of oscillation, and the walker has more time to respond to the wobbles and restore his balance. Compare how much easier it is to balance a one-metre stick on your finger compared with a 10-centimetre one.

3

Monkey Business

I have a spelling chequer
It came with my pee sea
It plainly marques four my revue
Miss takes I cannot see

I've run this poem threw it
I'm shore yaw pleased to no
It's letter perfect in its weigh
My chequer told me sew . . .

Barri Haynes

The legendary image of an army of monkeys typing letters at random and eventually producing the works of Shakespeare seems to have emerged gradually over a long period of time. In *Gulliver's Travels*, written in 1726, Jonathan Swift tells of a mythical Professor of the Grand Academy of Lagado who aims to generate a catalogue of all scientific knowledge by having his students continuously generate random strings of letters by means of a mechanical printing device. The first mechanical typewriter had been patented in 1714. After several eighteenth- and nineteenth-century French mathematicians used the example of a great book being composed by a random deluge of letters from a printing works as an example of extreme improbability, the monkeys appear first in 1909, when the French mathematician Émile Borel suggested that randomly typing monkeys would eventually produce every book in France's Bibliothèque Nationale. Arthur Eddington took up the analogy in his famous book *The Nature of the Physical World* in 1928, where he anglicised the library: 'If I let my fingers wander idly over the keys of a typewriter it *might* happen that my screed made an intelligible sentence. If an army of monkeys were strumming on typewriters they *might* write all the books in the British Museum.'

Eventually this oft-repeated example picked the 'Complete Works of Shakespeare' as the prime candidate for random recreation. Intriguingly, there was a website that once simulated an ongoing random striking of typewriter keys and then did pattern searches against the 'Complete Works of Shakespeare' to identify matching character strings. This simulation of the monkeys' actions began on 1 July 2003 with 100 monkeys, and the population of monkeys was effectively doubled every few days until recently. In that time they produced more than 10^{35} pages, each requiring 2,000 keystrokes.

A running record was kept of daily and all-time record strings until the Monkey Shakespeare Simulator Project site stopped updating in 2007. The daily records are fairly stable, around the 18- or 19-character-string range, and the all-time record inches steadily upwards. For example, one of the 18-character strings that the monkeys have generated is contained in the snatch:

. . . Theseus. Now faire UWfIlaNWSK2d6L;wb . . .

The first 18 characters match part of an extract from *A Midsummer Night's Dream* that reads

. . . us. Now faire Hippolita, our nuptiall houre . . .

For a while the record string was 21-characters long, with

. . . KING. Let fame, that
wtIA’”yh!”VYONOvwsFOsbhzkLH . . .

which matches 21 letters from *Love’s Labour’s Lost*

> KING. Let fame, that all hunt after in their lives,
> Live regist’red upon our brazen tombs,
> And then grace us in the disgrace of death; . . .

In December 2004 the record reached 23 characters with

Poet. Good day Sir
FhlOiX5a]OM,MLGtUGSxX4IfeHQbktQ . . .

which matched part of *Timon of Athens*

> Poet. Good day Sir
> Pain. I am glad y’are well
> Poet. I haue not seene you long, how goes the World?
> Pain. It weares sir, as it growes . . .

By January 2005, after 2,737,850 million billion billion billion monkey-years of random typing, the record stretched to 24 characters, with

RUMOUR. Open your ears; 9r”5j5&?OWTY Z0d ‘B-nEoF.vjSqj[. . .

which matches 24 letters from *Henry IV Part 2*

> RUMOUR. Open your ears; for which of you will stop
> The vent of hearing when loud Rumour speaks?

Which all goes to show: it is just a matter of time!

4

Independence Day

> I read that there's about 1 chance in 1000 that someone will board an airplane carrying a bomb. So I started carrying a bomb with me on every flight I take; I figure the odds against two people having bombs are astronomical.
>
> Anon.

Independence Day, 4 July 1977 is a date I remember well. Besides being one of the hottest days in England for many years, it was the day of my doctoral thesis examination in Oxford. Independence, albeit of a slightly different sort, turned out to be of some importance because the first question the examiners asked me wasn't about cosmology, the subject of the thesis, at all. It was about statistics. One of the examiners had found 32 typographical errors in the thesis (these were the days before word-processors and schpel-chequers). The other had found 23. The question was: how many more might there be which neither of them had found? After a bit of checking pieces of paper, it turned out that 16 of the mistakes had been found by both of the examiners. Knowing this information, it is surprising that you can give an answer as long as you assume that the two examiners work independently of each other, so that the chance of one finding a mistake is not affected by whether or not the other examiner finds a mistake.

Let’s suppose the two examiners found A and B errors respectively and that they found C of them in common. Now assume that the first examiner has a probability a of detecting a mistake while the other has a probability b of detecting a mistake. If the total number of typographical errors in the thesis was T, then A = aT and B = bT. But if the two examiners are proofreading *independently* then we also know the key fact that C = abT. So AB = abT^2 = CT and so the total number of mistakes is T = AB/C, irrespective of the values of a and b. Since the total number of mistakes that the examiners found (noting that we mustn’t double-count the C mistakes that they both found) was A + B – C, this means that the total number that they didn’t spot is just T – (A + B – C) and this is (A – C)(B – C)/C. In other words, it’s the product of the number that each found that the other didn’t divided by the number they both found. This makes good sense. If both found lots of errors but none in common then they are not very good proofreaders and there are likely to be many more that neither of them found. In my thesis we had A = 32, B = 23, and C = 16, so the number of unfound errors was expected to be (16 × 7)/16 = 7.

This type of argument can be used in many situations. Suppose different oil prospectors search independently for oil pockets: how many might lie unfound? Or if ecologists want to know how many animal or bird species might there be in a region of forest if several observers do a 24-hour census.

A similar type of problem arose in literary analysis. In 1976 two Stanford statisticians used the same approach to estimate the size of William Shakespeare’s vocabulary by investigating the number of different words used in his works, taking into account multiple usages. Shakespeare wrote about 900,000 words in total. Of these, he uses 31,534 different words, of which 14,376 appear only once, 4,343 appear only twice and 2,292 appear only three times.

They predict that Shakespeare knew at least 35,000 words that are not used in his works: he probably had a total vocabulary of about 66,500 words. Surprisingly, you know about the same number.

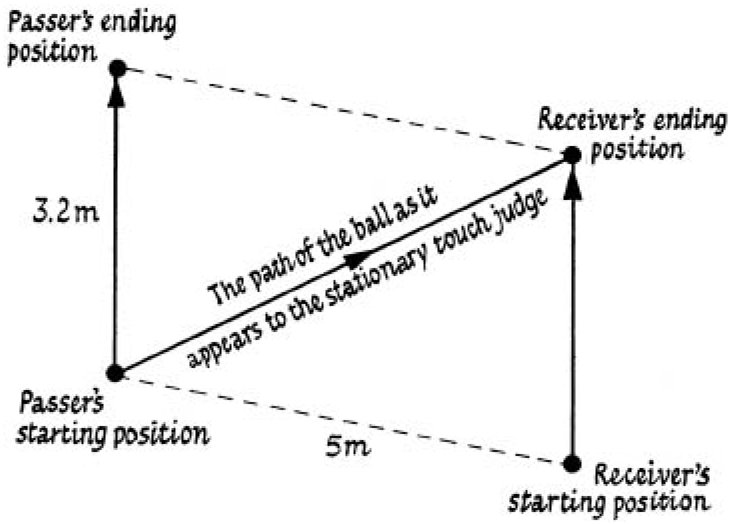
5

Rugby and Relativity

> Rugby football is a game I can’t claim absolutely to understand in all its niceties, if you know what I mean. I can follow the broad, general principles, of course. I mean to say, I know that the main scheme is to work the ball down the field somehow and deposit it over the line at the other end and that, in order to squalch this programme, each side is allowed to put in a certain amount of assault and battery and do things to its fellow man which, if done elsewhere, would result in 14 days without the option, coupled with some strong remarks from the Bench.
>
> P.G. Wodehouse, *Very Good, Jeeves*

Relativity of motion need not be a problem only for Einstein. Who has not had the experience of sitting in a stationary railway carriage at a station, then suddenly getting the sensation of being in motion, only to recognise that a train on the parallel track has just moved off in the other direction and your train is not moving at all?

Here is another example. Five years ago I spent two weeks visiting the University of New South Wales in Sydney during the time that the Rugby World Cup was dominating the news media and public interest. Watching several of these games on television I noticed an interesting problem of relativity that was unnoticed by the celebrities in the studio. What is a forward pass relative to? The written rules are clear: a forward pass occurs when the ball is thrown towards the opposing goal line. But when the players are moving the situation becomes more subtle for an observer to judge due to relativity of motion.



Imagine that two attacking players are running (up the page) in parallel straight lines 5 metres apart at a speed of 8 metres per sec towards their opponents' line. One player, the 'receiver', is a metre behind the other, the 'passer', who has the ball. The passer throws the ball at 10 metres per sec towards the receiver. The speed of the ball relative to the ground is actually $\sqrt{(10^2 + 8^2)} = 12.8$ metres per sec and it takes a time of 0.4 sec to travel the 5 metres between the players. During this interval the receiver has run a further distance of $8 \times 0.4 = 3.2$ metres. When the pass was thrown he was 1 metre behind the passer but when he catches the ball he is 2.2 metres in front of him from the point of view of a touch judge standing level with the original pass. He believes there has been a forward pass and waves his flag. But the referee is running alongside the play, doesn't see the ball go forwards, and so waves play on!

6

Wagons Roll

My heart is like a wheel.

Paul McCartney, ‘Let Me Roll It’

One weekend I noticed that the newspapers were discussing proposals to introduce more restrictive speed limits of 20 mph in built-up areas of the UK and to enforce them with speed cameras wherever possible. Matters of road safety aside, there are some interesting matters of rotational motion that suggest that speed cameras might end up catching large numbers of perplexed cyclists apparently exceeding the speed limit by significant factors. How so?

Suppose that a cycle is moving at speed V towards a speed detector. This means that a wheel hub or the body of the cyclist is moving with speed V with respect to the ground. But look more carefully at what is happening at different points of the spinning wheel. If the wheel doesn’t skid, then the speed of the point of the wheel in contact with the ground must be zero. If the wheel has radius R and is rotating with constant angular velocity Ω revolutions per second, then the speed of the contact point can also be written as $V - R\,\Omega$. This must be zero and therefore V equals $R\,\Omega$. The forward speed of the centre of the wheel is V, but the forward speed of the top of the wheel is the sum of V and the rotational speed. This equals $V + R\,\Omega$ and is therefore equal to 2V. If a camera determines the speed of an approaching or receding bicycle by measuring the speed of the top of the wheel, then it will register a speed twice as large as the cyclist is moving. An interesting one for m’learned friends perhaps, but I recommend you have a good pair of mudguards.

7

A Sense of Proportion

> You can only find truth with logic if you have already found truth without it.
>
> G.K. Chesterton

As you get bigger, you get stronger. We see all sorts of examples of the growth of strength with size in the world around us. The superior strength of heavier boxers, wrestlers and weightlifters is acknowledged by the need to grade competitions by the weight of the participants. But how fast does strength grow with increasing weight or size? Can it keep pace? After all, a small kitten can hold its spiky little tail bolt upright, yet its much bigger mother cannot: her tail bends over under its own weight.

Simple examples can be very illuminating. Take a short breadstick and snap it in half. Now do the same with a much longer one. If you grasped it at the same distance from the snapping point each time you will find that it is no harder to break the long stick than to break the short one. A little reflection shows why this should be so. The stick breaks along a slice through the breadstick. All the action happens there: a thin sheet of molecular bonds in the breadstick is broken and it snaps. The rest of the breadstick is irrelevant. If it were a hundred metres long it wouldn't make it any harder to break that thin slice of bonds at one point along its length. The strength of the