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Basic Math & Pre-Algebra All-in-One For Dummies®

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Introduction

Does math really need to be so hard? Nope.

I say this speaking as a guy who has struggled with math as much as, if not more than, you have. Believe me. And a big part of the struggle often has more to do with the lack of clarity in how math is explained than with the actual math.

This is too bad, because the whole idea behind math is supposed to be *clarity*. In a world where so many things are unclear, $2 + 2$ will always equal 4.

My second-greatest joy in teaching math is when a light breaks across a student's face as they suddenly understand something new. My greatest joy, though, is what often follows: a skepticism that it couldn't possibly be this easy.

When you approach math right, it's almost always easier than you think. And a lot of the stuff that hung you up when you first saw it probably isn't all that scary after all. Many students feel they got lost somewhere along the way on the road between learning to count to ten and their first day in an algebra class — and this may be true whether they're 14 or 104. If this is you, don't worry. You're not alone, and help is right here!

About This Book

This book brings together the four components you need to make sense of math:

- » Clear explanations of each topic
- » Example questions with step-by-step answers
- » Plenty of practice problems (with more available online!)
- » Chapter quizzes to test your knowledge at the end of most chapters

Although you can certainly work through this book from beginning to end, you don't have to. Feel free to jump directly to whatever chapter has the type of problems you want to practice. When you've worked through enough problems in a section to your satisfaction, feel free to jump to a different section. If you find the problems in a section too difficult, flip back to an earlier section or chapter to practice the skills you need — just follow the cross-references.

Foolish Assumptions

If you're planning to read this book, you likely fall into one of these categories:

- » A student who wants a solid understanding of the basics of math for a class or test you're taking
- » An adult who wants to improve skills in arithmetic, fractions, decimals, percentages, weights and measures, geometry, algebra, and so on for when you have to use math in the real world
- » Someone who wants a refresher so you can help another person understand math

My only assumption about your skill level is that you can add, subtract, multiply, and divide. So, to find out whether you're ready for this book, take this simple test:

$$4 + 7 = \underline{\quad} \quad 13 - 5 = \underline{\quad} \quad 9 \times 3 = \underline{\quad} \quad 35 \div 7 = \underline{\quad}$$

If you can answer these four questions correctly (the answers are 11, 8, 27, and 5), you're ready to begin.

Icons Used in This Book

You'll see the following five icons throughout the book:



EXAMPLE

Each example is a math question based on the discussion and explanation, followed by a solution. Work through these examples, and then refer to them to help you solve the practice problems that follow them, as well as the quiz questions at the end of the chapter.



REMEMBER

This icon points out important information that you need to focus on. Make sure you understand this information fully before moving on. You can skim through these icons when reading a chapter to make sure you remember the highlights.



TIP

This icon points out hints that can help speed you along when answering a question. You should find them useful when working on practice problems.



WARNING

This icon flags common mistakes that students make if they're not careful. Take note and proceed with caution!



YOUR
TURN

When you see this icon, it's time to put on your thinking cap and work out a few practice problems on your own. The answers and detailed solutions are available so you can feel confident about your progress.

Beyond the Book

In addition to the book you're reading right now, be sure to check out the free Cheat Sheet on Dummies.com. This handy Cheat Sheet covers some common “math demons” that students often stumble over. To access it, simply go to Dummies.com and type **Basic Math & Pre-Algebra All in One Cheat Sheet** in the Search box.

You'll also have access to online quizzes related to each chapter, starting with Chapter 3. These quizzes provide a whole new set of problems for practice and confidence-building. To access the quizzes, follow these simple steps:

- 1. Register your book or ebook at Dummies.com to get your PIN. Go to www.dummies.com/go/getaccess.**
- 2. Select your product from the drop-down list on that page.**
- 3. Follow the prompts to validate your product, and then check your email for a confirmation message that includes your PIN and instructions for logging in.**

If you do not receive this email within two hours, please check your spam folder before contacting us through our Technical Support website at <http://support.wiley.com> or by phone at 877-762-2974.

Now you're ready to go! You can come back to the practice material as often as you want — simply log on with the username and password you created during your initial login. No need to enter the access code a second time.

Your registration is good for one year from the day you activate your PIN.

Where to Go from Here

You can use this book in a variety of ways. If you're reading without immediate time pressure from a test or homework assignment, start at the beginning and keep going, chapter by chapter, to the end. If you do this, you'll be surprised by how much of the math you may have been dreading will be almost easy. Additionally, setting up some solid groundwork is a great way to prepare for what follows later in the book.

If your time is limited — especially if you're taking a math course and you're looking for help with your homework or an upcoming test — skip directly to the topic you're studying. Wherever you open the book, you can find a clear explanation of the topic at hand, as well as a variety of hints and tricks. Read through the examples and try to do them yourself, or use them as templates to help you with assigned problems.

1 Getting Started with Basic Math & Pre-Algebra

In This Unit . . .

CHAPTER 1: Playing the Numbers Game

Inventing Numbers
Understanding Number Sequences
Four Important Sets of Numbers

CHAPTER 2: The Big Four Operations

The Big Four Operations
Applying the Big Four Operations to Larger Numbers

IN THIS CHAPTER

- » Finding out how numbers were invented
- » Looking at a few familiar number sequences
- » Examining the number line
- » Understanding four important sets of numbers

Chapter 1

Playing the Numbers Game

One useful characteristic of numbers is that they're *conceptual*, which means that, in an important sense, they're all in your head. (This fact probably won't get you out of having to know about them, though — nice try!)

For example, you can picture three of anything: three cats, three baseballs, three tigers, three planets. But just try to picture the concept of three all by itself, and you find it's impossible. Oh, sure, you can picture the numeral 3, but *threeness* itself — much like love or beauty or honor — is beyond direct understanding. But when you understand the *concept* of three (or four, or a million), you have access to an incredibly powerful system for understanding the world: mathematics.

In this chapter, I give you a brief history of how numbers likely came into being. I discuss a few common *number sequences* and show you how these connect with simple math *operations* like addition, subtraction, multiplication, and division.

After that, I describe how some of these ideas come together with a simple yet powerful tool: the *number line*. I discuss how numbers are arranged on the number line, and I also show you how to use the number line as a calculator for simple arithmetic. Finally, I describe how the *counting numbers* (1, 2, 3, . . .) sparked the invention of more unusual types of numbers, such as *negative numbers*, *fractions*, and *irrational numbers*. I also show you how these *sets of numbers* are *nested* — that is, how one set of numbers fits inside another, which fits inside another.

Inventing Numbers

Historians believe that the first written number systems came into being at the same time as agriculture and commerce. Before that, people in prehistoric, hunter-gatherer societies were pretty much content to identify bunches of things as “a lot” or “a little.” They may have had concepts of small numbers, probably less than five or ten, but lacked a coherent way to think about, for example, the number 42.

Throughout the ages, the Babylonians, Egyptians, Greeks, Hindus, Romans, Mayans, Arabs, and Chinese (to name just a few) all developed their own systems of writing numbers.

Although Roman numerals gained wide currency as the Roman Empire expanded throughout Europe and parts of Asia and Africa, the more advanced system that was invented in India and adapted by the Arabs turned out to be more useful. Our own number system, the Hindu-Arabic numbers (also called decimal numbers), is mainly derived from these earlier number systems.

Understanding Number Sequences

Although humans invented numbers for counting commodities, as I explain in the preceding section, they soon put them to use in a wide range of applications. Numbers were useful for measuring distances, counting money, amassing armies, levying taxes, building pyramids, and lots more.

But beyond their many uses for understanding the external world, numbers have an internal order all their own. So numbers are not only an *invention*, but equally a *discovery*: a landscape reflecting fundamental truths about nature, and how humans think about it, that seems to exist independently, with its own structure, mysteries, and even perils.

One path into this new and often strange world is the *number sequence*: an arrangement of numbers according to a rule. In the following sections, I introduce you to a variety of number sequences that are useful for making sense of numbers.

Evening the odds

One of the first facts you probably heard about numbers is that all of them are either even or odd. For example, you can split an even number of marbles *evenly* into two equal piles. But when you try to divide an odd number of marbles the same way, you always have one *odd*, left-over marble. Here are the first few even numbers:

2 4 6 8 10 12 14 16 ...

You can easily keep the sequence of even numbers going as long as you like. Starting with the number 2, keep adding 2 to get the next number.

Similarly, here are the first few odd numbers:

1 3 5 7 9 11 13 15...

The sequence of odd numbers is just as simple to generate. Starting with the number 1, keep adding 2 to get the next number.

Patterns of even or odd numbers are the simplest number patterns around, which is why kids often figure out the difference between even and odd numbers soon after learning to count.

Counting by threes, fours, fives, and so on

When you get used to the concept of counting by numbers greater than 1, you can run with it. For example, here's what counting by threes, fours, and fives looks like:

Threes: 3 6 9 12 15 18 21 24 ...
Fours: 4 8 12 16 20 24 28 32 ...
Fives: 5 10 15 20 25 30 35 40 ...



TIP

Counting by a given number is a good way to begin learning the multiplication table for that number, especially for the numbers you're kind of sketchy on. (In general, people seem to have the most trouble multiplying by 7, but 8 and 9 are also unpopular.)

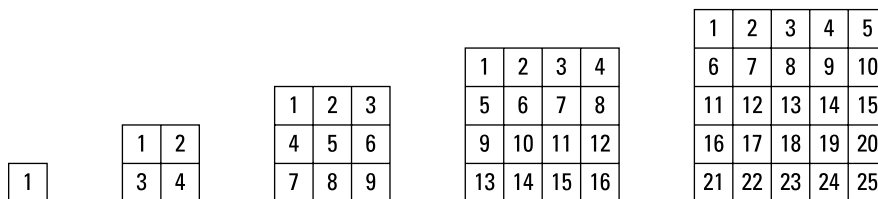
These types of sequences are also useful for understanding factors and multiples, which you get a look at in Chapter 9.

Getting square with square numbers

When you study math, sooner or later, you probably want to use visual aids to help you see what numbers are telling you. (Later in this book, I show you how one picture can be worth a thousand numbers when I discuss geometry in Chapter 19 and graphing in Chapter 25.)

The tastiest visual aids you'll ever find are those little square cheese-flavored crackers. (You probably have a box sitting somewhere in the pantry. If not, saltine crackers or any other square food works just as well.) Shake a bunch out of a box and place the little squares together to make bigger squares. Figure 1-1 shows the first few.

FIGURE 1-1:
Square
numbers.



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Voilà! The square numbers:

1 4 9 16 25 36 49 64 ...



TIP

You get a *square number* by multiplying a number by itself, so knowing the square numbers is another handy way to remember part of the multiplication table. Although you probably remember without help that $2 \times 2 = 4$, you may be sketchy on some of the higher numbers, such as $7 \times 7 = 49$. Knowing the square numbers gives you another way to etch that multiplication table forever into your brain.

Square numbers are also a great first step on the way to understanding exponents, which I introduce later in this chapter and explain in more detail in Chapter 5.

Composing yourself with composite numbers

Some numbers can be placed in rectangular patterns. Mathematicians probably should call numbers like these “rectangular numbers,” but instead they chose the term *composite numbers*. For example, 12 is a composite number because you can place 12 objects in rectangles of two different shapes, as in Figure 1-2.

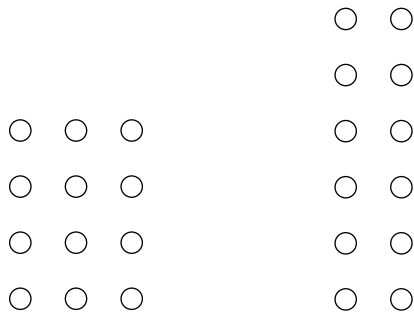


FIGURE 1-2:
The number 12
laid out in two
rectangular
patterns.

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As with square numbers, arranging numbers in visual patterns like this tells you something about how multiplication works. In this case, by counting the sides of both rectangles, you find out the following:

$$3 \times 4 = 12$$

$$2 \times 6 = 12$$

Similarly, other numbers such as 8 and 15 can also be arranged in rectangles, as in Figure 1-3.

As you can see, both these numbers are quite happy being placed in boxes with at least two rows and two columns. And these visual patterns show this:

$$2 \times 4 = 8$$

$$3 \times 5 = 15$$

FIGURE 1-3:
Composite numbers, such as 8 and 15, can form rectangles.



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The word *composite* means that these numbers are *composed of* smaller numbers. For example, the number 15 is composed of 3 and 5 — that is, when you multiply these two smaller numbers, you get 15. Here are all the composite numbers from 1 to 16:

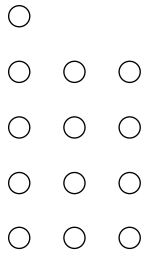
4 6 8 9 10 12 14 15 16

Notice that all the square numbers (see the section, “Getting square with square numbers”) also count as composite numbers because you can arrange them in boxes with at least two rows and two columns. Additionally, a lot of other non-square numbers are also composite numbers.

Stepping out of the box with prime numbers

Some numbers are stubborn. Like certain people you may know, these numbers — called *prime numbers* — resist being placed in any sort of a box. Look at how Figure 1-4 depicts the number 13, for example.

FIGURE 1-4:
Unlucky 13, a prime example of a number that refuses to fit in a box.



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Try as you may, you just can’t make a rectangle out of 13 objects. (That fact may be one reason why the number 13 got a bad reputation as unlucky.) Here are all the prime numbers less than 20:

2 3 5 7 11 13 17 19

As you can see, the list of prime numbers fills the gaps left by the composite numbers (see the preceding section). Therefore, every counting number is either prime or composite. The only exception is the number 1, which is neither prime nor composite. In Chapter 8, I give you a lot more information about prime numbers and show you how to *decompose* a number — that is, break down a composite number into its prime factors.

Multiplying quickly with exponents

Here's an old question whose answer may surprise you: Suppose you took a job that paid you just 1 penny the first day, 2 pennies the second day, 4 pennies the third day, and so on, doubling the amount every day, like this:

1 2 4 8 16 32 64 128 256 512 ...

As you can see, in the first ten days of work, you would've earned a little more than \$10 (actually, \$10.23 — but who's counting?). How much would you earn in 30 days? Your answer may well be, "I wouldn't take a lousy job like that in the first place." At first glance, this looks like a good answer, but here's a glimpse at your second ten days' earnings:

... 1,024 2,048 4,096 8,192 16,384 32,768 65,536 131,072 262,144 524,288 ...

By the end of the second 10 days, when you add it all up, your total earnings would be over \$10,000. And by the end of 30 days, your earnings would top out around \$10,000,000! How does this happen? Through the magic of exponents (also called *powers*). Each new number in the sequence is obtained by multiplying the previous number by 2:

$$2^1 = 2$$

$$2^2 = 2 \times 2 = 4$$

$$2^3 = 2 \times 2 \times 2 = 8$$

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

As you can see, the notation 2^4 means *multiply 2 by itself 4 times*.

You can use exponents on numbers other than 2. Here's another sequence you may be familiar with:

1 10 100 1,000 10,000 100,000 1,000,000 ...

In this sequence, every number is 10 times greater than the number before it. You can also generate these numbers using exponents:

$$10^1 = 10$$

$$10^2 = 10 \times 10 = 100$$

$$10^3 = 10 \times 10 \times 10 = 1,000$$

$$10^4 = 10 \times 10 \times 10 \times 10 = 10,000$$



REMEMBER

This sequence is important for defining *place value*, the basis of the decimal number system, which I discuss in Chapter 3. It also shows up when I discuss decimals in Chapter 13 and scientific notation in Chapter 17. You find out more about exponents in Chapter 5.