Manfred J. Holler Barbara Klose-Ullmann

Scissors and Rock

Game Theory for Those Who Manage







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Preface: Introduction and Warnings

Analytical statistics claim that there are two ways to make wrong decisions: A correct hypothesis is rejected or, alternatively, an incorrect hypothesis is accepted. In this book, you will learn about a third type of wrong decision and how to handle it. The essence of this type of failure is that decision makers either ignore that the results of their decisions depend on the decisions of others or that they cannot deal with this interdependency. The reason for the latter could be the complexity of the decision situation. However, it could also be the result of a lack of tools. Game theory is such a tool. It helps to understand the complexity of research decisions, and in many cases, it filters out inadequate decisions. International politics, parlor games like Chess, and the schoolvard game Rock-Scissors-Paper exhibit decision situations in which the results of decision making depend on the choice of more than one decision maker. The managing of game theory can support the managing of decision situations when decisions are interdependent and strategic reasoning is required, i.e., putting oneself into the shoes of the other. It is also of help in the designing and redesigning of decision situations, i.e., "changing the game," known more formally as mechanism design. The design of auctions is just one example; the writing of a constitution is another one. Obviously, mechanism design is an important instrument for politicians and business managers. However, it is also relevant for everybody who manages decision situations—which includes most of us. Game theory is the key. This is the focus of the present book.

The book has three major heroes: Niccolò Machiavelli, Adam Smith, and George Washington. In fact, Washington accomplished what Adam Smith suggested in the last page of his Wealth of Nations:

"If any of the provinces of the British empire cannot be made to contribute toward the support of the whole empire, it is surely time that Great Britain should free herself from the expence of defending those provinces in time of war, and of supporting any part of their civil or military establishments in time of peace, and accommodate her future views and design to the real mediocrity of her circumstances" (Smith 1981[1776/77]: 947).

King George III and his government did not follow Smith's recommendation, and much of the American colonies became independent after the War of Independence. We will not discuss George Washington any further in this book, but he is our prototype of "the man who managed." Much of what follows can be applied to his life and career.

In 1740, Voltaire arranged for the publication "The Refutation of Machiavelli's Prince or Anti-Machiavel" written by Frederick of Prussia, probably the most prominent Machiavelli critic. Prince Royal Frederick developed a model of an enlightened prince who considered himself a "first servant" to his State and a reliable agent in the interplay with fellow princes. However, when, in 1740, he succeeded his father as King of Prussia, his actual behavior was heavily influenced by the recipes suggested in Machiavelli's *Il Principe*. He may have been Machiavelli's most successful student and ardent follower.

There are a number of other heroes in this book: Johann Wolfgang von Goethe who applied the Vickrey's auction scheme when selling his manuscript of "Hermann and Dorothea" to the publisher Hans Friedrich Vieweg in Berlin; Joseph Heller and Peter Handke who contributed "strategic inspiration" by their novels Catch-22 and The Goalie's Anxiety at the Penalty Kick, respectively; the widely quoted Chinese military strategist Sun Tzu who suggested that "to a surrounded enemy you must leave a way of escape"; Napoleon who studied Machiavelli's *Il Principe* and sent his troops to Moscow where they died of hunger and cold; Émile Borel who was possibly the first to define the game of strategy "in which the winnings depend simultaneously on chance and the skill of the player"—who also proposed a thought experiment that entered popular culture under the name "infinite monkey theorem"; John von Neumann who proved the Minimax Theorem and thereby initiated the birth of game theory; and, of course, John Nash whose outstanding contributions to game theory not only earned him a Nobel Prize but also triggered a biography and, most prominently, a movie with the title Beautiful Mind. There is a long list of Nobel Prize winners who have been celebrated because of their work in game theory, and there is even

a much longer list of scholars who contributed to game theory's development and application—and thus to its popularity.

Specifically, this book is about strategic mistakes and how to avoid them. A first technique is: We have to think strategically; the right approach would be using game theory, the *theory of strategic thinking* in order to get a better understanding of the decision situation. However, to apply game theory, you have to learn it. This book will give you a well-structured introduction in game-theoretical thinking and basic methods and concepts. Every decision maker should study the basic concepts offered in this book. They are highly relevant not only in cases of *conflict* but also in cases of *cooperation*—and of *coordination problems*.

A better understanding of strategic problems and knowledge of possible solutions is extremely important to identify social or political conflicts, irrespective of whether the conflicts are between nations or family members, and to avoid them. While the German version of this book ("Spieltheorie für Manager"¹) focused on introducing game theory as a tool kit for solving strategic decision problems, the present version emphasizes the role of game theory as a means to identify the complexity of decision situations and to thereby obtain a better understanding of the world we live in and of the decisions we have to make. Of course, the latter does not exclude learning about tools which help to solve problems that involve strategic thinking. Needless to say: This book is not a literal translation of the German version.

In his *The Picture of Dorian Gray*, Oscar Wilde (1997[1890]: 30) characterized Sir Thomas Bordon, a radical member of the Parliament, with the notorious observation: "Like all people who try to exhaust a subject, he exhausted his listeners." In this book, we do not want to exhaust the subject as we do not want to exhaust our readers. We are sure that readers with some knowledge of game theory can easily find important issues that are missing in our text. We strongly suggest to the advanced reader studying what is offered here and then to verify whether he or she has learned something from it. However, readers who have so far been protected against game theory can sit down, enjoy the text, and get nervous about the thought experiments with which they will be confronted. Unfortunately, you have to *manage* game theory when you want to apply it.

¹Holler and Klose-Ullmann (2007), Spieltheorie für Manager: Handbuch für Strategen, 2nd edition, Munich: Verlag Franz Vahlen. Material in the present book also derives from Holler et al. (2019), Einführung in die Spieltheorie, 8th edition, Berlin: SpringerGabler.

Of course, we will not conclude our preface without illustrating the concept of strategic thinking and give an explanation to the title of this book: "Scissors and Rock." It is not unlikely that you played Rock-Scissors-Paper² in the schoolyard. It is a two-person game played with hands. Players have to choose whether they want to show a closed fist, representing a "rock"; a fist with two fingers sticking out forming a V, representing "scissors"; or a flat hand, representing "paper." The "rock" spoils the "scissors"; the "scissors" cut the "paper"; the "paper" wraps the "rock." Each alternative has the potential to "beat" another one, but is in danger of being defeated by a third alternative. These relations define losing and winning. If players choose identical alternatives, the particular round ends in a draw.

What alternative will you choose if choices are simultaneous and you want to win? Of course, if you find out that your opponent chooses "paper" more often than the two other alternatives, you will choose "scissors" more often than "paper" or "rock." If you decide to choose "scissors" all the time, the opponent will realize his own bias and perhaps switch to "rock" more often than you expected.

If you do not want to be exploited by your opponent, try to choose all three alternatives with equal probability. (The strategic decision problem is rather similar to the Penalty-kick game analyzed in Sect. 10.8.) In the equilibrium, both players choose each of the three alternatives with probability one-third. But there is Clever Mary who invites Sweet Paul to choose his alternative first and then she will choose hers. This is how "Scissors and Rock" prevailed. In fact, no matter what Sweet Paul chooses, Clever Mary always has a winning alternative—obviously, there is a second-mover advantage if the game is played sequentially. This is the reason why we see this game played simultaneously in schoolyards. Outside of schoolyards, again and again, decision makers try to slip into the role of Clever Mary and invite a Sweet Paul for a first move. It is not always a case of politeness, if somebody invites you to go first.

The example shows the possible power inherent in designing a game. A second move is not always to the disadvantage of the first mover. If Sweet Paul succeeds to reduce the set of alternatives to two elements, e.g., Scissor and Rock, then there is a first-mover advantage. Paul will choose Rock and win. If, different from Rock-Scissors-Paper, the game does not contain conflicting interests, a *sequential structure* may help to choose a successful solution to a coordination problem and implement an efficient outcome. Then,

²For details, see Sect. 10.9.

in general, the order of moves does not matter and—irrespective of whether a third or fourth move may exist—a cooperative outcome prevails.

Given this, we would like to thank Gregor Berz, Andreas Diekmann, Gudrun Keintzel-Schön, Norbert Leudemann, Hannu Nurmi, Florian Rupp, and Ernst Strouhal for their valuable support—and the inspiration which we received from them. We are grateful to Raymond Russ at the University of Maine who read the complete text and made very valuable propositions. Of course, many others inspired us while writing this text. Thank you very much!

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1

Playing for Susan

In the town hall of the German city of Augsburg, founded originally as Augusta Vindelicorum in the year 15 BC,¹ the ceiling of the central hall is decorated with a painting that shows Sapientia, the goddess of wisdom, in the center seated on a throne. A banner next to her, carried by some vassals, announces "per me reges regnant"—loosely translated, "it is through me that the kings rule." This book will demonstrate that it is not always easy to accomplish what Sapientia suggests. We will learn about the limits of her suggestions, but we will also see that the knowledge of game theory can extend the domain of Ratio, the enlightened companion of Sapientia.

In general, there are several competing, more or less convincing stories that explain an event, an outcome, or a fact—whether they are of today or of 500 years ago. Of course, we want to know why, say, a particular result prevailed, and how. What are the forces that produced this result, and not another? We want to learn from the story either to satisfy our natural curiosity or to avoid failures in our future actions. In fact, curiosity supports the learning of tools to avoid the traps waiting for us. Curious people can handle surprises much better than those who know all they want to know.

¹Norbert Leudemann informed us that the original name of Augsburg is "Augusta Vindelicum." In 15 BC, it was an army camp while the first civil settlement dated to 40 AC. The official name of the provincial capital was "Municipium Aelium Augustum," abbreviated as "Aelia Augusta."

1.1 Thinking Strategically

After reading Adam Smith's "The History of Astronomy," an article which comes as a surprise itself (Smith 1982 [1758]), we realize how dangerous surprises can be. The message is: We are involved in research and try to understand things in order to minimize surprises. Thinking strategically, putting oneself into the shoes of the other, helps to understand social interaction and resulting social situations. For many such situations, a reliable theory and the understanding that derives from it reduce the likelihood of surprises.

If decision making is strategic, then, typically, we can only hypothesize about the motivation, information, and reasoning producing the results that we see and want to explain to ourselves and, perhaps, to others. In the standard case, each decision maker can select one action only from a large set of alternatives without knowing what other decision makers will choose now or in the future, or, quite often, what they have chosen in the past. However, these choices specify the outcome that our decision maker wants to determine as he is likely to suffer or benefit from them. Sometimes we see the choices, and not the alternatives. Often, we only see the outcome—and nothing else—and we have to guess the choices and actions that caused it as well as those involved in the decision, and their motivations. Of course, in these cases, we have to seek refuge in very strong hypotheses about human behavior; typically, this entails the rationality hypothesis and some degree of selfishness that characterizes the homo economicus, which have become the trademark of modern microeconomics and of the sciences invaded by it: sociology, philosophy, psychology, etc.

In general, rationality and selfishness have to be further qualified to allow for deducting an explanation. In his "Essays: Moral, Political and Literary," David Hume (1985 [1777]: 42) recommended that "in convincing any system of government...every man ought to be supposed to be a knave and to have no other end, in all his actions, than private interest. By this interest we must govern him, and, by means of it, make him, notwithstanding his insatiable avarice and ambition, co-operate to public good." Are all men and women knaves or does this quotation merely imply that a successful government should be based on this assumption? Shall we imitate the government?

As for the government, according to Machiavelli (1952 [1532]: 92), it is

laudable...for a prince to keep faith and live with integrity, and not with astuteness, everyone knows. Still the experience of our times shows those princes to have done great things who have little regard for good faith, and

have been able by astuteness to confuse men's brains, and who have ultimately overcome those who made loyalty their foundation." He observes that for the prince "it is well to seem merciful, faithful, humane, sincere, religious, and also to be so," but the prince must have the mind so disposed that when it is needful to be otherwise you may be able to change to the opposite qualities," concluding that "it is not, therefore, necessary for a prince to have all the above-named qualities, but it is very necessary to seem to have them (Machiavelli 1952 [1532]: 93).

The shaping of expectations is essential to Machiavelli, even when it comes to architecture. How to build a fortress? In his *The Art of War*, he writes that he "would make the walls strong, and ditches...that everyone should understand that if the walls and the ditch were lost, the entire fortress would be lost" (Machiavelli 1882 [1521], Seventh Book). In the first step, it seems that walls have to be strong and enforced by ditches in order to motivate the spirit of those defending the fortress behind the walls. The next step, in Machiavelli's reasoning, is that those who attack strong walls have to expect a spirit of defense. But this spirit was sometimes lacking, and, as Machiavelli observed, people relied on strong walls and reduced their efforts of defense. Therefore, strong walls were not an unambiguous signal and a reliable solution for keeping the enemy away, as Machiavelli himself noted (Machiavelli 1882 [1521], Seventh Book).

A game-theoretical analysis could help to clarify this case. The history of game theory tells us that its success is, to a large extent, the result of its application to war and war-like situations. But if you are a pacifist, do not stop reading here. Strategic thinking is ubiquitous: It is an essential ingredient of "love and fear," but also of less dramatic core functions of life such as consumption. A large share of consumption is directed not to pleasure and satisfaction, but to create "social distance" by impressing others. In *The Theory of the Leisure Class*, Thorstein Veblen's world showed us an elite citizenry engaged in conspicuous consumption and honorific expenditures in search of pecuniary decency. A means to achieve this goal was to invest in delicate women, racing horses, and subduing dogs—and in Renaissance Art. The latter was thought to be most prestigious when it was transferred at large sums from an old English castle, owned by a semi-bankrupt lord, with the help of the most prestigious art dealer Joseph Duveen, who became himself a lord toward the end of his life.

We told this story in detail in our "Art Goes America" article (Holler and Klose-Ullmann 2010). In order to create and satisfy standards of excellence, to capture a shadow of aristocracy, and to impress their fellow citizens,

the American leisure class tried to imitate their British upper-class models. Veblen (1979 [1899]: 145) observed that the "English seat, and the peculiarly stressing gait which has made an awkward seat necessary, are a survival from the time when the English roads were so bad with mire and mud as to be virtually impassable for a horse traveling at a more comfortable gait; so that a person of decorous tastes in horsemanship to-day rides a punch with a cocked tail, in an uncomfortable posture and a distressing gait."

In art and architecture, American rusticity was not yet popular among the rich when Veblen published his leisure-class book in 1899. The rich may still try to buy a Raffaello out of some lord's castle. But soon they will demonstrate that, without social discounting, they can afford to show nineteenth-century American landscape painting of the Hudson River School, a group of artists around Thomas Cole and his student Frederic Edwin Church, in their prairie house homes. Of course, this counter-snobbery was meant to impress the snobs (Steiner and Weiss 1951), but it made identification rather complex as long as American paintings were at a low price and the butcher could buy them as well. Fortunately, due to the additional demand, prices went up. Consequently, counter-snobbery had to find new ways to manifest itself.

1.2 Why not Learn Game Theory?

As already said in the Introduction, we are sure that readers with some knowledge in game theory can easily find important issues that are missing in our text. However, readers who have so far been fully protected against game theory can sit down, enjoy the text, and get nervous with the thought experiments with which they are confronted. We strongly suggest that readers after having studied what is offered here ask themselves whether they have learned something from it—something that gives insights, something they can apply. Unfortunately, you have to learn game theory when you want to apply it. In general, it does not pay to hire a game theorist to do the job of strategic decision making for you. He or she does not know how much you like to win the battle and how strong your battleships, i.e., your resources, are. It is quite likely that, on the one hand, you cannot express your preferences and, on the other, you want to keep information concerning your resources as a secret. However, both items, your evaluations and your resources, are extremely important to model a game situation and to find a solution.

More specifically, let's put ourselves into the shoes of the head of the sales department of a large company who wants to apply game theory to outsmart the competitor. We were told that we have to know game theory if we want to apply it. This statement appears trivial at first sight. However, reflecting on the activities of the sales manager, it becomes evident that he² uses many skills in which he has not been formally trained—and which thus can hardly be reconstructed by an outsider. He continuously adopts results from the analyses of others, confiding in their reflections without being familiar with their principles. Why is it so?

Many management skills are almost impossible to learn. A great number of those skills are based on intuition or they are the result of a socially evolutionary process. For instance, future managers who conform to a certain behavioral codex have better prospects of attaining an executive position within a company than those candidates who show behavior that deviates from this codex and, therefore, are less successful in the given business culture. On the other hand, there are various problems which the manager expects to solve with the help of experts without understanding the methods applied in detail. Think about operation research analysis or the application of econometric models. If the manager applied identical methods and based his work on identical data, he would achieve the same results as the expert, although probably with a greater effort. This is likely to hold, e.g., for the prediction of economic growth or of the development of interest and exchange rates. However, this does not apply to forecasting the effect of a price reduction that a manager envisions for his company, especially if the company operates in a market with one or just a few competitors. Under these circumstances, decision making is, in general, much too complex to use an analytical (numeric) approach—not because of a shortage of data but due to the small number of competitors. In this instance, decision making is of a strategic nature: Competitors are likely to react to price reductions. But how do they react?

Game theory could provide an answer if the decision maker could interpret the market correctly. Therefore, the manager should not leave the application of game theory to a third party, although there are cases which can be molded in a more general framework. In principle, the manager must

 $^{^2}$ Not all managers are men as exemplified by one of the authors. We apologize exclusively using "he" in this text.

evaluate the market conditions himself and do his own analysis. Knowing game theory can be of help—especially when you have to explain your decision to others.

1.3 The Working of the Invisible Hand

In what follows, we illustrate the need for strategic reasoning of the manager using an example of a toy store which is meant to capture the stylized facts of some real-world markets. In the case of only two suppliers on the market, A and B, the effect of a price reduction by firm A is determined by the behavior of rival firm B and the demand of the customers. Thus, its effect depends on how B reacts to A's decision to reduce the price. The objective of A's price cut is to increase the demand for its own product. However, if the competitor reduces the price as well, the possible price cut effect is likely to be undermined. As a consequence, sales will not increase as much as expected and profits may even decrease. The decision on the price reduction by A will therefore be determined by A's expectation of B's reaction. B's reaction in turn will be affected by B's expectation of A's reaction. In order for A to predict B's reaction, A must take B's expectation in relation to its own behavior into account. Thus, B's expectation will depend on the expectations formed by both companies, A and B. The structure of this dependency is extremely complex.

As a result, both companies will face a severe problem. The managers have to develop some idea of the competitor's prices if they want to maximize their profits or to achieve a related goal (e.g., revenue maximization or increasing market shares). Moreover, in general, there is uncertainty about how the buyers will react to prices per se and also whether there is a potential entrant to the market. Let's abstract from such intricacies for the moment and use our toy example. For further simplification, we assume that the two suppliers to the market have just two modes of behavior: to choose a high or a low price. In the language of game theory, the modes of behavior are called strategies: They label the set of plans from which the decision makers can choose. As the decision situation is characterized by strategic interaction inasmuch as the outcome depends on the choices of both agents and, as assumed, the two agents know about it, it constitutes a *game situation*. As a consequence, the decision makers can be viewed as players.

| Matrix 1.1 | 1 The | competitive | trap |
|------------|-------|-------------|------|
|------------|-------|-------------|------|

| Player B A | high | Low |
|------------------|------------|-----------|
| high | (800,200) | (250,300) |
| low | (1500,100) | (500,150) |

The strategic interaction is obvious when we look at its representation by means of Matrix 1.1. If both players, A and B, choose strategy "high," then the matrix says that A and B will achieve payoffs of 800 and 200, respectively. In principle, the payoff numbers represent utility values, but for the given example profits seem to be good proxies. If player A chooses "high" and player B chooses "low," then the profits are represented by the payoff pair (250, 300). Is it better to choose low prices? If both sellers choose "low," the corresponding payoff pair is (500, 150). Obviously, it is not profitable for A to choose "high" when B chooses "low." Is it profitable for A to choose "high" when B chooses "high"? No! Irrespective of whether B chooses "high" or "low," it is always better for A to choose "low." The strategy "low" is a strictly dominant strategy for A. By a similar reasoning, we will find out that "low" is also a strictly dominant strategy for B: Irrespective of which strategy A chooses, it is always better for B to choose "low" instead of "high." To answer the question above, it seems that is better to choose low prices instead of high prices.

But is this answer correct? If both players choose "high," the payoffs are (800, 200), whereas if they choose "low" payoffs are (500, 150). Obviously, "low" prices are not profitable as both sellers are better off by choosing high prices. But above we have argued that low prices represent strictly dominant strategies for each player; that is, they are preferable irrespective of what the other player chooses. It seems that our players are trapped in a contradiction. Can we help them?

Matrix 1.1 does not illustrate a logical *contradiction*, but a trap called *competition*. The fact that payoffs (800, 200) result, if both players choose "high," is only of anecdotal value for an individual player, if he is solely interested in maximizing his own payoff. The latter objective suggests that he should choose his strictly dominant strategy: This is the *individual*

rational mode of behavior for both players in the decision situation described by Matrix 1.1; it results in the payoff pair (500, 150). This behavior is in conflict with collectively rational behavior—also labeled Pareto efficient behavior—that leads to the payoff pair (800, 200). But note that this behavior and its outcome are only efficient with respect to the sellers. From the point of view of the buyers, we should be happy about the low prices that result from the individual rational behavior of the sellers.

The game described by Matrix 1.1 constitutes a *Prisoners' Dilemma*—the most popular decision situation in game theory. In Chap. 3, we will learn why this game carries this name. It reflects a conflict between *individual rational behavior* and social efficiency (or *collective rationality*). But as the story goes, we should not feel sorry if the *invisible hand* of the competition works and drives the prices down. The merits of the invisible hand were already quoted by Adam Smith in his "Inquiry into the Nature and Causes of the Wealth of Nations," first published in 1776/77. In Book IV, Chapter II, we read that, in general, every individual

...neither intends to promote the public interest, nor knows how much he is promoting it. By preferring the support of domestic to that of foreign industry, he intends only his own security; and by directing that industry in such a manner as its produce may be of the greatest value, he intends only his own gain, and he is in this, as in many other cases, led by an invisible hand to promote an end which was no part of his intention. Nor is it always the worse for the society that it was no part of it. By pursuing his own interest he frequently promotes that of the society more effectually then when he really intends to promote it (Smith 1981 [1976/1977]: 456).

Please note the word "frequently." Adam Smith was quite aware that the invisible hand does not always work, because of cartels, or did not work properly because of institutional shortcomings—see his discussion of the banking sector—and the potential of free-riding in the provision of public goods. The emergence of externalities is another factor that makes the invisible hand tremble. From a game-theoretical point of view cartels are perhaps the most interesting handicap that hinders the successful working of the invisible hand. Adam Smith is very explicit that such cartels exist, for instance, on the labor market where wages depend on contracts, the parties' "...interests are by no means the same. The workmen desire to get as much, the masters to give as little as possible." Given this rather plain observation, Adam Smith (1981 [1776/1777], p 83) concludes, "The former are disposed to combine in order to raise, the latter in order to lower the wages of labour." And he goes on to reason:

It is not, however, difficult to foresee which of the two parties must, upon all ordinary occasions, have the advantage in the dispute, and force the other into a compliance with their terms. The masters, being fewer in number, can combine much more easily; and the law, besides, authorises, or at least does not prohibit their combinations, while it prohibits those of the workmen. We have no acts of parliament against combining to lower the price of work; but many against combining to raise it. In all such disputes the masters can hold out much longer. A landlord, a farmer, a master manufacturer, or merchant, though they did not employ a single workman, could generally live a year or two upon the stocks which they have already acquired. Many workmen could not subsist a week, few could subsist a month, and scarce any a year without employment. In the long—run the workman may be as necessary to his master as his master is to him; but the necessity is not so immediate (Smith 1981 [1776/77]: 83f).

But do these combinations really form? It seems that, in general, they are not made public and Adam Smith had to convince his readership that such combinations exist.

We rarely hear, it has been said, of the combinations of masters; though frequently of those of workmen. But whoever imagines, upon this account, that masters rarely combine, is as ignorant of the world as of the subject. Masters are always and every where in a sort of tacit, but constant and uniform combination, not to raise the wages of labour above their actual rate. To violate this combination is every where a most unpopular action, and a sort of reproach to a master among his neighbours and equals (Smith 1981 [1776/77]: 84).

Here, we have some interesting observations which we discuss in detail in Chap. 9: Agreements can be tacit and enforced by social and perhaps economic pressure.

The decision situation described by Adam Smith seems to imply a *Prisoners' Dilemma* with respect to the cooperation of the masters, as an individual master that deviates from the tacit contract could benefit by paying higher wages and thereby attracting better skilled "workmen"—if there were not the threat "of reproach to a master among his neighbours and equals." The relationship between the masters and the workmen constitutes a multi-person bargaining game which is, however, reduced to a market situation that has one agent, the "combination of masters," representing demand and many individual workmen on the other supply side as, by law, workmen were not allowed to collude. Economists call such market situation demand-side monopoly or, in a more sophisticated manner, monopsony.

In the modern language of game theory, combinations are called coalitions. They describe situations of conflict and coordination and are especially relevant for games with more than two players. In the course of this book, we will learn how they emerge and how the coalition surplus will be shared between its members.

1.4 The Real World and Its Models

From the interpretation of Matrix 1.1, we learned that a two-person Prisoners' Dilemma game is characterized by two features:

- (a) The two players have *strictly dominant strategies*, i.e., each player has a best strategy irrespective of the strategy choice of the other players.
- (b) The result, determined by the equilibrium in dominant strategies, is *socially inefficient* with respect to the players inasmuch as both players are better off if they either find a mode of cooperation or if cooperation is forced upon them.

Do such decision situations exist? Probably not in the abstract form as summarized by (a) and (b)! However, starting from the toy model described by Matrix 1.1, we can think of two gas stations that are close to each other on the same side of a highway. Their products are hardly differentiated. As a consequence, buyers will steer their car to the gas station with the lower price if prices differ. Similarly, many customers do not think that there is a quality difference between Coca-Cola and Pepsi Cola and buy the cheaper one if there is a choice at all. Often, the store decided already for the customer and offers either Coca-Cola or Pepsi Cola, but not both of them. Of course, the reasoning of the store manager is much more complicated because for him, in general, variables other than prices are relevant as well. Although there might be only negligible differences in the taste of the two drinks, the two suppliers can have very different marketing strategies directed to store managers that lead to a degree of monopolization inasmuch as a particular store only offers the brand that seems favorable to the manager.

To get an understanding of such more complex cases, let us describe the decision problem in way typical for a game-theoretical analysis. Let's assume we are one of the players and face a strategic decision situation. In order to manage such a situation, we have two basic concerns: (a) to find an adequate

description of the situation and (b) to find a solution to our decision problem. There are three steps to help us in this project.

Step One: Identification of a decision situation as a game-theoretical problem. A decision situation is strategic if (a) the outcome is the result of the decisions of more than one decision maker, (b) each decision maker is aware of this interdependency, (c) each decision maker assumes that the other decision makers are aware of this interdependency, and (d) each decision maker takes (a), (b), and (c) into consideration.

Of course, this only makes sense if the number of players is small such that the interdependency can be considered as relevant and being handled accordingly. However, what is a small number? In a way, this is defined by our behavior in such a decision situation. If we take (a), (b), (c), and (d) into consideration, then we think that the number of agents is small enough—and we see ourselves in a *strategic decision situation*.

Step Two: Formulation of the adequate game model. A game consists of the following building blocs: (a) Decision makers, agents, etc., called players. (b) Strategy sets: Each player chooses his or her strategy out of a corresponding set of strategies that are given by the resources and defined by the rules of the game. (c) Payoffs—are utilities that the players assign to the possible outcomes determined by corresponding choices on strategies.

Note that the outcomes (or events) do not show up in the game, but their evaluations in the form of payoffs do. In Matrix 1.1, we assumed that profits are a good proxy for payoffs and did not distinguish between the two concepts which is the regular approach procedure in standard microeconomics with respect to firms. But how shall we proceed if the outcomes are apples, pears, and bananas? We evaluate them in accordance with our preferences and assume that the other players will do the same. Of course, the problem is that, in general, we can only guess the other players' preferences. To give our preferences in the form of numbers can be difficult enough as modern utility theory, referring to introspection and experiments, tells us. In Chap. 10, we will discuss some extreme cases of "misrepresentation."

With respect to strategies, we should keep in mind that they represent plans often in the form of a sequence of moves. Moves can be contingent in the form of "If player A does x, I will choose y; if A does z, I will choose v." Think about chess, which is a popular illustration of a game, but note that the strategies of the game are certainly numerous. Nobody can formulate a plan that lists the suggested moves from the beginning to the end of the game. Still, the example may help us to understand that the set of strategies

depends on the rules of the game. Outside the game arena, such rules are often given by laws and public regulations, but also by behavioral standards. If we violate them, we may be eliminated from the standard games of the society we live in.

Step Three: Selection of the *solution concept*. Applying a particular solution concept or, in short, a *solution* to a game is meant to determine the strategies that the players are expected to choose, and thus determine the outcome and the corresponding payoffs of the players.

Often, the selected outcome is not unique, and for some solution concepts and a particular game, an outcome may not even exist. In the Prisoners' Dilemma game, the solution concept, i.e., equilibrium in dominant strategies, is defined by the strategies that the players are expected to choose. Alternatively, we may define the set of Pareto efficient outcomes as a solution which corresponds to the payoff pairs (800, 200), (250, 300), and (1500, 100) in Matrix 1.1. Note that given one of these payoff pairs no player can be made better off without making the other worse off. Given this set, of course, we have to discuss how one of its elements can be achieved, given the game situation and self-interested players. A favorite answer has recourse to altruism. However, if we introduce Adam Smith's "fellow-feelings," proposed on the first page of his Theory of Moral Sentiments (Smith 1982 [1759]), into Matrix 1.1 and these fellow-feelings are strong enough so that at least one of the players has no strictly dominant strategy available, then the game is no longer a Prisoners' Dilemma. Moreover, fellow-feelings among the managers of gas stations are not very likely. If we see that they choose high prices and thus deviate from the equilibrium of dominant strategies, we have to look for another explanation. Chap. 9 offers such an answer to this problem.

1.5 Winner-Takes-It-All and the Chicken Game

Now let us apply our just developed scheme to a real-world case, but described in terms of its stylized characteristics. Let us take a historical case: the *Browser War* between *Microsoft*, on the one side, and *Netscape*, on the other. Time Magazine of September 16, 1996 (p. 53ff.), reported that a dramatic battle between *Microsoft* and *Netscape* developed. Each of the two suppliers of browser programs wanted to help us find our way on the Internet. The winner of this battle could expect to earn billions of dollars,

while the loser would become marginalist on the market and perhaps would even have to close down the business. This looked like a *winner-takes-it-all game*.³

An important component of any strategy in this battle was the compatibility of a particular browser program. In the beginning, the older program of the two, Netscape's *Navigator* had the advantage to be widely used, and therefore, its net effects were larger than the net effects of the newcomer's program. Netscape could expect that the users of *Navigator* would be loyal to their browser program. It seems that there was a strong *first-mover advantage* embedded in the net effects—and the routine of the users. However, this was challenged by the fact that the *Internet Explorer* of *Microsoft* was easier to handle for newcomers and it was offered for free. Of course, with a zero price, Microsoft could not expect to make profits out of the sale of browser programs. But it was expected that "buyers" of the Microsoft browser would also buy other programs and services supplied by Microsoft, and this is what happened. As a result of the zero-price policy of Microsoft, *Netscape's Navigator* vanished from the market.

To describe the set of intertemporal strategies that were available to the players is rather difficult in this case. Moreover, the decisions were driven by expectations, and we have as yet not the instruments to deal with expected values.⁴ So far we simply do not have the capacity to represent this situation adequately. However, we can look at a toy model of this case that nevertheless might be useful to illustrate the decision problem and to derive some preliminary conclusions. Let us start with Matrix 1.1. We identify Microsoft and Netscape by the players A and B, respectively.

The entries in the cells of the matrix represent expected profits for A and B. So, if A chooses "high" and B chooses "low," the payoffs will be 250 for A and 300 for B. However, Matrix 1.2 assumes the payoffs (-50, -100) for the case that both players choose low prices, while Matrix 1.1 assumed the payoffs (500, 150). Obviously, the underlying decision situations are different and the payoff pair (-50, -100) suggests that an ongoing price war will be hazardous to both suppliers in the long run.

³From *The Winner Takes It All* lyrics by *ABBA*: "The winner takes it all/The loser's standing small/Beside the victory/That's her destiny."

⁴For expected values, see Chap. 10.

Would you choose a high or a low price if you were player A?

Matrix 1.2 The Chicken Game

| Player B A | high | Low |
|------------------|------------|------------|
| high | (800,250) | (250,300) |
| low | (1500,100) | (-50,-100) |

A comparison of the games in Matrices 1.1 and 1.2 shows that a perhaps minor change in the payoffs can have tremendous consequences for the decision situation. In Matrix 1.2, none of the players has a strictly dominant strategy. Therefore, the solution concept of an equilibrium in strictly dominant strategies does not apply. Whether a strategy is a good choice for A depends on which strategy B chooses, and vice versa. If we assume that players choose their strategies simultaneously, so that A does not know the strategy which B selects and B does not know the strategy which A chooses, then we see that the decision problem of the two players is nontrivial. In the course of this game, we will learn several solution concepts that should help players A and B to make rational choices—and to help us understand decisions made in game situations. Without going into detail, we see that the strategy pairs (high, low) and (low, high) are characterized by some stability as neither player is motivated to revise his or her strategy, given the strategy of the other player. In Chap. 3, we will learn that this property defines a Nash equilibrium. However, the strategy pairs (high, low) and (low, high) cannot be satisfied at the same time; that is, they are alternatives that exclude each other. Even though it would be beneficial for player A to see the strategy pair (low, high) put into reality, A cannot force B to choose a high price.

Note that in a strategic decision situation a player cannot choose an outcome, independent of what the other player does—in fact, *a player chooses a strategy and not an outcome*. The game in Matrix 1.2 is known as *Chicken Game*. Different from the Prisoners' Dilemma game in Matrix 1.1, it represents a rather complex decision situation as we will see in Chap. 4. In Chap. 4, we will hear of James Dean and learn why this game is called a "chicken" and how this game can be applied to analyze the lovers' battle in the Kamasutra.