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Introduction

So, you've chosen to study maths for A level? Fantastic. Welcome to the course!

It's a bit of a cliché that A level is a big step up from GCSE – but only because it's at least partly true. That said, if you know everything from your GCSE, you could probably pick up about a third of the marks in Core 1 without doing any more study.

In reality, though, not many people know *everything* from GCSE, and if your teacher assumes you do when you don't, well, then you have a problem. Worse still, it's a problem that's up to you to put right!

Luckily, you've done something smart and got hold of a book to help you. You're not going to find exercises in it or nice pictures of Carl Friedrich Gauss, but you will find plenty of advice and instructions on how to tackle various types of questions.

As for me, I've been tutoring A level maths since 2008, so I know what students tend to struggle with – and the kinds of explanations that tend to make complicated ideas make sense. Throughout the book, I try to break everything down into simple, repeatable steps that will work even if the question is a bit different from the example I've chosen.

About This Book

This book is for you if you're about to start studying for your AS- or A level in maths – or if you're already doing the course and need to improve your grade.

I take you through the topics that you can expect to come up in the exams, starting with the GCSE knowledge you're expected to carry through to A level and ending with differential equations – which sound scarier than they really are.

The book is in five parts:

- **Getting started:** This part covers the things that will make your life easier if you know them before you start studying. If you're already some way through your course, they're still useful to learn or revise.
- **Algebra:** Here, you develop your algebra techniques further, building on what you learnt at GCSE to solve more-involved problems. The biggest new ideas are, I think, infinite series and logarithms.
- **Geometry:** Similarly, the geometry you need for A level builds on what you've done before. In this part, you take trigonometry to new levels and do more interesting things with vectors than you ever imagined.
- **Calculus:** For most students, calculus is a new topic (although if you've done a Further Maths GCSE, Additional Maths or even the IGCSE, you may be familiar with some of the concepts). Calculus is loosely the study of slopes and areas, and it's probably the most important mathematical concept you get from A level maths if you're planning to study a science at university. (For some subjects, statistical reasoning may push it close – but that's not in this book, so it's obviously not *that* important!)
- **The Part of Tens:** Lastly, I offer some quick hints and tips to help you get started on evil problems and avoid some of the mistakes everybody else makes.

Because the topics covered in the book are complicated enough, I've tried to keep the writing conventions simple. Here are the ones you should know:

- *Italics* are used for emphasis or to highlight new words or phrases in each chapter. They also indicate variables.
- **Boldfaced text** indicates key words in bulleted lists or the key steps in action lists. Vectors also appear in bold.
- Internet and email addresses appear in `monofont` to help them stand out.

Foolish Assumptions

Making assumptions is always risky, but it's unavoidable in this kind of book. Knowing where I'm coming from should help put you at ease, though. Here's what I've assumed:

- You've done reasonably well at GCSE, or (if you're coming back to maths after a break) you remember enough of your secondary school maths that you'd get at least a B. Don't worry, though – in Part I of the book, I review the skills you may have missed.

- You're competent with the basics of algebra (solving linear equations), arithmetic (you know how to add, subtract, multiply, divide and take powers of numbers), and geometry (you know the names of shapes, and you won't be surprised by the equations attached to them).
- You've chosen to do Maths A level, you have some enthusiasm for the subject, and you want to do well in it.

Icons Used in This Book

Here are the icons I use to draw your attention to particularly noteworthy paragraphs:

Theories are fine, but anything marked with a Tip icon in this book tells you something practical to help you get to the right answer.

Paragraphs marked with the Remember icon contain the key takeaways from the book and the essence of each subject.

The Warning icon highlights mistakes that can cost you marks or your sanity – or both!

You can skip anything marked with the Technical Stuff icon without missing out on the main message, but you may find the information useful for a deeper understanding of the subject.

Beyond the Book

In addition to all the great content provided in this book, you can find even more of it online. Check out `www.dummies.com/cheatsheet/asalevelmaths` for a free Cheat Sheet that provides you with a quick reference to trigonometric tricks and calculus rules.

You can also find several bonus articles on topics such as the link between arithmetic series and straight lines, and why the derivatives of the trigonometric functions are what they are, at `www.dummies.com/extras/asalevelmathsuk`.

Where to Go from Here

Head to Chapter 1 for an overview of what you can expect in your A level course. Or if you want to get your study habits straight first, Chapter 2 ought to be your first port of call.

You can also use the table of contents and index to find the specific areas you want to work on. This book is meant as a reference guide, so keep it handy with your study gear and look things up whenever you're confused.

I wish you all the best with your studies! If there's something that isn't clear, get in touch – I'm always happy to try to make things make sense. The best way to reach me is on Twitter (I'm `@icecolbeveridge`) or through my website (`www.flyingcoloursmaths.co.uk`).

Part I
Getting Started

For Dummies has great info on lots of different topics. Check out www.dummies.com to find out how you can learn more and do more with *For Dummies*.

In this part . . .

- Set off towards mathematical mastery.
- Put yourself in a position to do well on the exam.
- Refresh your arithmetic and algebra.
- Get your shapes ship-shape.
- Sketch graphs with aplomb.

Chapter 1

Moving towards Mathematical Mastery

In This Chapter

- Understanding the overlap with GCSE
- Doing advanced algebra
- Building on geometry
- Diving into calculus

It's a big step up from GCSE to A level – especially if you're coming in with a B or a marginal A. The pace is pretty frenetic, and there's a fair amount of A and A* material from GCSE that's assumed knowledge at A level. If you're not especially happy about algebraic fractions or sketching curves, for example, you're likely to have a bit of catching up to do.

Luckily, this book has a whole part devoted to catching up with the top end of GCSE, as well as the stuff you'll need to learn completely fresh. In this chapter, I note where the content overlaps with GCSE and introduce you to A level algebra, trigonometry and calculus.

Reviewing GCSE

The good news is that if you've got a solid understanding of everything in your GCSE, quite a lot of Core 1 and a fair amount of Core 2 will be old news to you. Possibly less good news is that if you've got gaps in your knowledge, you need to fill them in pretty sharpish.

The four key areas where there's an overlap between the two qualifications are algebra, graphs and powers (all in Core 1) and trigonometry (which comes up a lot in Core 2). There are other bits and pieces, too – your arithmetic needs to be pretty decent in Core 1, where you don't have a calculator, and parts of Core 2 are likely to test your knowledge of shapes other than triangles – but generally speaking, these are the big four. The first part of this book is all about making sure you're up to speed with them.

Setting up for study success

Forgive me if you think it's patronising to tell you how to study – after all, you must have done pretty well with exams to get this far. I go into studying because A level is a much tougher beast than GCSE – it's possible for a reasonably smart student to coast through GCSE and get a good grade without needing to work too hard; by contrast, it's unusual to see someone glide through A level. And presumably, if you were finding it straightforward, you wouldn't be buying books like this to help you through it.

In this kind of scenario, everything you can do to optimise your working environment, your note-taking and your revision translates to quicker understanding and more marks in the exam.

Also, if you're studying at sixth form, there's likely to be a bit more going on socially than at secondary school. The more quickly you can absorb your studies, the sooner you can get out to absorb the odd lemonade with your friends. That is what you're drinking, isn't it?

All about the algebra

You've probably been manipulating algebraic expressions for years by now, and some of it will be second nature. However, just as a checklist, here are some of the topics you need to have under your belt:

- Solving linear equations (such as $7x+9=3x-7$)
- Expanding quadratic brackets (for example, $(x+3)(2x-5)$)
- Factorising and solving quadratics (such as $x^2-5x-36=0$)
- Solving linear and nonlinear simultaneous equations
- Simplifying algebraic fractions

I recap all of these in Chapter 3.

All this algebra isn't just for the sake of jumbling letters around and feeling super-smug when your answer matches the one in the mark scheme (although that can be a nice motivator). Algebraic competence underpins just about everything in A level. Even in places where you'd normally expect to use only numbers (for example, Pythagoras's theorem), you may be asked to work with named constants (such as k) instead of given numbers (such as 3).

Grabbing graphs by the horns

Somewhat related to algebra are graphs. You rarely need to draw an *accurate* graph at A level; it's far more common to be asked to *sketch* a graph.

That's good news: sketching is much quicker and more generously marked than plotting. However, you can no longer rely on painstakingly working out the coordinates and joining them up with a nice curve. Instead, you need to know the shapes of several kinds of graphs you've come across at GCSE: the straight line, the quadratic and the cubic graphs as well as the reciprocal and squared-reciprocal graphs.

You'll frequently be asked to work out where a graph crosses either of the coordinate axes (which is really an algebra question), and you'll be expected to be on top of curve transformations. I cover all these in Chapters 4 and 10.

Taming triangles and other shapes

Triangles, obviously, are the best shape of all, which is why you spend so much time on SOH CAH TOA, the sine and cosine rules, and finding areas at GCSE.

Oh, and Pythagoras's theorem. If there were a usefulness scale, Pythagoras's theorem would be *way* off it. I can't think of a more important equation at A level, and you can read about it in Chapter 4.

Those skills are extremely useful in A level maths. Pretty much every Core 2 paper I've ever seen has used a triangle somewhere, and triangles frequently crop up in other modules. (If you're doing Mechanics, having strong trigonometry skills is a massive help.)

It's not just triangles you need to know about, though. Be sure you know the areas and perimeters of basic two-dimensional shapes like rectangles, trapeziums and circles as well as the surface areas and volumes of three-dimensional shapes such as cuboids and prisms. I recap these shapes in Chapter 15.

Attacking Advanced Algebra

As you'd expect, the algebra you're expected to do at A level gets a bit more involved than what you did at GCSE. It comes down to learning some new techniques and linking some new notation to ideas you may already have a decent grasp of.

You start with powers and surds, a GCSE topic that sometimes gets glossed over. You'll need to be fairly solid on these, as they come up over and over again in A level. In later modules, you have a calculator that will happily tell you that the square root of 98 can be written as $7\sqrt{2}$, but in Core 1, you need to be able to work that out on paper.

You also deal with sequences and series (extending the work you've done in the past), solidify the ideas of factorising polynomials, and do some work on functions – one of the most important ideas in maths.

Picking over powers and surds

Working out combinations of powers is one of the most critical skills for A level maths. I wouldn't say it's more important than topics like algebra, but a student's skill here is a strong indicator of how easy a student is going to find A level. If you're a bit rusty on the power laws, you're going to have to sort that out in fairly short order.

You also need to be pretty hot on your surds, especially in Core 1, when you're one calculator short of a pencil case. Throughout your course, you'll need to work square roots out in *simplified surd* form or, more generally, in *exact* form – examiners want to see things like $\pi\sqrt{2}$ rather than 4.443.

A step up from powers and surds are *logarithms*, which are handy functions for turning equations with unknown powers into equations with unknown multipliers. For example, without logarithms, $3^x = 100$ is hard to solve (you know the answer is 4-and-a-bit but not necessarily what the bit is), but with logarithms, getting the answer is a simple bit of algebra: $x\log(3) = \log(100)$, where log(3) and log(100) are just numbers you can get from your calculator.

Lastly, under the 'powers' heading, you need to work with one of the most interesting numbers in all of maths, e. It's a constant (exactly $1+\frac{1}{1}+\frac{1}{2\times 1}+\frac{1}{3\times 2\times 1}+\ldots$, or roughly $2.718281828459045\ldots$) with the lovely property that if you work out the tangent line of $y = e^x$ at any point, you find its gradient is equal to e^x.

Sorting out sequences and series

A *sequence* is simply a list of mathematical objects (often numbers, sometimes expressions). A *series* is what you get if you add them up.

You probably did some work on sequences in the past (finding the *n*th term, for instance, or deciding whether a term belonged to a sequence), and that will stand you in good stead. At A level, though, there's a lot more to it (who would have thought?).

As well as the arithmetic sequences you know and love, there are geometric sequences (where each term is a constant multiple of the one before). There are also explicitly and recursively defined functions, which *sound* like a horrible wild-card but in fact are quite nice because you're told precisely how they behave in the question.

And there are binomial expansions, which are a really neat way of expanding expressions like $(1+2x)^6$ without needing to multiply out huge numbers of brackets. It's a particularly useful technique when you get to Core 4 and have to expand monsters like $\frac{1}{\sqrt{4+3x}}$ and use the result to approximate $\frac{1}{\sqrt{4.03}}$.

Of course, the binomial expansion is one of many things you do much more often in exams than you ever will in the outside world. (We have machines for that.) However, the idea of approximating things using polynomial series is a powerful tool for doing serious maths if you take the subject beyond A level. Oh, and it's handy for doing mental arithmetic tricks that make you look like a god, too.

Finding factors

I keep coming back to a theme in this book: things in brackets are (usually) happy things. In most cases, if you can put something in brackets – a quadratic expression, or a cubic, or a fraction – you almost certainly should. If you have to solve for where an expression is 0, the factorising makes it very easy; if you need to sketch a curve, the bracketed form is much easier to work with than the expanded one.

All the work you did in learning to factorise quadratics over the last few years will serve you quite well with this – although, as you may expect, you take it a few steps further at A level.

In Core 2, you learn to identify factors of cubics (and higher-degree polynomials) using the factor theorem. You use polynomial division to take these factors out so you can factorise the remaining expression. You also use its cousin, the remainder theorem, to find out what's left over without having to work through the whole division.

Sometimes, though, you need to do the whole division or, depending on your preferred method, find a way to work around it. I like to turn the problem on its head by coming up with a template answer and seeing which numbers have to go in the template, but your mileage may vary.

All this factor fun shows up in Chapter 7, along with the Core 4 topic of partial fractions. Since you started working with fractions, you've been adding and subtracting them using common denominators. Partial fractions is the reverse process of taking a fraction that's been combined and splitting it up into the parts that once made it up. Why would you do such a thing? Two reasons: it makes things much easier to integrate, and it means you can apply the binomial expansion much more easily.

Functions

A mathematical *function* is, roughly speaking, a recipe for taking one or more values and spitting out another. They're a big deal, mathematically speaking: being able to talk about functions in the abstract, without explaining what the recipe is, means you can do interesting things with graphs and calculus without getting bogged down in the details. For example, if you compare the graph of $y = f(x-2)$ with the graph of $y = f(x)$, you can say, 'The graph has moved two units to the right' without caring whether the function is quadratic, reciprocal, trigonometric or other – quite a handy trick!

In Chapter 9, you learn about the slightly esoteric notation you use for defining functions. You find out how to combine functions with each other, how to *invert* (undo) functions and how to solve equations involving functions.

You also get to play with *iteration*, which falls into the dull-but-usually-straightforward category. The idea is that you set up a recursive process, doing the same thing over and over again, until it converges on a specific value.

Getting to Grips with Geometry

Geometry – literally 'measuring the Earth' – has developed over time to mean the study of shapes. At A level, the most important shapes are triangles (clearly the best shape) and circles (which are really triangles in disguise), although you will need to deal with rectangles and trapeziums and all manner of three-dimensional shapes in good time.

The four main areas of A level geometry are

- **Coordinate geometry,** which is about dealing – as you might expect – with coordinates; that includes midpoints, distances and equations of curves
- **Circles,** including their equations and some theorems
- **Triangles**, including advanced trigonometry
- **Vectors**, including vector lines, angles between vectors, and triangles in three dimensions

Conquering coordinate geometry

Coordinate geometry is a big topic at A level. You need to be super-confident with your *x*s and *y*s. You've covered the basics at GCSE – the equation of a line, finding midpoints and distances between points using Pythagoras's theorem, and so on – but it all gets taken a bit further at A level.

There are curves to sketch and shapes whose areas need to be known. After you do some differentiation, there are tangents and normals to find the equations of.

Setting up circles and triangles

I'm always surprised when I ask for the equation of a circle and someone says, 'πr^2'. First of all, that's not an equation (he or she means $A = \pi r^2$, where A is the area and r is the radius), and second, that's the *area* of the circle, not the circle itself.

The equation of a circle, like the equation of a line, gives you a relationship between its coordinates. If the equation is $(x-3)^2 + (y-4)^2 = 25$, one of A level's favourite circles, you can tell whether a particular point is on the circle by putting its coordinates into the equation as x and y.

In Chapter 11, I show you how to work out the equation of a circle as well as work out the area of a sector, the length of an arc and other things related to a circle.

Circles are closely linked to triangles; apart from all the trigonometry you know from GCSE, you'll also need to be able to do it all in *radians*, a much better measurement of angles than the degree. Fortunately, much of it is just a case of switching your calculator mode and relabelling your graphs.

Taking trigonometry further

If you split the word *trigonometry* up into its parts, you get 'tri', meaning three; 'gon', meaning 'knee' or corner; 'o', which means nothing; and 'metry', meaning 'measuring.' Trigonometry is about measuring things with three corners.

However, that's not all you use it for. It also has applications in any situation where things are periodic – measuring tides, modelling daylight lengths and analysing sounds, just off the top of my head. For that reason, you need to be able to take trigonometry further. Some of the things you'll be doing include

- Finding all the possible solutions to simple trigonometric equations in a given interval
- Using trigonometric identities to turn trigonometric equations into something you can solve
- Exploring what happens to sine, cosine and their friends when you add angles together
- Adding sine and cosine waves together to get another sine or cosine wave
- Working with the minor trigonometric functions – the reciprocals of sine, cosine and tangent, which are the cosecant, secant and cotangent (respectively); usually, they're denoted $\text{cosec}(x)$, $\sec(x)$ and $\cot(x)$

You also need to be up to speed on proving that two trigonometric expressions are equivalent. That generally involves combining fractions, applying identities, factorising things cleverly and understanding the symmetries of the various functions. Because these problems bring so many areas together, they're one of the most demanding (but also most rewarding) bits of Core 3.

Vanquishing vectors

You've done some work with vectors at GCSE – although you may not have done as much as you'd like, because vectors are usually in the A* questions at the end of the paper. It's OK, though: your GCSE vectors work isn't essential to your A level studies.