



Jeffrey T. Barton

SOLUTIONS MANUAL TO ACCOMPANY

MODELS FOR LIFE

An Introduction to Discrete Mathematical
Modeling with Microsoft® Office Excel®

WILEY

**SOLUTIONS MANUAL TO ACCOMPANY
MODELS FOR LIFE**

**SOLUTIONS MANUAL
TO ACCOMPANY
MODELS FOR LIFE:
AN INTRODUCTION
TO DISCRETE
MATHEMATICAL
MODELING WITH
MICROSOFT[®]
OFFICE EXCEL[®]**

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PREFACE

Included in this manual are complete, step-by-step solutions to all odd-numbered problems from the text *Models for Life: An Introduction to Discrete Mathematical Modeling with Microsoft® Office Excel®*. As is the case in many areas of mathematics, in mathematical modeling there are often several reasonable ways to approach a problem, and an effort has been made in this manual to mention alternate approaches to problems where appropriate. Please keep in mind that if a proposed solution does not exactly match the one given here it does not necessarily mean that solution is incorrect. Exploring and making sense of such differences can be an excellent way to deepen one's understanding of the material, and I encourage the sharing of alternate solutions with classmates and/or the instructor.

I also encourage the use of this manual as a tool for getting unstuck on a problem, but to go no further in the solution than is necessary to get past the sticking point. It can be tempting to turn to the solution and read through it in its entirety; however, to get the most out of this manual I suggest working through the solution only far enough to see how to continue. Finally, at some point, especially during exam preparation, it is important to make sure that all assigned problems can be worked from scratch without assistance. Reviewing solutions is valuable, but a much deeper level of understanding is required to reproduce solutions independently, and that is what is usually required on an exam. Exercises in this manual are referenced by section number. Thus "Exercise 1.2.5" refers to Exercise 5 from Section 1.2.

JEFFREY T. BARTON
December 2015

ABOUT THE COMPANION WEBSITE

This book is accompanied by a companion website:
www.wiley.com/go/barton/solutionsmanual_modelsforlife

The website includes:

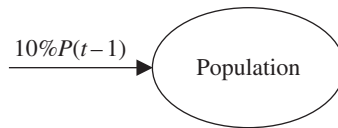
- Instructor's Solutions Manual
- Instructor Excel Spreadsheets for Models in Text
- Test Bank
- Instructor Notes and Project Ideas
- Student Excel Spreadsheets
- Selected Excel Spreadsheets Used in Exercises for Instructor

1

DENSITY INDEPENDENT POPULATION MODELS

1.1 EXPONENTIAL GROWTH

1 Consider the flow diagram in Text Figure 1.21.



TEXT FIGURE 1.21 Flow diagram for Exercise 1.1.1.

a. Find the corresponding DDS.

The flow diagram tells us how the population changes from one year to the next. Inward pointing arrows represent additions while outward pointing arrows represent subtractions. Here there is only one arrow, and it represents an addition. Thus the DDS is given by

$$P(t) = P(t-1) + 10\%P(t-1).$$

We can also write the DDS as $P(t) = P(t-1) + 0.10P(t-1)$, or $P(t) = 1.10P(t-1)$.

- b. Use a calculator to predict the population after 2 years if $P(0) = 50$.

If $P(0) = 50$, then by using the DDS we can predict the population 1 year later:

$$\begin{aligned} P(1) &= P(0) + 10\%P(0) \\ &= 50 + 0.10 \cdot 50 \\ &= 55. \end{aligned}$$

Applying the DDS once more gives us the model prediction for year 2:

$$\begin{aligned} P(2) &= P(1) + 10\%P(1) \\ &= 55 + 0.10 \cdot 55 \\ &= 60.5. \end{aligned}$$

After 2 years we predict the population will be 60.5.

- c. Use Excel to project the population in year 10.

Since the model in this problem is the exponential growth model, we can save time by using the same spreadsheet we created for the Yellowstone grizzly population. We only need to change the growth rate to $r = 10\%$ and the initial population to $P(0) = 50$. Figure 1.1 shows the result with the projection for year 10 highlighted. The model predicts a population of about 129.7 in year 10.

	A	B	C
1	Exercise 1.1.1		
2			
3	Growth rate, $r =$		10%
4			
5	t	Population	
6	0	50	
7	1	55.0	
8	2	60.5	
9	3	66.6	
10	4	73.2	
11	5	80.5	
12	6	88.6	
13	7	97.4	
14	8	107.2	
15	9	117.9	
16	10	129.7	

FIGURE 1.1 Excel output for Exercise 1.1.1.

3 Consider the flow diagram in Text Figure 1.23.



TEXT FIGURE 1.23 Flow diagram for Exercise 1.1.3.

a. Find the corresponding DDS.

The flow diagram tells us how the population changes from one year to the next. Inward pointing arrows represent additions while outward pointing arrows represent subtractions. Here we have two arrows: one an addition and one a subtraction. Thus the DDS is given by

$$\begin{aligned} P(t) &= P(t-1) + 8\%P(t-1) - 5\%P(t-1) \\ &= P(t-1) + 3\%P(t-1). \end{aligned}$$

We can also write the DDS as $P(t) = P(t-1) + 0.03P(t-1)$, or $P(t) = 1.03P(t-1)$.

b. Use a calculator to predict the population after 2 years if $P(0) = 100$.

If $P(0) = 100$, then by using the DDS we can predict the population 1 year later:

$$\begin{aligned} P(1) &= P(0) + 3\%P(0) \\ &= 100 + 0.03 \cdot 100 \\ &= 103. \end{aligned}$$

Applying the DDS once more gives us the model prediction for year 2:

$$\begin{aligned} P(2) &= P(1) + 3\%P(1) \\ &= 103 + 0.03 \cdot 103 \\ &= 106.09. \end{aligned}$$

After 2 years we predict the population will be about 106.1.

c. Use Excel to project the population in year 10.

We see from the DDS that this model is still an exponential growth model with $r = 3\%$. Thus we can use the Yellowstone grizzly spreadsheet with the new growth rate and the initial population set to 100. The result is given in Figure 1.2 with the projection for year 10 highlighted. The model predicts a population of about 134.4 in year 10.

	A	B	C
1	Exercise 1.1.3		
2			
3	Growth rate, $r =$		3%
4			
5	t	Population	
6	0	100	
7	1	103.0	
8	2	106.1	
9	3	109.3	
10	4	112.6	
11	5	115.9	
12	6	119.4	
13	7	123.0	
14	8	126.7	
15	9	130.5	
16	10	134.4	

FIGURE 1.2 Excel output for Exercise 1.1.3.

- 5 Draw a flow diagram that corresponds to the following DDS:

$$P(t) = P(t-1) + 4\%P(t-1).$$

The addition of 4% of the previous year's population is represented by an inward pointing arrow in the flow diagram, given in Figure 1.3.

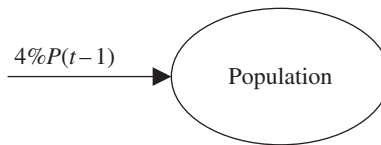


FIGURE 1.3 Flow diagram for Exercise 1.1.5.

- 7 Draw a flow diagram that corresponds to the following DDS:

$$P(t) = P(t-1) - 0.30P(t-1).$$

The DDS indicates a subtraction of 30% of the previous year's population. We account for this subtraction with an outward pointing arrow in the flow diagram, given in Figure 1.4. Note that there is no minus sign in front of the arrow label.

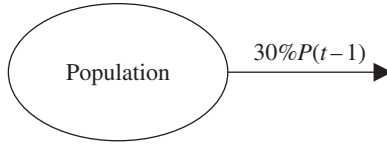


FIGURE 1.4 Flow diagram for Exercise 1.1.7.

- 9 Give the flow diagram and corresponding DDS for a grizzly population that is growing by 8% per year and has 5 bears illegally poached annually.

We represent the 8% growth by an inward pointing arrow and the poaching by an outward pointing arrow. The result is Figure 1.5. Note that there is no minus sign in front of the 5.

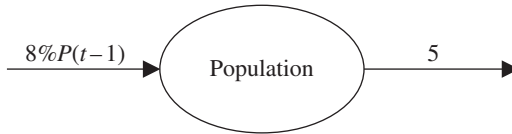


FIGURE 1.5 Flow diagram for Exercise 1.1.9.

The corresponding DDS is given by $P(t) = P(t-1) + 0.08 \cdot P(t-1) - 5$.

- 11 Suppose you know that the DDS for a population is given by

$$P(t) = P(t-1) + 3\%P(t-1) - 50.$$

- a. Draw a flow diagram that would lead to this DDS.

The 3% increase is represented by an inward pointing arrow while the removal of 50 from the population is represented by an outward pointing arrow. The result is given in Figure 1.6.



FIGURE 1.6 Flow diagram for Exercise 1.1.11.

- b. Explain in a complete sentence how the population is changing from year to year.

The population is experiencing growth of 3% of the previous year's population while at the same time 50 members of the population are leaving each year.

- 13 Suppose that the 1993 *Grizzly Bear Recovery Plan* had never been implemented and that the 1993 estimate of a 1% growth rate continued to hold. How long would it have taken for the population to reach 416 bears?

We use the Yellowstone grizzly population Excel model with $r=1\%$ and $P(0)=197$. We are looking for the year that the population reaches 416 bears, so we drag the model formulas down until we see the population meet or exceed 416 for the first time. This happens 76 years from the initial population estimate, and the population of bears is projected to be about 419.7 at that time.

- 15 Suppose that the numbers of adult females with cubs sighted in Yellowstone were 52 in 2003, 60 in 2004, and 65 in 2005. Estimate the total grizzly population in 2005.

The 3-year total of adult female grizzlies is $52+60+65=177$. No known deaths are mentioned, so we assume 0 known deaths. Thus we have 177 adult females, representing about 27.4% of the total population of bears. This total is given by $\frac{177}{0.274} \approx 645.99$, or about 646 bears.

- 17 Text Table 1.2 contains more population data for the wild California condor population from the 1996 *Recovery Plan for the California Condor* (U.S. Fish and Wildlife Service, 1996).

TEXT TABLE 1.2 The Number of California Condors Remaining in the Wild between 1982 and 1985 (U.S. Fish and Wildlife Service, 1996)

Year	Number Wild California Condors
1982	21
1983	19
1984	15
1985	9

- a. Compare the population values in the table to what our model would predict using the rate of decline found in Example 1.5 and an initial population of 50 condors. In general, how well did our model do?

Here we use the California condor Excel spreadsheet that we already created, where $P(0)=50$ in 1968, and the rate of decline from Example 1.5 is $r=6.6\%$. Next we drag the model formulas down until we reach the year 1985, or $t=17$. The projected values for years 1982-5 for our model are 19, 18, 17, and 16. We compare the model projections to the data in Text Table 1.2, which recorded condor populations of 21, 19, 15, and 9 for the years 1982-5. Our model seems to have done reasonably well, though from the data it appears as though something happened in 1985 that caused a larger than predicted decline in the population.

b. Can you think of possible reasons for any discrepancies?

As noted above, the most striking difference between our model projections and the actual population data seems to be for the year 1985. There could be any number of reasons for the larger than predicted decline in 1985, including accidents, poaching, or disease.

19 Recall that our estimate for the California condor’s rate of decline was based on the lower population estimates given by Sibley, Mailed, and Wilbur. Re-estimate the rate of decline from 1968 to 1978 using three other combinations from the population estimates:

a. The lower value from 1960’s and the higher value from 1978.

The range of values for the California condor population was given as 50–60 in the late 1960’s and 25–30 in 1978. Taking the lower value from the 1960’s (with the assumption of 1968 for our starting year), we use $P(0) = 50$. Using the higher estimate, 30 condors, in 1978 gives us $P(10) = 30$. Thus we repeat the trial-and-error approach from Example 1.5 in order to estimate the rate of decline from 1968 to 1978. We use the already created California condor Excel model and type in different values for r until we get 30 condors in 1978. The result is shown in Figure 1.7 with the value for r highlighted. Our new estimate for the rate of decline is about 5% per year. Note that it makes sense for the rate of decline to be lower than in Example 1.5 because the assumed population in 1978 is higher – there was less of an assumed decline.

b. The higher value from 1960’s and the lower value from 1978.

	A	B	C
1	Exercise 1.19		
2			
3	Rate of decline, $r =$		5.0%
4			
5	t	Population	
6	0	50	
7	1	48	
8	2	45	
9	3	43	
10	4	41	
11	5	39	
12	6	37	
13	7	35	
14	8	33	
15	9	32	
16	10	30	

FIGURE 1.7 Excel output for Exercise 1.1.19.