

Gopal B. Saha

Basics of PET Imaging

Physics, Chemistry,
and Regulations

Third Edition

 Springer

Basics of PET Imaging

Gopal B. Saha

Basics of PET Imaging

Physics, Chemistry, and Regulations

Third Edition



Springer

Gopal B. Saha, PhD
Emeritus Staff
Cleveland Clinic
Cleveland, OH, USA

ISBN 978-3-319-16422-9 ISBN 978-3-319-16423-6 (eBook)
DOI 10.1007/978-3-319-16423-6

Library of Congress Control Number: 2015945822

Springer Cham Heidelberg New York Dordrecht London
© Springer International Publishing Switzerland 2016

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made.

Printed on acid-free paper

Springer International Publishing AG Switzerland is part of Springer Science+Business Media (www.springer.com)

Dedication

With an ocean of love to my granddaughter Samira



who has been a perpetual source of my joy and inspiration

Preface

Stipulation of the third edition of the book, *Basics of PET Imaging*, has been prompted by many new developments in PET technology, equipment, and products since its second edition published in 2010. In this edition, new materials have been added, obsolete topics deleted, and pertinent contents updated. The purpose of the book remains the same as was for the previous editions—to fulfill the need for radiology and nuclear medicine professionals in their board examinations and practice.

The content of the book has been organized concisely in 14 chapters. Chapters 1 and 12 have no major change. In Chapter 2, information on detectors has been updated, and a new section on MR scanners and the principle of their operation has been added. Updated tables for specifications of PET, PET/CT, PET/MR, and animal scanners from different manufacturers have been incorporated. Chapter 3 contains a new section on PET/MR imaging. Iterative reconstruction method has been elaborated in Chapter 4. A table of PACS from different vendors has been added in Chapter 5. Chapter 6 contains a section on quality control of MR scanners. New positron emitting radionuclides with potential for human use have been included in Chapter 7, and similarly new PET radiopharmaceuticals have been added in Chapter 8. The original Chapter 9 has been split into two separate chapters to include FDA regulations in Chapter 9 and NRC regulations in Chapter 10 with updated information. Included in Chapter 11 is current updated information on reimbursement in healthcare. A new procedure for amyloid imaging in Alzheimer's patients using ^{18}F -florbetapir PET/CT imaging has been detailed in Chapter 13. A new Chapter 14 contains special topics of interest, such as absorbed doses from ^{18}F -FDG and ^{82}Rb -RbCl, SUV calculation in ^{18}F -FDG tumor imaging, and the use of infusion pump in ^{82}Rb -RbCl myocardial perfusion imaging, which were presented as appendices in past editions. In addition, four old appendices have been kept as before with minimal updating.

I would like to thank William C. Franz for his kind help in providing updated information on reimbursement in healthcare, which are included in Chapter 11. My special thanks are due to Ms Janet Foltin, Senior Editor, Clinical Medicine, of Springer Science and Business Media, Inc., who graciously encouraged and supported me in pursuing the third edition. I also sincerely thank Mr. Patrick Carr, production editor, for kind support in the production of the book. I would like to thank the SPi-Global of Chennai, India for the excellent production of the book.

Cleveland, OH, USA

Gopal B. Saha, PhD

Contents

1	Radioactive Decay and Interaction of Radiation with Matter	1
	Atomic Structure	1
	Radioactive Decay	2
	α Decay	2
	β^- Decay	3
	Positron (β^+) Decay	3
	Electron Capture	4
	Isomeric Transition	4
	Radioactive Decay Equations	5
	General Decay Equations	5
	Successive Decay Equations	8
	Units of Radioactivity	10
	Units of Radioactivity in System Internationale	10
	Calculations	10
	Interaction of Radiation with Matter	11
	Interaction of Charged Particles with Matter	11
	Interaction of γ Radiation with Matter	13
	Questions	17
	References and Suggested Reading	18
2	PET Scanning Systems	19
	Background	19
	Solid Scintillation Detectors	20
	Semiconductor Detectors	23
	Photomultiplier Tube	26
	Pulse Height Analyzer	27
	Arrangement of Detectors	27
	Coincidence Timing Window	30
	PET Scanner	31
	Scintillation Camera for PET Imaging	33

PET/CT Scanner	35
PET/MR Scanner	41
Principles of MR Imaging	41
MR Scanner	45
Commercial PET/MR Scanner	46
Small-Animal PET/CT and PET/MR Scanner	50
Mobile PET or PET/CT Scanner	52
Questions	53
References and Suggested Reading	54
3 Data Acquisition and Corrections	55
PET Data Acquisition	55
Time-of-Flight Method	60
Two-Dimensional vs. Three-Dimensional Data Acquisition	61
Factors Affecting Acquired PET Data	63
Normalization	63
Photon Attenuation	64
Random Coincidences	67
Scatter Coincidences	69
Dead-Time Loss	72
Depth of Interaction	74
PET/CT Data Acquisition	76
Factors Affecting PET/CT Data	78
Positioning of Patient	78
Metal Object	78
Contrast Agent	78
Truncation Effect	80
Respiratory Movement	82
PET/MR Data Acquisition	83
Factors Affecting PET/MR Data	84
Attenuation Correction	85
Respiratory Movement	86
Metal Object	86
Truncation Effect	87
Questions	87
References and Suggested Reading	89
4 Image Reconstruction	91
Introduction	91
Simple Backprojection	91
Filtered Backprojection	92
The Fourier Method	93
Types of Filters	94

Iterative Reconstruction	97
3D Reconstruction	103
Partial Volume Effect	105
Questions	106
References and Suggested Reading	107
5 Storage, Display, and PACS	109
Introduction	109
Storage	109
Display	111
Software and DICOM	113
Picture Archiving and Communication Systems	113
Electronic Health Record	118
Teleradiology	119
Questions	119
References and Suggested Reading	120
6 Performance Characteristics of PET Scanners	121
Introduction	121
Spatial Resolution	121
Sensitivity	124
Noise Equivalent Count Rate	127
Scatter Fraction	127
Contrast	128
Quality Control of PET Scanner	129
Daily Quality Control Tests	129
Weekly Quality Control Tests	130
Quality Control of CT Scanner	131
Quality Control of MR Scanner	132
Acceptance Tests for PET Scanner	133
Spatial Resolution	135
Scatter Fraction	136
Sensitivity	138
Count Rate Loss and Random Coincidence	139
Questions	140
References and Suggested Reading	142
7 Cyclotron and Production of PET Radionuclides	143
Introduction	143
Cyclotron Operation	143
Medical Cyclotron	146
Nuclear Reaction	146
Target and Its Processing	149
Equation for Production of Radionuclides	151
Specific Activity	153

Production of Positron-Emitting Radionuclides	153
Fluorine-18	153
Carbon-11	155
Nitrogen-13	155
Oxygen-15	156
Iodine-124	156
Strontium-82	156
Technetium-94m	157
Germanium-68	157
Gallium-68	157
Copper-64	158
Copper-62	158
Yttrium-86	158
Zirconium-89	158
Questions	159
References and Suggested Reading	159
8 Synthesis of PET Radiopharmaceuticals	161
Introduction	161
Automated Synthesis Device	161
PET Radiopharmaceuticals	163
¹⁸ F-Sodium Fluoride	163
¹⁸ F-Fluorodeoxyglucose	164
6- ¹⁸ F-L-Fluorodopa	165
¹⁸ F-Fluorothymidine	167
¹⁸ F-O-(2-Fluoroethyl)-L-Tyrosine	167
¹⁸ F-Fluoromisonidazole	167
¹⁸ F-1-(5-Fluoro-5-Deoxy- α -Arabinofuranosyl)-2-Nitroimidazole	168
¹⁸ F-Florbetapir	168
¹⁵ O-Water	168
<i>n</i> - ¹⁵ O-Butanol	169
¹³ N-Ammonia	169
¹¹ C-Sodium Acetate	169
¹¹ C-Flumazenil	170
¹¹ C-Methylspiperone	170
¹¹ C-L-Methionine	171
¹¹ C-Raclopride	171
¹¹ C-Choline	171
⁶² Cu-Pyruvaldehyde-Bis(N ⁴ -Methylthiosemicarbazonato)	
Copper(II)	172
⁶⁸ Ga-DOTA-Peptides	172
⁸² Rb-Rubidium Chloride	172
Quality Control of PET Radiopharmaceuticals	173
Methods of Quality Control	174
Questions	177
References and Suggested Reading	177

9	FDA Regulations for PET Radiopharmaceuticals	179
	Food and Drug Administration	179
	Investigational New Drug	179
	New Drug Application	180
	Exploratory IND	181
	Radioactive Drug Research Committee	183
	Difference Between RDRC and Exploratory IND	183
	Expanded Access IND for PET Radiopharmaceutical	184
	Compounding of PET Radiopharmaceuticals	184
	Personnel	185
	Facility and Equipment	186
	Components, Materials, and Supplies	186
	Compounding Procedure Verification	186
	Dispensing of PET Radiopharmaceuticals	187
	Aseptic Technique	187
	Legal Requirements for Practicing PET	187
	License or Registration	188
	PET Certification for Technologists	190
	Accreditation of PET Facility	192
	Questions	195
	References and Suggested Reading	195
10	NRC Regulations for Radiation Protection in PET	197
	NRC Regulations for PET Radiopharmaceuticals	197
	Definitions	198
	Caution Signs and Labels	200
	Radiation Safety Officer	201
	Occupational Dose Limits	201
	Personnel Monitoring	202
	Receiving and Monitoring of Radioactive Packages	202
	ALARA Program	203
	Radioactive Waste Disposal	203
	Surveys for Radiation Exposure and Contamination	204
	Syringe and Vial Shields	204
	Use of Dose Calibrator	205
	Radioactive Spill	205
	Record Keeping	206
	Principles of Radiation Protection	206
	Time	206
	Distance	206
	Shielding	207
	Activity	209
	Do's and Don'ts in Radiation Protection Practice	209

Department of Transportation	210
Distribution of ^{18}F -FDG	212
Questions	213
References and Suggested Reading	214
11 Reimbursement for PET Procedures	215
Background	215
Coverage	215
Coding	216
CPT, HCPCS, and APC Codes	216
Diagnosis Codes	217
Payment	217
Hospital Inpatient Services: Medicare Part A	219
Hospital Outpatient Services: Medicare Part B	219
Payment for PET Radiopharmaceuticals	220
Physician's Payment by Medicare	220
Freestanding Facilities: Medicare	221
Non-Medicare Payers: All Settings	221
Billing	221
Billing Process	222
Chronology of Reimbursement for PET Procedures	223
National Oncologic PET Registry	224
Questions	225
References and Suggested Reading	225
12 Design and Cost of PET Center	227
Introduction	227
Site Planning	228
Passage	229
PET Center	229
Scanner Section	229
Cyclotron Section	230
Office Area	231
Caveat	232
Shielding	232
Case Study	236
Cost of PET/CT and Cyclotron Operation	237
Questions	238
References and Suggested Reading	239
13 Sample Procedures for PET Studies	241
Introduction	241
Whole-Body PET Imaging with ^{18}F -FDG	242
Physician's Directive	242
Patient Preparation	242
Dosage Administration	242

Scan	242
Reconstruction and Storage	243
Whole-Body PET/CT Imaging with ^{18}F -FDG	243
Physician Directive	243
Patient Preparation	244
Dosage Administration	244
Scan	244
Reconstruction and Storage	245
Amyloid-Plaque Imaging in Alzheimer's Patients Using ^{18}F -Florbetapir PET/CT	245
Patient Preparation	245
Dosage Administration	245
Scan	245
Reconstruction and Storage	246
Myocardial Metabolic PET or PET/CT Imaging with ^{18}F -FDG	246
Patient Preparation	246
Dosage Administration	247
Scan	247
Reconstruction and Storage	247
Myocardial Perfusion PET or PET/CT Imaging with ^{82}Rb -RbCl	248
Patient Preparation	248
Dosage Administration and Scan	248
Reconstruction and Storage	249
References and Suggested Reading	249
14 Topics of Interest	251
Estimated Absorbed Dose from Intravenous Administration of ^{18}F -FDG and ^{82}Rb -RbCl in Humans	251
Evaluation of Tumor Uptake of ^{18}F -FDG by PET	251
Infusion Pump for ^{82}Sr - ^{82}Rb Generator	255
Calculation of ^{82}Sr and ^{85}Sr Breakthrough in ^{82}Rb Eluate	256
Questions	257
References and Suggested Reading	257
Appendix A: Abbreviations Used in the Text	259
Appendix B: Terms Used in the Text	263
Appendix C: Units and Constants	269
Appendix D: Answers to Questions	271
Index	275

Chapter 1

Radioactive Decay and Interaction of Radiation with Matter

Atomic Structure

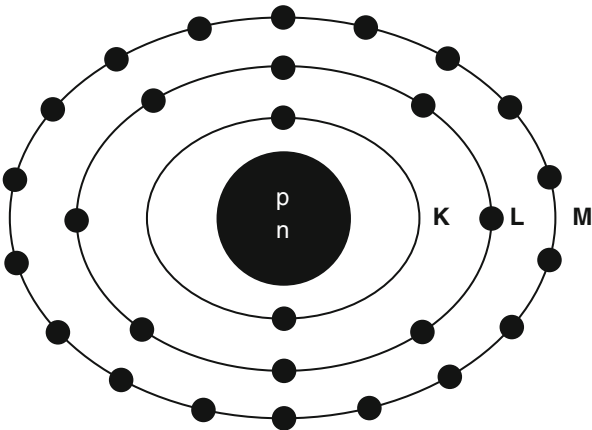
Matter is composed of atoms. An atom consists of a nucleus containing protons (Z) and neutrons (N), collectively called nucleons, and electrons rotating around the nucleus. The sum of neutrons and protons (total number of nucleons) is the mass number denoted by A . The properties of neutrons, protons, and electrons are listed in Table 1.1. The number of electrons in an atom is equal to the number of protons (atomic number Z) in the nucleus. The electrons rotate along different energy shells designated as K-shell, L-shell, M-shell, etc. (Fig. 1.1). Each shell further consists of subshells or orbitals, e.g., the K-shell has s orbital; the L-shell has s and p orbitals; the M-shell has s , p , and d orbitals, and the N-shell has s , p , d , and f orbitals. Each orbital can accommodate only a limited number of electrons. For example, the s orbital contains up to 2 electrons; the p orbital, 6 electrons; the d orbital, 10 electrons; and the f orbital, 14 electrons. The capacity number of electrons in each orbital adds up to give the maximum number of electrons that each energy shell can hold. Thus, the K-shell contains 2 electrons; the L-shell 8 electrons, the M-shell 18 electrons, and so forth.

A combination of a given number of protons and neutrons in a nucleus leads to an atom called the nuclide. A nuclide X is represented by A_ZX_N . Some nuclides (280 or so) are stable, while others (more than 3400) are unstable. The unstable nuclides are termed the radionuclides, most of which are artificially produced in the cyclotron or reactor, with a few naturally occurring. The nuclides having the same number of protons are called the isotopes, e.g., ${}^{12}_6\text{C}$ and ${}^{13}_6\text{C}$; the nuclides having the same number of neutrons are called the isotones, e.g., ${}^{16}_8\text{O}$ and ${}^{15}_7\text{N}$; the nuclides having the same mass number are called the isobars, e.g., ${}^{131}_{53}\text{I}$ and ${}^{131}_{54}\text{Xe}$; and the nuclides with the same mass number but differing in energy are called the isomers, e.g., ${}^{99\text{m}}_{44}\text{Tc}$ and ${}^{99}_{44}\text{Tc}$.

Table 1.1 Characteristics of electrons and nucleons				
Particle	Charge	Mass (amu) ^a	Mass (kg)	Mass (MeV) ^b
Electron	−1	0.000549	0.9108×10^{-30}	0.511
Proton	+1	1.00728	1.6721×10^{-27}	938.78
Neutron	0	1.00867	1.6744×10^{-27}	939.07

^aamu = 1 atomic mass unit = 1.66×10^{-27} kg = 1/12 of the mass of ¹²C
^b1 atomic mass unit = 931 MeV

Fig. 1.1 Schematic structure of a ²⁸Ni atom. The nucleus containing protons and neutrons is at the center. The K-shell has 2 electrons, the L-shell 8 electrons, and the M-shell 18 electrons

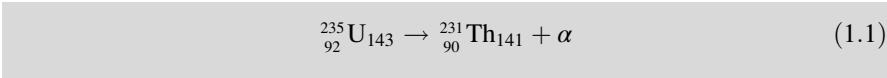


Radioactive Decay

Radionuclides are unstable due to the unsuitable composition of neutrons and protons or excess energy and, therefore, decay by emission of radiations such as α particles, β^- particles, β^+ particles, electron capture, and isomeric transition.

α Decay

This decay occurs in heavy nuclei such as ²³⁵U and ²³⁹Pu. For example,



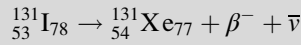
Alpha particles are a nucleus of helium atom ⁴He₂ having two protons and two neutrons in the nucleus with two orbital electrons stripped off from the K-shell. The α particles are emitted with discrete energy and have a very short range in matter, e.g., about 0.03 mm in human tissues.

β^- Decay

β^- Decay occurs in radionuclides that are neutron rich. In the process, a neutron in the nucleus is converted to a proton along with the emission of a β^- particle and an antineutrino, $\bar{\nu}$.

$$n \rightarrow p + \beta^- + \bar{\nu} \quad (1.2)$$

For example,



The energy difference between the two nuclides (i.e., between ${}^{131}\text{I}$ and ${}^{131}\text{Xe}$ in the above example) is called the decay energy or transition energy, which is shared between the β^- particle and the antineutrino $\bar{\nu}$. Because of the random nature of decay, β^- particles are emitted with a spectrum of energy with the transition energy as the maximum energy and with an average energy equal to one-third of the maximum energy.

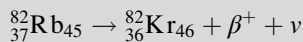
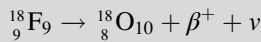
Positron (β^+) Decay

When a radionuclide is proton rich, it decays by the emission of a positron (β^+) along with a neutrino ν . In essence, a proton in the nucleus is converted to a neutron in the process.

$$p = n + \beta^+ + \nu \quad (1.3)$$

Since a neutron is one electron mass heavier than a proton, the right-hand side of Eq. (1.3) is two electron mass more than the left-hand side, i.e., $2 \times 0.511 \text{ MeV} = 1.022 \text{ MeV}$ more on the right side. For conservation of energy, therefore, the radionuclide must have a transition energy of at least 1.022 MeV to decay by β^+ emission. The energy beyond 1.022 MeV is shared as kinetic energy by the β^+ particle and the neutrino.

Some examples of positron-emitting nuclides are:



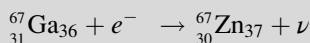
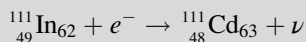
Positron emission tomography (PET) is based on the principle of coincidence detection of the two 511-keV photons arising from positron emitters, which will be discussed in detail later.

Electron Capture

When a radionuclide is proton rich, but has energy less than 1.022 MeV, then it decays by electron capture. In the process, an electron from the nearest shell, i.e., K-shell, is captured by a proton in the nucleus to produce a neutron, and a neutrino ν is emitted to conserve energy.

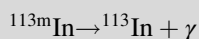
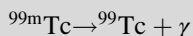


Note that when the transition energy is less than 1.022 MeV, the radionuclide definitely decays by electron capture. However, when the transition energy is more than 1.022 MeV, the radionuclide can decay by positron emission and/or electron capture. The greater the transition energy above 1.022 MeV, the more likely the radionuclide will decay by positron emission. Some examples of radionuclides decaying by electron capture are:



Isomeric Transition

When a nucleus has excess energy above the ground state, it can exist in excited (energy) states, which are called the isomeric states. The lifetimes of these states normally are very short ($\sim 10^{-15}$ to 10^{-12} s); however, in some cases, the lifetime can be longer from minutes to years. When an isomeric state has a longer lifetime, it is called a metastable state and is represented by “*m*.” Thus, having an energy state of 140 keV above ^{99}Tc and decaying with a half-life of 6 h, $^{99\text{m}}\text{Tc}$ is an isomer of ^{99}Tc .



A radionuclide may decay by α , β^{-} , β^{+} emissions, or electron capture to different isomeric states of the product nucleus, if allowed by the rules of quantum physics.

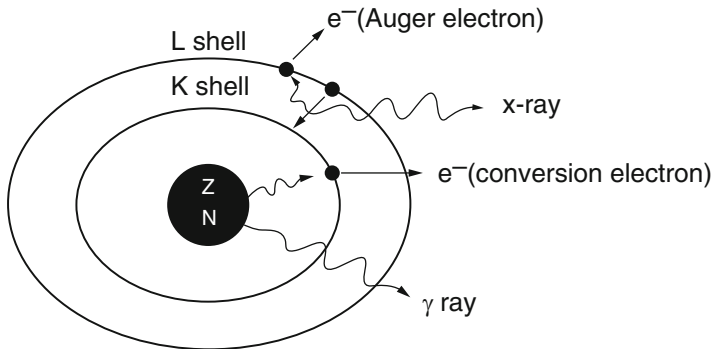


Fig. 1.2 γ -Ray emission and internal conversion process. In internal conversion process, the excitation energy of the nucleus is transferred to a K-shell electron, which is then ejected, and the K-shell vacancy is filled by an electron from the L-shell. The energy difference between the L-shell and K-shell appears as the characteristic K X-ray. The characteristic K X-ray energy may again be transferred to an L-shell electron, which is then ejected in the Auger process

Naturally, these isomeric states decay to lower isomeric states and finally to the ground states of the product nucleus, and the energy differences appear as γ -ray photons.

As an alternative to γ -ray emission, the excitation energy may be transferred to an electron, preferably in the K-shell, which is then ejected with energy $E_\gamma - E_B$, where E_γ and E_B are the γ -ray energy and binding energy of the electron, respectively (Fig. 1.2). This process is called the internal conversion, and the ejected electron is called the conversion electron. The vacancy created in the K-shell is filled by the transition of an electron from an upper shell. The energy difference between the two shells appears as a characteristic K X-ray. Similarly, characteristic L X-ray and M X-ray can be emitted if the vacancy in the L- or M-shell is filled by electron transition from upper shells. Like γ rays, the characteristic X-ray energy can be emitted as photons or be transferred to an electron in a shell which is then ejected, if energetically possible. The latter is called the Auger process, and the ejected electron is called the Auger electron.

The decay of radionuclides is represented by a decay scheme, an example of which is given for ^{68}Ga in Fig. 1.3.

Radioactive Decay Equations

General Decay Equations

The atoms of a radioactive sample decay randomly, and one cannot tell which atom will decay when. One can only talk about an average decay of the atoms in the sample. This decay rate is proportional to the number of radioactive atoms present. Mathematically,

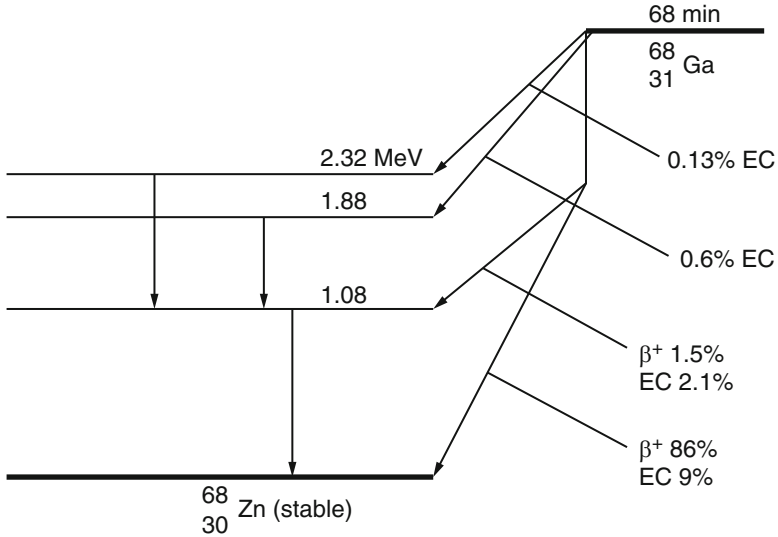


Fig. 1.3 The decay scheme of ^{68}Ga . The 87.5% of positrons are annihilated to give rise to 175% of 511-keV photons

$$-\frac{dN}{dt} = \lambda N, \quad (1.5)$$

where $-dN/dt$ is the rate of decay denoted by the term activity or radioactivity A , λ is the decay constant, and N is the number of atoms of the radionuclide present. Thus,

$$A = \lambda N, \quad (1.6)$$

Integrating Eq. (1.5) gives the activity A_t at time t as

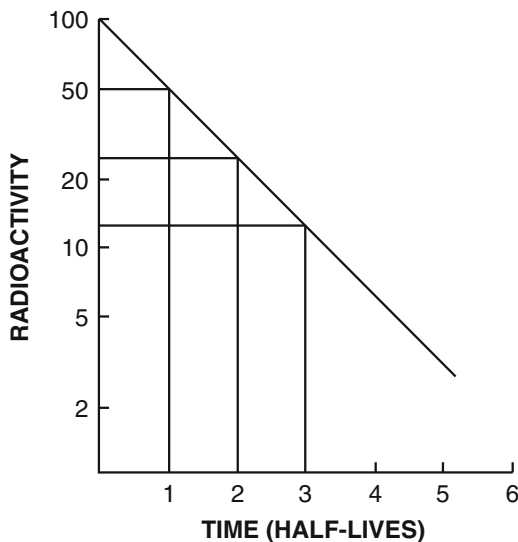
$$A_t = A_0 e^{-\lambda t}, \quad (1.7)$$

where A_0 is the activity at time $t = 0$. The plot of A_t vs. t on a semilog scale is shown in Fig. 1.4. If one knows activity A_0 at a given time, the activity A_t at time t before or later can be calculated by Eq. (1.7).

Half-life ($t_{1/2}$): The half-life of a radionuclide is defined as the time required to reduce the initial activity to one-half. It is unique for every radionuclide and is related to the decay constant as follows:

$$\lambda = \frac{0.693}{t_{1/2}}. \quad (1.8)$$

Fig. 1.4 Plot of activity A_t against time on a semilogarithmic graph indicating a *straight line*. The slope of the line is the decay constant λ of the radionuclide. The half-life $t_{1/2}$ is calculated from λ using (1.8). Alternatively, the half-life is determined by reading an initial activity and half its value and their corresponding times. The difference in time between the two readings is the half-life



The half-life of a radionuclide is determined by measuring the radioactivity at different time intervals and plotting them on semilogarithmic paper, as shown in Fig. 1.4. An initial activity and half its value are read from the straight line, and the corresponding times are noted. The difference in time between the two readings gives the half-life of the radionuclide.

The mean life τ of a radionuclide is defined by

$$\tau = \frac{1}{\lambda} = \frac{t_{1/2}}{0.693} = 1.44t_{1/2}. \quad (1.9)$$

A radionuclide decays by 63% in one mean life.

Effective half-life: Each radionuclide decays with a definite half-life, called the physical half-life, which is denoted by T_p or $t_{1/2}$. When radiopharmaceuticals are administered to patients, analogous to physical decay, they are eliminated from the body by biological processes such as fecal excretion, urinary excretion, and perspiration. This elimination is characterized by a biological half-life (T_b) which is defined as the time taken to eliminate a half of the administered activity from the biological system. It is related to the decay constant λ_b by

$$\lambda_b = \frac{0.693}{T_b}.$$

Thus, in a biological system, the loss of a radiopharmaceutical is related to λ_p and λ_b . The net effective rate of loss (λ_e) is characterized by

$$\lambda_e = \lambda_p + \lambda_b. \quad (1.10)$$

Since $\lambda = 0.693/t_{1/2}$,

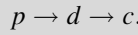
$$\frac{1}{T_e} = \frac{1}{T_p} + \frac{1}{T_b}, \quad (1.11)$$

$$T_e = \frac{T_p \times T_b}{T_p + T_b}. \quad (1.12)$$

The effective half-life is always less than the shorter of T_p or T_b . For a very long T_p and a short T_b , T_e is almost equal to T_b . Similarly, for a very long T_b and a short T_p , T_e is almost equal to T_p .

Successive Decay Equations

In a successive decay, a parent radionuclide p decays to a daughter nuclide d , and d in turn decays to another nuclide c , and we are interested in the decay rate of d over time. Thus,



Mathematically,

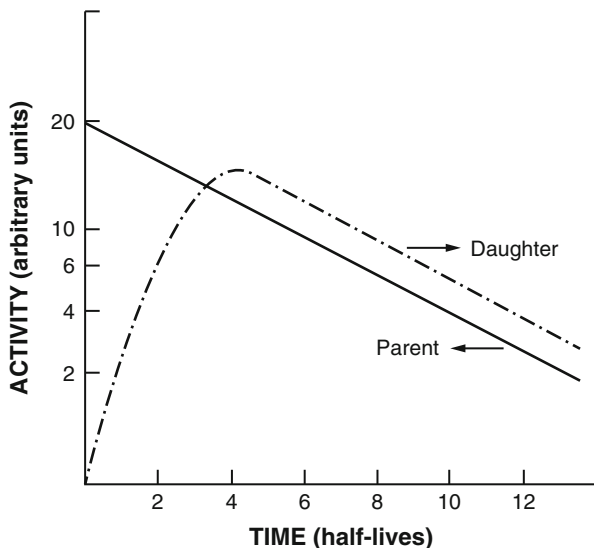
$$-\frac{dN_d}{dt} = \lambda_p N_p - \lambda_d N_d. \quad (1.13)$$

On integration,

$$A_d = \frac{\lambda_d (A_p)_0}{\lambda_d - \lambda_p} [e^{-\lambda_p t} - e^{-\lambda_d t}]. \quad (1.14)$$

If the parent half-life is greater than the daughter half-life (say, by a factor of 10–100), that is, $(t_{1/2})_p > (t_{1/2})_d$ or $\lambda_d > \lambda_p$ and also if the time of decay (t) is very long, then $e^{-\lambda_d t}$ is almost zero compared to $e^{-\lambda_p t}$. Then

Fig. 1.5 The transient equilibrium is illustrated in the plot of activity vs. time on a semilogarithmic graph. The daughter activity increases initially with time, reaches a maximum, then transient equilibrium, and finally appears to decay following the half-life of the parent. Note that the daughter activity is higher than the parent activity in equilibrium



$$(A_d)_t = \frac{\lambda_d}{\lambda_d - \lambda_p} (A_p)_t. \quad (1.15)$$

Equation (1.15) represents a *transient equilibrium* between the parent p and the daughter d radionuclides, which is achieved after several half-lives of the daughter. The graphical representation of this equilibrium is shown in Fig. 1.5. It can be seen that after equilibrium, the daughter activity is greater than the parent activity and the daughter appears to decay following the half-life of the parent. The principle of transient equilibrium is applied to many radionuclide generators such as the ^{99}Mo – $^{99\text{m}}\text{Tc}$ generator.

If the parent half-life is much greater than the daughter half-life (by a factor of hundreds or thousands), then λ_p is negligible compared to λ_d , that is, $\lambda_d \gg \lambda_p$. Then Eq. (1.15) becomes

$$(A_d)_t = (A_p)_t. \quad (1.16)$$

This equation represents a *secular equilibrium* in which the daughter activity becomes equal to the parent activity, and the daughter decays with the half-life of the parent. The ^{82}Sr – ^{82}Rb generator is an example of secular equilibrium.

Units of Radioactivity

$$1 \text{ Ci} = 3.7 \times 10^{10} \text{ disintegration per second (dps)}$$

$$1 \text{ mCi} = 3.7 \times 10^7 \text{ dps}$$

$$1 \mu\text{Ci} = 3.7 \times 10^4 \text{ dps}$$

Units of Radioactivity in System Internationale

$$1 \text{ Becquerel (Bq)} = 1 \text{ dps}$$

$$1 \text{ kBq} = 10^3 \text{ dps} = 2.7 \times 10^{-8} \text{ Ci}$$

$$1 \text{ Mbq} = 10^6 \text{ dps} = 2.7 \times 10^{-5} \text{ Ci}$$

$$1 \text{ GBq} = 10^9 \text{ dps} = 2.7 \times 10^{-2} \text{ Ci}$$

Calculations

Problem 1.1 A dosage of ^{18}F -FDG has 20 mCi at 10 a.m. Wednesday. Calculate the activity of the dosage at 7 a.m. and 2 p.m. that day. The half-life of ^{18}F is 110 min.

Answer:

$$\lambda \text{ for } ^{18}\text{F} = \frac{0.693}{110} \text{ min}^{-1}$$

$$\text{Time from 7 a.m. to 10 a.m.} = 43 \text{ h} = 180 \text{ min}$$

$$\text{Time from 10 a.m. to 2 a.m.} = 4 \text{ h} = 240 \text{ min}$$

$$\text{Activity of } ^{18}\text{F}\text{-FDG at 7 a.m.} = 20 \times e^{+\frac{0.693}{110}} \times 180$$

$$= 20 \times e^{+1.134}$$

$$= 62 \text{ mCi (2.29 GBq)}$$

$$\text{Activity of } ^{18}\text{F}\text{-FDG at 2 p.m.} = 20 \times e^{-\frac{0.693 \times 240}{110}}$$

$$= 20 \times e^{-1.512}$$

$$= 20 \times 0.22 = 4.4 \text{ mCi (163.1 MBq)}$$

Problem 1.2 A radioactive sample decays 40% per hour. What is the half-life of the radionuclide?

Answer:

$$\lambda = 0.4 \text{ h}^{-1} = \frac{0.693}{t_{1/2}}$$

$$t_{1/2} = \frac{0.693}{0.4} = 1.73 \text{ h.}$$

Interaction of Radiation with Matter

Radiations are either particulate type, such as α particle and β particle, or nonparticulate type, such as high-frequency electromagnetic radiation (e.g., γ rays, X-rays), and both kinds are ionizing radiations. The mode of interaction of these two types of radiations with matter is different.

Interaction of Charged Particles with Matter

The energetic charged particles such as α particles and β particles, while passing through matter, lose their energy by interacting with the orbital electrons of the atoms in the matter. In these processes, the atoms are ionized in which the electron in the encounter is ejected or the atoms are excited in which the electron is raised to a higher energy state. In both excitation and ionization processes, chemical bonds in the molecules of the matter may be ruptured, forming a variety of chemical entities.

The lighter charged particles (e.g., β particles) move in a zigzag path in the matter, whereas the heavier particles (e.g., α particles) move in a straight path, because of the heavy mass and charge. The straight line path traversed by the charged particles is called the range R . The range of a charged particle depends on the energy, charge, and mass of the particle as well as the density of the matter it passes through. It increases with increasing charge and energy, while it decreases with increasing mass of the particle and increasing density of the matter. The range of positrons and other properties of common positron-emitters are given in Table 1.2.

A unique situation of the passage of positrons through an absorber is that as a positron loses its energy by interaction with electrons of the absorber atoms and comes to almost rest, it combines with an electron of an absorber atom. At this instant, both particles (β^+ and e^-) are annihilated as a result of matter–antimatter encounter to produce two photons of 511 keV, which are emitted in opposite directions ($\sim 180^\circ$) (Fig. 1.6). This process is called the *annihilation* process. Because the positrons have a residual momentum at the time of annihilation, the

Table 1.2 Properties of common positron emitters

Radionuclide range	Half-life	$E_{\beta^{+},\text{max}}$ (MeV)	Max. β^{+} range (mm) in water	Average β^{+} range (mm) in water
^{11}C	20.4 min	0.97	3.8	0.85
^{13}N	10 min	1.20	5.0	1.15
^{15}O	2 min	1.74	8.0	1.80
^{18}F	110 min	0.64	2.2	0.46
^{68}Ga	68 min	1.90	9.0	2.15
^{82}Rb	75 s	3.35	15.5	4.10

Adapted by the permission of the Society of Nuclear Medicine from Brown TF, Yasillo NJ (1997) Radiation safety considerations for PET centers. J Nucl Med Technol 25:98

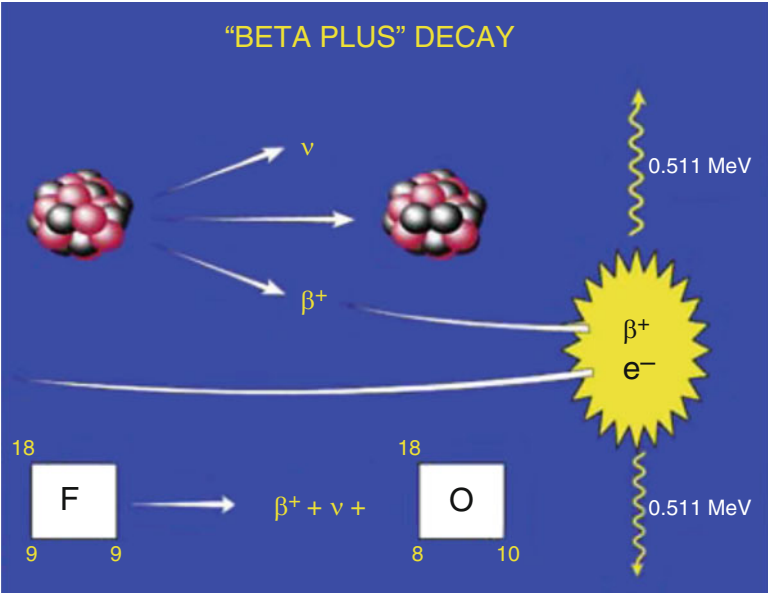
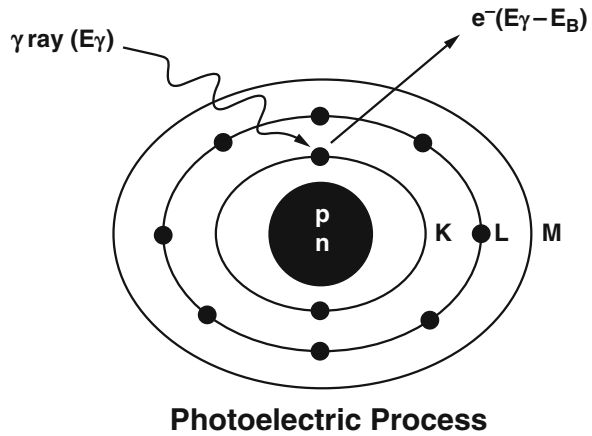


Fig. 1.6 A schematic illustration of the annihilation of a positron and an electron in the medium. Two 511-keV photons are produced and emitted in opposite directions (180°) (Reprinted with the permission of The Cleveland Clinic Center for Medical Art & Photography © 2009. All Rights Reserved)

two annihilation photons are not emitted exactly at 180°. Detection of the two 511-keV photons in coincidence by two opposite detectors is the basis of PET.

An important parameter related to the interaction of radiations with matter is linear energy transfer (LET). It is the energy deposited by a radiation per unit length of the path in the absorber and is normally given in units of kiloelectron volt per micrometer (keV/ μm). The LET varies with the energy, charge, and mass of the particle. The γ radiations and β^- particles interact with matter depositing relatively

Fig. 1.7 An illustration of photoelectric effect, where a γ ray transfers all its energy E_γ to a K-shell electron and the electron is ejected with $E_\gamma - E_B$, where E_B is the binding energy of the electron in the K-shell. The characteristic K X-ray emission or the Auger process can follow, as described in Fig. 1.2



less amount of energy per unit length and so have low LET. On the other hand, α particles and protons deposit more energy per unit length because of their greater mass and charge and so have higher LET.

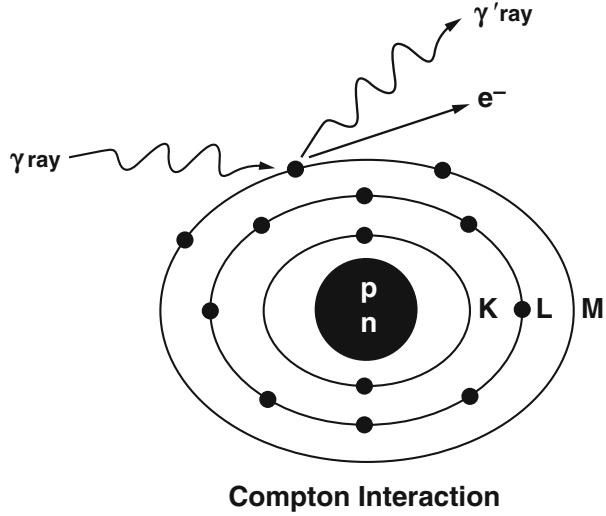
Interaction of γ Radiation with Matter

In the spectrum of electromagnetic radiations, γ radiations are high-frequency radiations and interact with matter by three mechanisms: photoelectric, Compton, and pair production.

Photoelectric Process: In this process, a γ radiation, while passing through an absorber, transfers its entire energy primarily to an inner shell electron (e.g., the K-shell) of an absorber atom and ejects the electron (Fig. 1.7). The ejected electron will have the kinetic energy equal to $E_\gamma - E_B$, where E_γ is the γ -ray energy and E_B is the binding energy of the electron in the shell. The probability of this process decreases with increasing energy of the γ ray, but increases with increasing atomic number of the absorber. It is roughly given by Z^5/E_γ^3 . The vacancy in the shell is filled in by the transition of an electron from the upper shell, which is followed by emission of the energy difference between the two shells as characteristic X-rays or by the Auger process described in the internal conversion process.

Compton Scattering Process: In a Compton scattering process, a γ radiation with somewhat higher energy interacts with an outer shell electron of the absorber atom transferring only part of its energy to the electron and ejecting it (Fig. 1.8). The ejected electron is called the Compton electron and carries a part of the γ -ray energy minus its binding energy E_B in the shell, i.e., $E'_\gamma - E_B$, where E'_γ is a part of the original γ ray energy E_γ that is transferred to the electron. The scattered photon carries energy equal to $E_\gamma - E'_\gamma - E_B$. Thus, in Compton scattering, a scattered photon and a Compton electron are produced. The scattered photon may again,

Fig. 1.8 The Compton scattering process in which a γ ray transfers only a part of its energy to an electron in a shell and is itself scattered with reduced energy. The electron is ejected from the shell with energy, $E'_\gamma - E_B$, where E'_γ is the partial energy transferred by the γ ray and E_B is the binding energy of the electron in the shell. The remaining γ -ray energy appears as a scattered photon



depending on the energy and location of interaction, encounter a photoelectric process or another Compton scattering process, or leave the absorber without interaction. As the energy of the γ radiation increases, the photoelectric process decreases and the Compton scattering process increases, but the latter also decreases with photon energy above 1.0 MeV or so. The probability of Compton scattering is independent of the atomic number Z of the absorber.

Pair Production: When the γ -ray energy is higher than 1.022 MeV, the photon interacts with the nucleus of an absorber atom during its passage through it and produces a positron and an electron. This is called pair production. The excess energy beyond 1.022 MeV is shared as kinetic energy between the two particles. The probability of pair production increases with increasing photon energy above 1.022 MeV. The positron produced will undergo annihilation in the absorber as described earlier.

Attenuation of γ Radiations: When γ radiations pass through the absorber medium, they undergo one or a combination of the above three processes (photoelectric, Compton, and pair production) depending on their energy, or they are transmitted out of the absorber without any interaction. The combined effect of the three processes is called the *attenuation* of the γ radiations (Fig. 1.9). For a γ radiation passing through an absorber, the linear attenuation coefficient (μ_ℓ) of the γ radiation is given by

$$\mu_\ell = \tau + \sigma + \kappa, \quad (1.17)$$

where τ is the photoelectric coefficient, σ is the Compton coefficient, and κ is the pair production coefficient (Fig. 1.10). The linear attenuation coefficient of a