LEARNING MADE EASY



Calculus Essentials



The "must-know" formulas and equations

Exactly what you need to know to conquer calculus

Core calculus topics in quick, focused lessons

Mark Ryan

Founder and owner of The Math Center, author of Calculus For Dummies and Calculus Workbook For Dummies



Calculus Essentials

by Mark Ryan



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Introduction

he mere thought of having to take a required calculus course is enough to make legions of students break out in a cold sweat. Others who have no intention of ever studying the subject have this notion that calculus is impossibly difficult unless you happen to be a direct descendant of Einstein.

Well, I'm here to tell you that you *can* master calculus. It's not nearly as tough as its mystique would lead you to think. Much of calculus is really just very advanced algebra, geometry, and trig. It builds upon and is a logical extension of those subjects. If you can do algebra, geometry, and trig, you can do calculus. Read this jargon-free book, get a handle on calculus, and join the happy few who can proudly say, "Calculus? Oh, sure, I know calculus. It's no big deal."

About This Book

Calculus Essentials For Dummies is intended for three groups of readers: students taking their first calculus course, students who need to brush up on their calculus to prepare for other studies, and adults of all ages who'd like a good introduction to the subject. For those who'd like a fuller treatment of the subject, check out *Calculus For Dummies*.

If you're enrolled in a calculus course and you find your textbook less than crystal clear, *Calculus Essentials For Dummies* is the book for you. It covers the two most important topics in the first year of calculus: differentiation and integration.

If you've had elementary calculus, but it's been a couple of years and you want to review the concepts to prepare for, say, some graduate program, *Calculus Essentials For Dummies* will give you a quick, no-nonsense refresher course.

Nonstudent readers will find the book's exposition clear and accessible. *Calculus Essentials For Dummies* takes calculus out of the ivory tower and brings it down to earth.

This is a user-friendly math book. Whenever possible, I explain the calculus concepts by showing you connections between the calculus ideas and easier ideas from algebra and geometry. I then show you how the calculus concepts work in concrete examples. Only later do I give you the fancy calculus formulas. All explanations are in plain English, not math-speak.

Conventions Used in This Book

The following conventions keep the text consistent and oh-soeasy to follow.

- >> Variables are in *italics*.
- Calculus terms are italicized and defined when they first appear in the text.
- In the step-by-step problem-solving methods, the general action you need to take is in **bold**, followed by the specifics of the particular problem.

Foolish Assumptions

Call me crazy, but I assume . . .

- >> You know at least the basics of algebra, geometry, and trig.
- If you're rusty, you might want to brush up a bit on these pre-calculus topics. Actually, if you're not currently taking a calculus course, and you're reading this book just to satisfy a general curiosity about calculus, you can get a good conceptual picture of the subject without the nitty-gritty details of algebra, geometry, and trig. But you won't, in that case, be able to follow all the problem solutions. In short, without the pre-calculus stuff, you can see the calculus forest, but not the trees. If you are enrolled in a calculus course, you've got no choice — you've got to know the trees.

- You're willing to do some w___.
- No, not the dreaded w-word! Yes, that's w-o-r-k, work. I've tried to make this material as accessible as possible, but it is calculus after all. You can't learn calculus by just listening to a tape in your car or taking a pill not yet anyway.

Icons Used in This Book

Keep your eyes on the icons:



Next to this icon are the essential calculus rules, definitions, and formulas you should definitely know.



These are things you need to know from algebra, geometry, or trig, or things you should recall from earlier in the book.

REMEMBER



The bull's-eye icon appears next to things that will make your life easier. Take note.



This icon highlights common calculus mistakes. Take heed.



In contrast to the Critical Calculus Concepts, you generally don't need to memorize the fancy-pants formulas next to this icon unless your calc teacher insists.

Where to Go from Here

Why, Chapter 1, of course, if you want to start at the beginning. If you already have some background in calculus or just need a refresher course in one area or another, then feel free to skip around. Use the table of contents and index to find what you're looking for. If all goes well, in a half a year or so, you'll be able to check calculus off your bucket list:

___ Run a marathon

- ___ Go skydiving
- ____ Write a book
- <u>x</u>Learn calculus
- ____ Swim the English Channel
- ___ Cure cancer
- ____ Write a symphony
- ____ Pull an inverted 360° at the X-Games

For the rest of your list, you're on your own.

- » Calculus it's just souped-up regular math
- » Zooming in is the key
- » Delving into the derivative: It's a rate or a slope
- » Investigating the integral addition for experts

Chapter **1** Calculus: No Big Deal

n this chapter, I answer the question "What is calculus?" in plain English and give you real-world examples of how it's used. Then I introduce you to the two big ideas in calculus: differentiation and integration. Finally, I explain why calculus works. After reading this chapter, you *will* understand what calculus is all about.

So What Is Calculus Already?

Calculus is basically just very advanced algebra and geometry. In one sense, it's not even a new subject — it takes the ordinary rules of algebra and geometry and tweaks them so that they can be used on more complicated problems. (The rub, of course, is that darn *other* sense in which it *is* a new and more difficult subject.)

Look at Figure 1–1. On the left is a man pushing a crate up a straight incline. On the right, the man is pushing the same crate up a curving incline. The problem, in both cases, is to determine the amount of energy required to push the crate to the top. You can do the problem on the left with regular math. For the one on the right, you need calculus (if you don't know the physics shortcuts).







FIGURE 1-1: The difference between regular math and calculus: In a word, it's the curve.

For the straight incline, the man pushes with an *unchanging* force, and the crate goes up the incline at an *unchanging* speed. With some simple physics formulas and regular math (including algebra and trig), you can compute how many calories of energy are required to push the crate up the incline. Note that the amount of energy expended each second remains the same.

For the curving incline, on the other hand, things are constantly *changing*. The steepness of the incline is *changing* — and it's not like it's one steepness for the first 3 feet and then a different steepness for the next 3 — it's *constantly changing*. And the man pushes with a *constantly changing* force — the steeper the incline, the harder the push. As a result, the amount of energy expended is also *changing*, not just every second or thousandth of a second, but *constantly*, from one moment to the next. That's what makes it a calculus problem. It should come as no surprise to you, then, that calculus is called "the mathematics of change." Calculus takes the regular rules of math and applies them to fluid, evolving problems.

For the curving incline problem, the physics formulas remain the same, and the algebra and trig you use stay the same. The difference is that — in contrast to the straight incline problem, which you can sort of do in a single shot — you've got to break up the curving incline problem into small chunks and do each chunk separately. Figure 1-2 shows a small portion of the curving incline blown up to several times its size.

When you zoom in far enough, the small length of the curving incline becomes practically straight. Then you can solve that small chunk just like the straight incline problem. Each small chunk can be solved the same way, and then you just add up all the chunks.



FIGURE 1-2: Zooming in on the curve — voilà, it's straight (almost).

That's calculus in a nutshell. It takes a problem that can't be done with regular math because things are constantly changing — the changing quantities show up on a graph as curves — it zooms in on the curve till it becomes straight, and then it finishes off the problem with regular math.

What makes calculus such a fantastic achievement is that it does what seems impossible: it zooms in *infinitely*. As a matter of fact, everything in calculus involves infinity in one way or another, because if something is constantly changing, it's changing infinitely often from each infinitesimal moment to the next.

Real-World Examples of Calculus

So, with regular math you can do the straight incline problem; with calculus you can do the curving incline problem. With regular math you can determine the length of a buried cable that runs diagonally from one corner of a park to the other (remember the Pythagorean Theorem?). With calculus you can determine the length of a cable hung between two towers that has the shape of a *catenary* (which is different, by the way, from a simple circular arc or a parabola). Knowing the exact length is of obvious importance to a power company planning hundreds of miles of new electric cable.

You can calculate the area of the flat roof of a home with regular math. With calculus you can compute the area of a complicated, nonspherical shape like the dome of the Minneapolis Metrodome. Architects need to know the dome's area to determine the cost of materials and to figure the weight of the dome (with and without snow on it). The weight, of course, is needed for planning the strength of the supporting structure.

With regular math and simple physics, you can calculate how much a quarterback must lead a pass receiver if the receiver runs in a *straight* line and at a *constant* speed. But when NASA, in 1975, calculated the necessary "lead" for aiming the Viking I at Mars, it needed calculus because both the Earth and Mars travel on *ellip*-*tical* orbits, and the speeds of both are *constantly changing* — not to mention the fact that on its way to Mars, the spacecraft was affected by the different and *constantly changing* gravitational pulls of the Earth, moon, Mars, the sun. See Figure 1–3.



FIGURE 1-3: B.C.E. (Before the Calculus Era) and C.E. (the Calculus Era).

Differentiation

Differentiation is the first big idea in calculus. It's the process of finding a *derivative* of a curve. And a derivative is just the fancy calculus term for a curve's slope or steepness.

In algebra, you learned the slope of a line is equal to the ratio of the *rise* to the *run*. In other words, $Slop = \frac{rise}{run}$. In Figure 1–4, the *rise* is half as long as the *run*, so segment AB has a slope of 1/2. On a curve, the slope is constantly *changing*, so you need calculus to determine its slope.



FIGURE 1-4: Calculating the slope of a curve isn't as simple as *rise* over *run*.

The slope of segment AB is the same at every point from A to B. But the steepness of the curve is changing between A and B. At A, the curve is less steep than the segment, and at B the curve is steeper than the segment. So what do you do if you want the exact slope at, say, point C? You just zoom in. See Figure 1–5.



FIGURE 1-5: Zooming in on the curve.

When you zoom in far enough — actually infinitely far — the little piece of the curve becomes straight, and you can figure the slope the old-fashioned way. That's how differentiation works.

Integration

Integration, the second big idea in calculus, is basically just fancy addition. Integration is the process of cutting up an area into tiny sections, figuring out their areas, and then adding them up to get the whole area. Figure 1–6 shows two area problems — one that you can do with geometry and one where you need calculus.



FIGURE 1-6: If you can't determine the area on the left, hang up your calculator.

The shaded area on the left is a simple rectangle, so its area, of course, equals length times width. But you can't figure the area on the right with regular geometry because there's no area formula for this funny shape. So what do you do? Why, zoom in, of course. Figure 1-7 shows the top portion of a narrow strip of the weird shape blown up to several times its size.



FIGURE 1-7: For the umpteenth time: When you zoom in, curves become straight.

When you zoom in, the curve becomes practically straight, and the further you zoom in, the straighter it gets — with integration, you actually zoom in *infinitely* close, sort of. You end up with the shape on the right in Figure 1-7, an ordinary trapezoid — or a triangle sitting on top of a rectangle. Because you can compute the areas of rectangles, triangles, and trapezoids with ordinary geometry, you can get the area of this and all the other thin strips and then add up all these areas to get the total area. That's integration.