

ALGORITHMIC DIFFERENTIATION IN FINANCE **EXPLAINED**

Marc Henrard

FINANCIAL
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Financial Engineering Explained

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Preface

The book's title originally proposed to the editor was "Forty-nine shades of Algorithmic Differentiation"; a title that he duly refused as not fitting the series guideline very well and potentially distracting the target audience. The proposed title was not only a marketing exercise; there may not be as many as forty-nine variations of Algorithmic Differentiation (AD) presented in the book, nevertheless it is one of the goals of the book to present as many variations, shades or faces of the subject as possible, and not a one-size-fit-all approach. While exaggerating the number of shades, why not going to fifty? I'm expecting the readers to implement their own version of the technique, so for each reader it will be "*fifty shades of AD*." I also removed one from the number in the title to avoid confusion between AD and grey; AD is a very colorful subject!

Algorithmic Differentiation (AD) has been used in engineering and computer science for a long time. The term *Algorithmic Differentiation* can be explained as "the art of calculating the differentiation of functions with a computer."

When I first heard about it, it sounded to me like some kind of dark art or black magic where you get plenty of results from your computer for (almost) free. As a beginner in finance I was told that "there is no free lunch." Certainly this technique could not be applied to finance.

I nevertheless scrutinized the technique carefully, having heard about it several times from serious quants in serious conferences. What I discovered is that at the same time, it is not magic – it is mathematics –, it is not free – you have to invest in development time – and it appears like magic and (almost) free at run time.

Six years later, I have invested a lot of time understanding the technique and developing quantitative finance libraries which implement AD. Now I ask myself: "Why is not everybody in finance using AD?" I don't have an answer to this last question! But I'm very biased in this. I'm like a newly converted faithful to a religion, I don't understand how I was not a believer in the past and I don't understand how others don't believe.

It took me a while to write this book. Between my first article on the subject in 2010 and this book, more than 6 years have passed.

Why it took me so long?

Chacun sa méthode . . . Moi, je travaille en dormant et la solution de tous les problèmes, je la trouve en rêvant. (Personal translation: Each his own method . . . Myself, I work sleeping and the solution to all problems, I find it dreaming).

Drôle de drame (1937) – Marcel Carné

This means a lot of working nights to dream all those pages.

Obviously I cannot write something without referring to my previous book: *Interest Rate Modelling in the Multi-curve Framework: Foundations, Evolution, and Implementation* Henrard (2014). Most of the examples in this book are related to interest rate modeling and the multi-curve framework.

In my previous book I tried to follow the steps of giants in the art of relevant and irrelevant quotes. For this book, I changed the style. Each chapter starts with a couple of sentences summarizing some of the chapter's discoveries, like in old detective fictions. Those of the readers who know me personally, may have notice my zealous collection of (paper) books and the personal library to collect them. That collection contains a lot of detective fictions.

This book grew from seminars, lectures and training I presented on Algorithmic Differentiation. The first of those seminars was probably a “pizza seminar” at OpenGamma in March 2012. Pizza have disappeared from OpenGamma diet since, but seminars are still going on . . . Training are also continuing, at conference workshops or in-house in some banks, and I see more and more interest in the AD subject. Professional finance magazines, like *Risk* and *Structured Product*, have proposed articles on the subject – see for example Sherif (2015) and Davidson (2015) – and interviewed me for them.

This book has benefitted of the valuable feedback from numerous people in the financial “quant” community, among them Wim Schoetens, Luca Capriotti, Yupeng Jiang, and Andrea Macrina

In the book I use the term “we” with the general and vague meaning of the author, the finance community and the readers. The terms “I,” “me” or “my” are used with the precise meaning of *the author personally with its opinions and biases*. The term “I” should be used as a warning sign that the sentence contains opinions and maybe not only facts. You have been warned!

The book was written astride 2014 and 2015. Among my resolutions for 2015 was to be more clear and direct in expressing my opinions. You may find some vapor of that resolution in the book. Some opinions may appear strong or lacking nuance. It is for clarity sake, not to be rude to others opinions. Even if I have spent some time in England, I have not learned the art of *understatement* yet!

Enjoy!

Marc Henrard

London, Brussels and Wépion – September 2016

*Supplied with cleverness of every imaginable type,
Man ventures once towards evil, and then towards good.*

Sophocles, Chorus in Antigone

Use Algorithmic Differentiation for the good!