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# ALL the MATH YOU'LL EVER NEED

ALL-NEW  
EDITION OF THE  
BESTSELLING  
GUIDE

A Self-Teaching Guide

*Steve Slavin*



# **All the Math You'll Ever Need**

**A Self-Teaching Guide Revised Edition**

Steve Slavin



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## How to Use This Book

How long has it been since you've worked out math problems? When I was growing up in Brooklyn in the 1940s, my mother bought fruit and vegetables from Mr. Levine, who added sums on a paper bag. Today we've got cashiers in fast-food restaurants who've got pictures of each item right on the cash register, so they don't even have to punch in the dollars and cents each item costs. No wonder so many of us have forgotten how to use numbers.

This book is organized by chapter with periodic self-tests throughout each chapter. Their purpose is to make sure you comprehend material before moving on. If you find that you have made an error, look back at the preceding material to make sure you understand the correct answer. The information is arranged so that it builds on what comes before. To fully understand the information at the end of a chapter, you must first have completed all of the preceding self-tests.

Before you even start, please put away your pocket calculator. That's right! You're going to be able to go through this entire book, working out hundreds of problems, without once having to use a calculator.

Admit it—you have an addiction. You have a pocket calculator habit. And just look at how that dependency has affected your mathematical skills. The more you came to depend on your calculator, the harder it became to do simple multiplication and division, either in your head or on paper. Do you want to go through the rest of your life in this condition, or do you want to break out of this vicious circle?



Recently, my friend Donna bought \$1,536.40 worth of groceries at the supermarket. Usually her bills are somewhat lower, but once a year she's entitled to a 10% discount. That day, however, she was out of luck, because the cashier had forgotten to bring her calculator to work. In exasperation Donna blurted out, "Ten percent of \$1,536.40 is \$153.64." The cashier was so amazed that she called over all her fellow cashiers *and* the store manager to meet this great math whiz.

The trick to learning math is moving ahead at just the right pace. The format of this book lends itself to proper pacing. When you're going too slowly, you'll say to yourself, "This stuff is so easy—I'm getting bored." So you'll be able to skip a few sections and move on to new material. But when you find yourself pounding your fists against the wall and despairing of ever learning math, that may mean you've been moving ahead a bit too quickly.

Beginning with chapter 3, every chapter has at least two self-tests. If you feel that you don't need to read a particular chapter, you may want to take the self-tests anyway. These provide not only a quick review of the subject matter covered in the chapter, but also a good way of gauging what you already know.

Should you find, on the other hand, that you're having trouble doing a certain type of problem, it will be made clear to you that you need to review an earlier section. For example, no one can do simple division without knowing the multiplication table. So everyone who gets stuck at this point will be sent back to learn that table once and for all. Once that's accomplished, it will be clear sailing through the next few chapters.

This book provides a fast-paced review of arithmetic and elementary algebra, with a smattering of statistics thrown in. It is intended to refresh the memory of the high school or college graduate, the person whose mathematical thought processes have atrophied over the years.

The main emphasis here is on getting you to rely on your own mathematical skills. After completing this book, you will have risen above the ranks of the mathematically illiterate.

Don't be ashamed to carry this book around with you. Sure, people will stop you in the street and ask you questions. No, they won't ask you to multiply 798,999 by 516,204. They'll probably want to know where *they* can get a copy of the book.

So you'll meet a lot of people and become very popular. And as your math skills come back to you, you'll find your whole life has changed. You'll show off in restaurants with your friends; when the check comes, you'll calculate everyone's share in your head. But the main thing you'll find is that you'll have lost your fear of numbers. New horizons will open

up to you. And all of this will happen because you have overcome your dependency on your pocket calculator.

This book will do the same thing for you that the Charles Atlas course did for 97-pound weaklings. No longer will you be intimidated trying to calculate tips. No longer will you need to whip out your pocket calculator to do simple arithmetic. And you won't have to wait months to see tangible results. You won't even have to wait weeks. In just a few days your friends and colleagues will notice your new mathematical muscles. That's right! No one will ever dare to call you a mathematical illiterate again! And when you walk in the street, your neighbors will look at you with new respect. So don't delay another minute. Turn to chapter 1 and just watch those brain cells start to grow.

## Acknowledgments

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I owe a large debt of gratitude to my family, especially to my nephews, Jonah and Eric Zimiles. Jonah provided me with a blow-by-blow critique of the strengths and weaknesses of my previous book, *Economics: A Self-Teaching Guide* (Wiley, 1988), on which I was able to build while writing *this* book. And Eric, after having read that book, recognized its format lent itself best to my writing style and encouraged me to write another book. Eric's daughters, Eleni, 11, Justine, 7, and Sophie, 5, have contributed to the new edition by helping me with my math whenever I happened to get stuck.

My father, Jack, a retired math teacher, provided inspiration of another kind. As the oldest living academic perfectionist, he upholds such unattainable standards that one cannot help but feel tolerance for one's own shortcomings and those of just about everyone else. And finally, I wish to thank my sister, Leontine Temsky, for her rationality and common sense in the most uncommon and irrational of times.

## 1— Getting Started

Tens of millions of Americans are mathematically illiterate. They have trouble doing even third- and fourth-grade arithmetic. And many of these people are college graduates. One cannot help but wonder how so many people managed to get so far in school without having mastered basic arithmetic, let alone simple algebra.

Math phobia seems to have spread across the country. Whenever I mention more than one or two numbers in the same sentence, I notice that my listeners' eyes begin to glaze over. Like counting sheep, just the mention of a couple of numbers is a surefire way to put an audience to sleep.

As an economics professor, one of the strangest phenomena I've encountered is that so many college freshmen are unable to multiply and divide. After all, they studied that back in elementary, middle, or junior high school; since then they've had a couple of years of high school algebra.

So how come they can't do third- and fourth-grade arithmetic? And even more to the point, what can be done to make up for this deficiency?

That is how I came to write this book. I needed results, and I needed them fast, because you can't understand economics if you can't crunch a few numbers.

But you don't need to be an economics student just to deal with numbers. We all have to deal with numbers *sometime*—to balance our

checkbooks, to do our income tax, and to decide whether we want a fixed- or variable-interest mortgage.

Like it or not, numbers are an important part of our lives, and it is time they stopped intimidating us.

There's an apocryphal story that made the rounds back in the 1950s. A big Cadillac pulls up outside Grossinger's, a fancy resort located in the Catskill Mountains. Out steps a very well dressed couple. And right behind them comes their chauffeur carrying a five- or six-year-old boy.

Jenny Grossinger herself rushes out to greet them. She tells them how sorry she is about their little boy.

"Sorry?" replies the boy's mother. "Why are you sorry?"

"Because he can't walk."

"Of *course* he can walk," replies the mother. "Thank God he doesn't *have* to."

What does this have to do with mathematical illiteracy? Just as that little boy who never has to walk will lose the use of his legs, so too will the person who doesn't use his or her mathematical abilities lose them. When you rely on something else to help you, in this case your pocket calculator, your skills grow rusty.

Clearly the villain is the pocket calculator. That's right! Why bother to multiply and divide when your calculator will save you the trouble? In fact, why bother to do any arithmetic at all when all you need to do is press a couple of buttons and read the answer shown in the window?

So what we'll be doing here is going back to basics. Since nearly everyone who can count past 10 knows how to add and subtract, we'll start off with the multiplication table. You'll need to memorize it. If you need to go over addition and subtraction, you need to consult an even more basic text, such as *Quick Arithmetic* by Robert A. Carmen and Marilyn J. Carmen (Wiley, 1987).

If you don't know the multiplication table, you make an awful lot of mistakes in multiplication *and* division. And even if you once knew the multiplication table, if you came to depend on a calculator, your skills at multiplication and division have eroded.

In fact, what has probably happened is that you have developed a math phobia. Numbers make a lot of people nervous. I often hear, "When I see numbers, I freeze . . . or panic . . . or just give up."

So what it all comes down to is learning the multiplication table, using that knowledge to multiply and divide, and then going on to even bigger and better things.

In this text, the use of complex formulas is purposely avoided. Although such formulas have an honored place in mathematics, they clearly don't belong in a book that is read by people who are trying to get over their fear of numbers.

The use of Greek letters will also be eliminated in this book. A simple  $x$  for an unknown will generally suffice.

Finally, the use of technical terms is avoided whenever possible. You will rarely encounter such terms as "numerator" or "denominator" in this book, and the term "exponent" will only be used three times on the first page of chapter 13—it will not be mentioned again after that. There are no quadratic formulas, logarithmic tables, integrals, or derivatives, and there are only a handful of very simple graphs. As long as you learn the math, we'll be happy to skip the vocabulary lesson.

Before we get too carried away, a word of caution is in order. This is not quite the mathematical equivalent of Lourdes. You will not be asked to throw away your calculator as some have thrown away their crutches and wheelchairs. Just put it in a safe place for now, to be taken out and used only on special occasions. A calculator is most effectively used for three tasks: (1) to do calculations that need to be done rapidly, (2) to do repetitive calculations, and (3) to do sophisticated calculations that would take up to a couple of hours to do without a calculator.

The trick is to use our calculators for these specific tasks and not for arithmetic functions that we can do in our heads. Just as many suburbanites drive everywhere—even to the corner to buy a paper—so do many of us depend on our calculators to do simple addition, subtraction, multiplication, and division. So put away your calculator and start using your innate mathematical ability.

2—

**Simple Multiplication**

Once you can multiply without using a calculator, you will find the entire world of mathematics opened up to you. The point of this chapter is to make sure you know the entire multiplication table from  $1 \times 1$  to  $10 \times 10$ .

If you are sure that you do, then go directly to Table 2.1 (page 7) and fill it in completely. Then use Table 2.2 (page 8) to check your work. If you get everything right, then go on to chapter 3. But if you get even two wrong, return to frame 1.

1—

**What Is Multiplication?**

Multiplication is addition. For instance, how much is  $4 \times 3$ ? You know it's 12 because you searched your memory for that multiplication fact. There's nothing wrong with that.

Another way to calculate  $4 \times 3$  is to add them:

$$3 + 3 + 3 + 3 = 12 \quad \text{or} \quad 4 + 4 + 4 = 12$$

Solve the problem  $5 \times 7$ . Maybe you know it's 35. You could always do this:

$$7 + 7 + 7 + 7 + 7 = 35 \quad \text{or} \quad 5 + 5 + 5 + 5 + 5 + 5 + 5 = 35$$

We do multiplication instead of addition because it's shorter—sometimes

much shorter. Suppose you needed to multiply  $78 \times 95$ . If you set this up as an addition problem, you'd be calculating for an hour.

Let's set this up as a regular multiplication problem and take the very first step toward solving it:

$$\begin{array}{r} 95 \\ \times 78 \\ \hline \end{array}$$

The first set of numbers we'd multiply would be  $8 \times 5$ . Then we'd multiply  $8 \times 9$ . Next would come  $7 \times 5$  and  $7 \times 9$ . That's a lot easier than writing out a column of 78 95s and adding them up.

As you can see, a long multiplication problem can be broken down into a series of simple multiplication problems. In this chapter, we will concentrate on these short, or simple, multiplication problems (nothing past  $10 \times 10$ ). In the next chapter, we'll tackle longer multiplication.

2—

### The Multiplication Table

The bare bones are laid out in Table 2. 1. You'll be filling in the blanks.

The table shown here goes from  $1 \times 1$  to  $10 \times 10$ . When I learned the table way back in the third grade, shortly after the Civil War, we had to learn up to  $12 \times 12$ . But I'll let you off the hook. After all, how many times do you need to multiply  $12 \times 11$ ?

How well do you know the table? There's only one way to find out. Fill it in. Multiply each number in the vertical column by each number in the horizontal row. I've done a few to get you started.

Once you've finished, use Table 2.2 to check your work. Circle any wrong answers. The problems most commonly missed are  $9 \times 6$  and  $8 \times 7$ .

If you got everything right, you may advance to the next chapter. If you got just one or two of these wrong, then you'll need to go over them several times, until you're sure you know them.

What if you just don't know the multiplication table at all? There are no shortcuts. Just keep reading. And by the time you've finished this chapter, you'll be able to do the multiplication table in your sleep.

3—

### Learning the Multiplication Table

Let's return to a concept discussed in frame 1—multiplication as addition. Fill in the missing numbers of these five series:



Table 2.1 Blank multiplication table										
Please fill in completely. If you're not too sure of yourself, do it in pencil first.										
	1	2	3	4	5	6	7	8	9	10
1	1	2								
2	2	4								
3			9	12						
4										
5										
6										
7										
8										
9										
10										

1	2	3	—	—	—	—	—	—	—	—
2	4	6	—	—	—	—	—	—	—	—
3	6	9	—	—	—	—	—	—	—	—
4	8	12	—	—	—	—	—	—	—	—
5	10	15	—	—	—	—	—	—	—	—

Now check your work:

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30

<b>Table 2.2 Completed multiplication table</b>										
	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

4      8      12      16      20      24      28      32      36      40

5      10      15      20      25      30      35      40      45      50

If you got everything right, then you're ready for the next set. If not, you'll need to go over the ones you got wrong until you've mastered them.

We'll begin with sixes. Even if you need to count on your fingers, that's okay.

6      12      18      \_\_\_\_      \_\_\_\_      \_\_\_\_      \_\_\_\_      \_\_\_\_      \_\_\_\_

Next we'll do sevens:

7      14      21      \_\_\_\_      \_\_\_\_      \_\_\_\_      \_\_\_\_      \_\_\_\_      \_\_\_\_

Now check your work:

6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70

How did you do? If you got them all right, go on to the next set. If not, continue working on those you missed.

We're ready to do eights:

8	16	24	___	___	___	___	___	___	___
---	----	----	-----	-----	-----	-----	-----	-----	-----

Now try nines:

9	18	27	___	___	___	___	___	___	___
---	----	----	-----	-----	-----	-----	-----	-----	-----

Finally, fill in the missing numbers for tens:

10	20	30	___	___	___	___	___	___	___
----	----	----	-----	-----	-----	-----	-----	-----	-----

Now check your work:

8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Need more practice? Return to the beginning of frame 3 and redo all the exercises. Fill in Table 2.3 and check your work against Table 2.2.

If you're still getting some of these wrong, then you're ready for flash cards. Write the problem on one side of an index card and the answer on the other side. Practice them until you have memorized the correct answer to each of these problems.

Don't get discouraged. Once you've mastered these problems, you'll own them for the rest of your life. The only catch is that you'll have to keep using them. It's like learning how to type. Once you know the multiplication table, your math skills will keep improving.

As you get used to working with numbers, you'll find things getting much easier. Numbers will seem a lot less intimidating, and you'll find yourself gaining confidence. It's really a cumulative process that began

<b>Table</b> <b>2.3</b> <b>Another Blank Multiplication Table</b>										
	1	2	3	4	5	6	7	8	9	10
1	1	2								
2	2	4								
3			9	12						
4										
5										
6										
7										
8										
9										
10										

when you discarded your calculator and learned or relearned the multiplication table.

We're now ready to apply what we've learned here by using the multiplication table in the next chapter. Remember, if you don't use it, you'll lose it.

3—

**Long Multiplication**

We'll be doing some longer, more complex multiplication problems in this chapter. But first you should determine whether you really should be reading this chapter.

1—

**Testing Your Knowledge**

We need to separate the advanced multipliers from the nonmultipliers. If you don't know your multiplication table, you're not yet ready for this chapter. Please go back to the beginning of chapter 2. On the other hand, you may already know how to calculate more complex multiplication problems. There's only one way to find out—sink or swim. Please work out these three problems:

89	195	7064
<u>× 57</u>	<u>× 473</u>	<u>× 3978</u>

*Solutions:*

89	195	7064
<u>× 57</u>	<u>× 473</u>	<u>× 3978</u>
623	585	56512
445	1365	49448
5073	780	63576
	92235	21192
		28100592

Did you notice that our answers had no commas? Certainly 28100592 looks better this way: 28,100,592. But we've left out the commas so our numbers would align vertically. To avoid mistakes, when *you* do long multiplication problems, make sure that *your* figures are aligned. And since you're not typesetting a book, be sure to put in all the commas.

If you got just the first problem right, then you're definitely in the right chapter. If all three problems were correct, then proceed to frame 6. Don't worry, we won't be covering any new material before then.

If you answered two out of three problems correctly, keep reading for the next few frames, work out a few more problems, and then, if you feel you really know how to do long multiplication, you may want to skip to frame 6.

2—

### About Long Multiplication

Long multiplication is just simple multiplication combined with addition. Let's analyze the first problem. When we multiply  $89 \times 57$ , we're multiplying  $89 \times 7$ , then  $89 \times 5$ , and then adding the two products.

Let's walk through all the steps:

1. 
$$\begin{array}{r} 89 \\ \times 57 \\ \hline \end{array}$$

2.  $7 \times 9 = 63$

3. Write down the 3 and carry the 6.

4. 
$$\begin{array}{r} 89 \text{ carry } 6 \\ \times 57 \\ \hline 3 \end{array}$$

5.  $7 \times 8 = 56$

6.  $56 + \text{the carried } 6 = 62$

7. Write down 62.

8. 
$$\begin{array}{r} 89 \\ \times 57 \\ \hline 623 \end{array}$$

9.  $5 \times 9 = 45$

10. Write down the 5 and carry the 4.

11. 89 carry 4

$$\begin{array}{r} \times 57 \\ 623 \\ 5 \end{array}$$

- 12.
- $5 \times 8 = 40$

- 13.
- $40 + \text{the carried } 4 = 44$

14. Write down 44.

15. 89

$$\begin{array}{r} \times 57 \\ 623 \\ \underline{445} \\ 5073 \end{array}$$

16. Add
- $623 + 445 = 5073$
- .

Notice how straight my columns are. One number right under another. I do have a slight advantage, since the typesetter did this for me. But it should be obvious that you can avoid a lot of mistakes if you keep your columns straight.

Also notice that we need to indent when we go from one row of numbers to the next. When we're ready for the second row, the 5 goes under the 2 of 623. Each new row requires an indentation.

Why put the 5 under the 2? Why not write it under the 3 like this?

$$\begin{array}{r} 89 \\ \times 57 \\ 623 \\ 445 \end{array}$$

When we start a new row, we always indent one place to the left because that number is ten times the value of a number one place to the right. In this case, we've placed that 5 in the tens column, making it 50. When we multiply, we are rarely, if ever, conscious that when we indent, we are arranging our numbers in columns of units, tens, hundreds, and thousands. But when we get to the answer, we read it as five thousand and seventy-three (five thousands, zero hundreds, five tens, and three units).

3—

**Practice Exercises with Two Digits**

For further practice work out the next two problems and check the solutions:

$$\begin{array}{r} 64 \\ \times 94 \\ \hline \end{array}$$

$$\begin{array}{r} 59 \\ \times 30 \\ \hline \end{array}$$

*Solutions:*

$$\begin{array}{r} 64 \\ \times 94 \\ \hline 256 \\ 576 \\ \hline 6016 \end{array}$$

$$\begin{array}{r} 59 \\ \times 30 \\ \hline 00 \\ 177 \\ \hline 1770 \end{array}$$

How did you do? If you got these two right, go on to frame 4. If not, then let's spend some time going over both problems.

The first one is straightforward:  $4 \times 4 = 16$ , carry the 1;  $4 \times 6 = 24$ ;  $24 + 1 = 25$ . So we have 256 in the first row.

We begin the second row by multiplying  $9 \times 4 = 36$ . Place the 6 directly under the 5 of 256. We must always indent by one place when we start a new row. So we write down the 6 of 36 and carry the 3. Next we multiply  $9 \times 6 = 54 + 3 = 57$ . This gives us 576 in the second row. Then we add.

Now we'll examine the second problem. How much is zero times any number? It's zero. Since zero has no value, there's nothing wrong with adding a few zeros to the left of any number. (You may add zeros to the right of the number zero without changing its value, but if you added zero to the right of a whole number, such as 17, for example, you'd be multiplying this number by 10, ending up with 170.)

So we'll put two zeros in the first row. Remembering to indent, we multiply  $3 \times 9 = 27$ , placing the 7 in the second row and carrying the 2. Then  $3 \times 5 = 15 + 2 = 17$ , giving us 177 in the second row. The rest is addition. In this way, complex multiplication is broken down into a series of simple multiplication and addition steps.

Solve these problems:

$$\begin{array}{r} 80 \\ \times 97 \\ \hline \end{array}$$

$$\begin{array}{r} 63 \\ \times 50 \\ \hline \end{array}$$