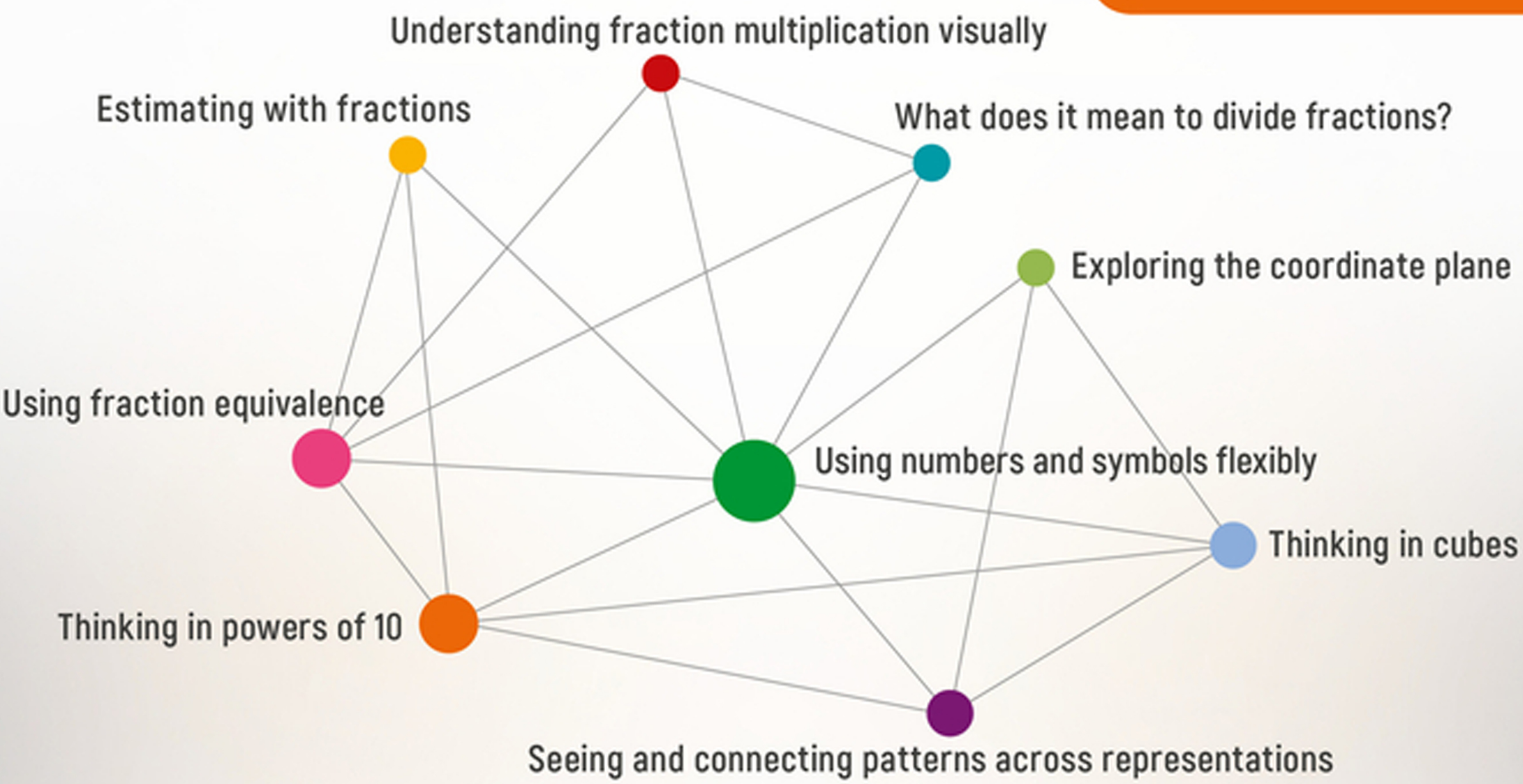
mindset mathematics

Visualizing and Investigating **Big Ideas**

GRADE 5



JO BOALER JEN MUNSON CATHY WILLIAMS

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Visualizing and Investigating Big Ideas

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Contents

To all those teachers pursuing a mathematical mindset journey with us.

Introduction

I still remember the moment when Youcubed, the Stanford center I direct, was conceived. I was at the Denver NCSM and NCTM conferences in 2013, and I had arranged to meet Cathy Williams, the director of mathematics for Vista Unified School District. Cathy and I had been working together for the past year improving mathematics teaching in her district. We had witnessed amazing changes taking place, and a filmmaker had documented some of the work. I had recently released my online teacher course, called How to Learn Math, and been overwhelmed by requests from tens of thousands of teachers to provide them with more of the same ideas. Cathy and I decided to create a website and use it to continue sharing the ideas we had used in her district and that I had shared in my online class. Soon after we started sharing ideas on the Youcubed website, we were invited to become a Stanford University center, and Cathy became the codirector of the center with me.

In the months that followed, with the help of one of my undergraduates, Montse Cordero, our first version of youcubed.org was launched. By January 2015, we had managed to raise some money and hire engineers, and we launched a revised version of the site that is close to the site you may know today. We were very excited that in the first month of that relaunch, we had five thousand visits to the site. At the time of writing this, we are now getting three million visits to the site each month. Teachers are excited to learn about the new research and to take the tools, videos, and activities that translate research ideas into practice and use them in their teaching.

Low-Floor, High-Ceiling Tasks

One of the most popular articles on our website is called "Fluency without Fear." I wrote this with Cathy when I heard from many teachers that they were being made to use timed tests in the elementary grades. At the same time, new brain science was emerging showing that when people feel stressed—as students do when facing a timed test—part of their brain, the working memory, is restricted. The working memory is exactly the area of the brain that comes into play when students need to calculate with math facts, and this is the exact area that is impeded when students are stressed. We have evidence now that suggests strongly that timed math tests in the early grades are responsible for the early onset of math anxiety for many students. I teach an undergraduate class at Stanford, and many of the undergraduates are math traumatized. When I ask them what happened to cause this, almost all of them will recall, with startling clarity, the time in elementary school when they were given timed tests. We are really pleased that "Fluency without Fear" has now been used across the United States to pull timed tests out of school districts. It has been downloaded many thousands of times and used in state and national hearings.

One of the reasons for the amazing success of the paper is that it does not just share the brain science on the damage of timed tests but also offers an alternative to timed tests: activities that teach math facts conceptually and through activities that students and teachers enjoy. One of the activities—a game called How Close to 100—became so popular that thousands of teachers tweeted photos of their students playing the game. There was so much attention on Twitter and other media that Stanford noticed and decided to write a news story on the damage of speed to mathematics learning. This was picked up by news outlets across the United States, including *US News & World Report,* which is part of the reason the white paper has now had so many downloads and so much impact. Teachers themselves caused this mini revolution by spreading news of the activities and research.

How Close to 100 is just one of many tasks we have on youcubed.org that are extremely popular with teachers and students. All our tasks have the feature of being "low floor and high ceiling," which I consider to be an extremely important quality for engaging all students in a class. If you are teaching only one student, then a mathematics task can be fairly narrow in terms of its content and difficulty. But whenever you have a group of students, there will be differences in their needs, and they will be challenged by different ideas. A low-floor, high-ceiling task is one in which everyone can engage, no matter what his or her prior understanding or knowledge, but also one that is open enough to extend to high levels, so that all students can be deeply challenged. In the last two years, we have launched an introductory week of mathematics lessons on our site that are open, visual, and low floor, high ceiling. These have been extremely popular with teachers; they have had approximately four million downloads and are used in 20% of schools across the United States.

In our extensive work with teachers around the United States, we are continually asked for more tasks that are like those on our website. Most textbook publishers seem to ignore or be unaware of research on mathematics learning, and most textbook questions are narrow and insufficiently engaging for students. It is imperative that the new knowledge of the ways our brains learn mathematics is incorporated into the lessons students are given in classrooms. It is for this reason that we chose to write a series of books that are organized around a principle of active student engagement, that reflect the latest brain science on learning, and that include activities that are low floor and high ceiling.

Youcubed Summer Camp

We recently brought 81 students onto the Stanford campus for a Youcubed summer math camp, to teach them in the ways that are encouraged in this book. We used open, creative, and visual math tasks. After only 18 lessons with us, the students improved their test score performance by an average of 50%, the equivalent of 1.6 years of school. More important, they changed their relationship with mathematics and started believing in their own potential. They did this, in part, because we talked to them about the brain science showing that

- There is no such thing as a math person—anyone can learn mathematics to high levels.
- Mistakes, struggle, and challenge are critical for brain growth.
- Speed is unimportant in mathematics.
- Mathematics is a visual and beautiful subject, and our brains want to think visually about mathematics.

All of these messages were key to the students' changed mathematics relationship, but just as critical were the tasks we worked on in class. The tasks and the messages about the brain were perfect complements to each other, as we told students they could learn anything, and we showed them a mathematics that was open, creative, and engaging. This approach helped them see that they could learn mathematics and actually do so. This book shares the kinds of tasks that we used in our summer camp, that make up our week of inspirational mathematics (WIM) lessons, and that we post on our site.

Before I outline and introduce the different sections of the book and the ways we are choosing to engage students, I will share some important ideas about how students learn mathematics.

Memorization versus Conceptual Engagement

Many students get the wrong idea about mathematics—exactly the wrong idea. Through years of mathematics classes, many students come to believe that their role in mathematics learning is to memorize methods and facts, and that mathematics success comes from memorization. I say this is exactly the wrong idea because there is actually very little to remember in mathematics. The subject is made up of a few big, linked ideas, and students who are successful in mathematics are those who see the subject as a set of ideas that they need to think deeply about. The Program for International Student Assessment (PISA) tests are international assessments of mathematics, reading, and science that are given every three years. In 2012, PISA not only assessed mathematics achievement but also collected data on students' approach to mathematics. I worked with the PISA team in Paris at the Organisation for Economic Co-operation and Development (OECD) to analyze students' mathematics approaches and their relationship to achievement. One clear result emerged from this analysis. Students approached mathematics in three distinct ways. One group approached mathematics by attempting to memorize the methods they had met; another group took a "relational" approach, relating new concepts to those they already knew; and a third group took a self-monitoring approach, thinking about what they knew and needed to know.

In every country, the memorizers were the lowest-achieving students, and countries with high numbers of memorizers were all lower achieving. In no country were memorizers in the highest-achieving group, and in some high-achieving countries such as Japan, students who combined self-monitoring and relational strategies outscored memorizing students by more than a year's worth of schooling. More detail on this finding is given in this *Scientific American* Mind article that I coauthored with a PISA analyst: https://www.scientificamerican.com/article/why-math-education-in-the-u-s-doesn-t-add-up/.

Mathematics is a conceptual subject, and it is important for students to be thinking slowly, deeply, and conceptually about mathematical ideas, not racing through methods that they try to memorize. One reason that students need to think conceptually has to do with the ways the brain processes mathematics. When we learn new mathematical ideas, they take up a large space in our brain as the brain works out where they fit and what they connect with. But with time, as we move on with our understanding, the knowledge becomes compressed in the brain, taking up a very small space. For first graders, the idea of addition takes up a large space in their brains as they think about how it works and what it means, but for adults the idea of addition is compressed, and it takes up a small space. When adults are asked to add 2 and 3, for example, they can quickly and easily extract the compressed knowledge. William Thurston (1990), a mathematician who won the Field's Medal—the highest honor in mathematics—explains compression like this:

> Mathematics is amazingly compressible: you may struggle a long time, step by step, to work through the same process or idea from several approaches. But once you really understand it and have the mental perspective to see it as a whole, there is often a tremendous mental compression. You can file it away, recall it quickly and completely when you need it, and use it as just one step in some other mental process. The insight that goes with this compression is one of the real joys of mathematics.

You will probably agree with me that not many students think of mathematics as a "real joy," and part of the reason is that they are not compressing mathematical ideas in their brain. This is because the brain only compresses concepts, not methods. So if students are thinking that mathematics is a set of methods to memorize, they are on the wrong pathway, and it is critical that we change that. It is very important that students think deeply and conceptually about ideas. We provide the activities in this book that will allow students to think deeply and conceptually, and an essential role of the teacher is to give the students time to do so.

Mathematical Thinking, Reasoning, and Convincing

When we worked with our Youcubed camp students, we gave each of them journals to record their mathematical thinking. I am a big fan of journaling—for myself and my students. For mathematics students, it helps show them that mathematics is a subject for which we should record ideas and pictures. We can use journaling to encourage students to keep organized records, which is another important part of mathematics, and help them understand that mathematical thinking can be a long and slow process. Journals also give students free space—where they can be creative, share ideas, and feel ownership of their work. We did not write in the students' journals, as we wanted them to think of the journals as their space, not something that teachers wrote on. We gave students feedback on sticky notes that we stuck onto their work. The images in Figure I.1 show some of the mathematical records the camp students kept in their journals.

Another resource I always share with learners is the act of color coding—that is, students using colors to highlight different ideas. For example, when working on an algebraic task, they may show the *x* in the same color in an expression, in a graph, and in a picture, as shown in Figure I.2. When adding numbers, color coding may help show the addends (Figure I.3).

Color coding highlights connections, which are a really critical part of mathematics.

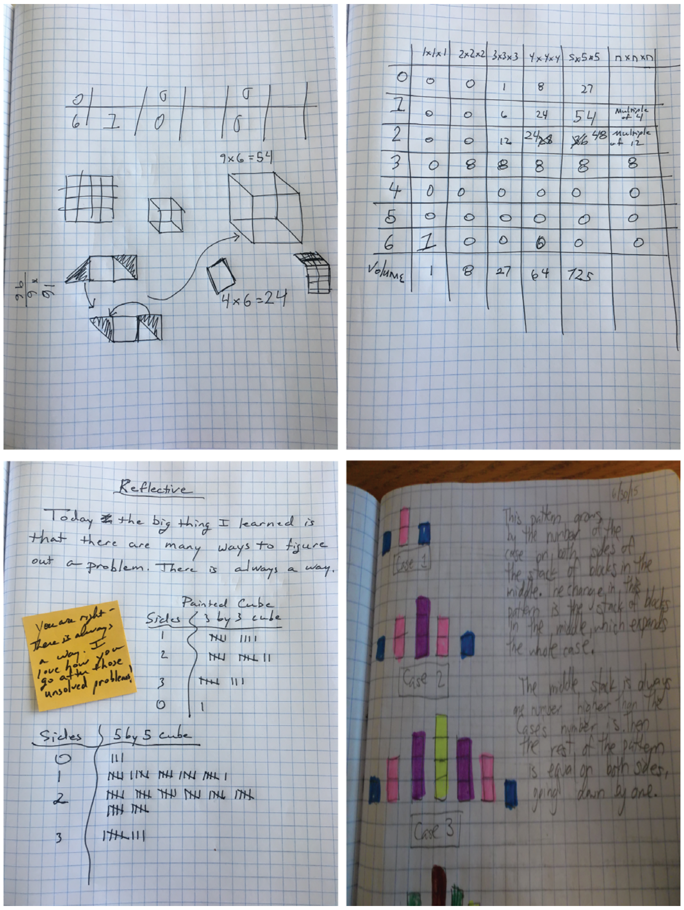

Figure I.1

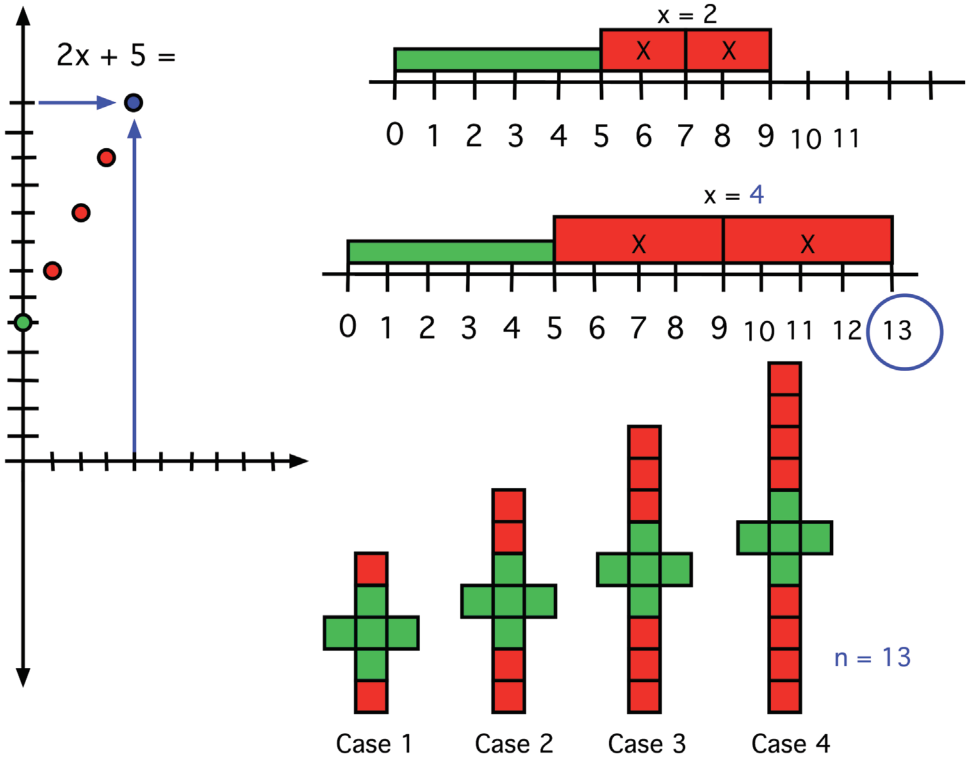

Figure I.2

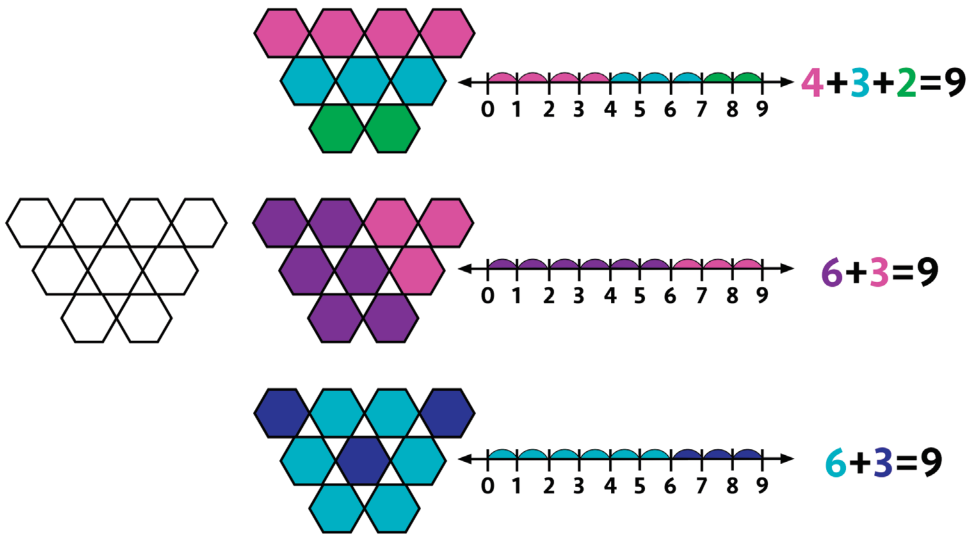

Figure I.3

Another important part of mathematics is the act of reasoning—explaining why methods are chosen and how steps are linked, and using logic to connect ideas. Reasoning is at the heart of mathematics. Scientists prove ideas by finding more cases that fit a theory, or countercases that contradict a theory, but mathematicians prove their work by reasoning. If students are not reasoning, then they are not really doing mathematics. In the activities of these books, we suggest a framework that encourages students to be convincing when they reason. We tell them that there are three levels of being convincing. The first, or easiest, level is to convince yourself of something. A higher level is to convince a friend. And the highest level of all is to convince a skeptic. We also share with students that they should be skeptics with one another, asking one another why methods were chosen and how they work. We have found this framework to be very powerful with students; they enjoy being skeptics, pushing each other to deeper levels of reasoning, and it encourages students to reason clearly, which is important for their learning.

We start each book in our series with an activity that invites students to reason about mathematics and be convincing. I first met an activity like this when reading Mark Driscoll's teaching ideas in his book *Fostering Algebraic Thinking.* I thought it was a perfect activity for introducing the skeptics framework that I had learned from a wonderful teacher, Cathy Humphreys. She had learned about and adapted the framework from two of my inspirational teachers from England: mathematician John Mason and mathematics educator Leone Burton. As well as encouraging students to be convincing, in a number of activities we ask students to prove an idea. Some people think of proof as a formal set of steps that they learned in geometry class. But the act of proving is really about connecting ideas, and as students enter the learning journey of proving, it is worthwhile celebrating their steps toward formal proof. Mathematician Paul Lockhart (2012) rejects the idea that proving is about following a set of formal steps, instead proposing that proving is "abstract art, pure and simple. And art is always a struggle. There is no systematic way of creating beautiful and meaningful paintings or sculptures, and there is also no method for producing beautiful and meaningful mathematical arguments" (p. 8). Instead of suggesting that students follow formal steps, we invite them to think deeply about mathematical concepts and make connections. Students will be given many ways to be creative when they prove and justify, and for reasons I discuss later, we always encourage and celebrate visual as well as numerical and algebraic justifications. Ideally, students will create visual, numerical, and algebraic representations and connect their ideas through color coding and through verbal explanations. Students are excited to experience mathematics in these ways, and they benefit from the opportunity to bring their individual ideas and creativity to the problem-solving and learning space. As students develop in their mathematical understanding, we can encourage them to extend and generalize their ideas through reasoning, justifying, and proving. This process deepens their understanding and helps them compress their learning.

Big Ideas

The books in the Mindset Mathematics Series are all organized around mathematical "big ideas." Mathematics is not a set of methods; it is a set of connected ideas that need to be understood. When students understand the big ideas in mathematics, the methods and rules fall into place. One of the reasons any set of curriculum standards is flawed is that standards take the beautiful subject of mathematics and its many connections, and divide it into small pieces that make the connections disappear. Instead of starting with the small pieces, we have started with the big ideas and important connections, and have listed the relevant Common Core curriculum standards within the activities. Our activities invite students to engage in the mathematical acts that are listed in the imperative Common Core practice standards, and they also teach many of the Common Core content standards, which emerge from the rich activities. Student activity pages are noted with a and teacher activity pages are noted with a .

Although we have chapters for each big idea, as though they are separate from each other, they are all intrinsically linked. Figure I.4 shows some of the connections between the ideas, and you may be able to see others. It is very important to share with students that mathematics is a subject of connections and to highlight the connections as students work. You may want to print the color visual of the different connections for students to see as they work. To see the maps of big ideas for all of the grades K through 8, find our paper "What Is Mathematical Beauty?" at youcubed.org.

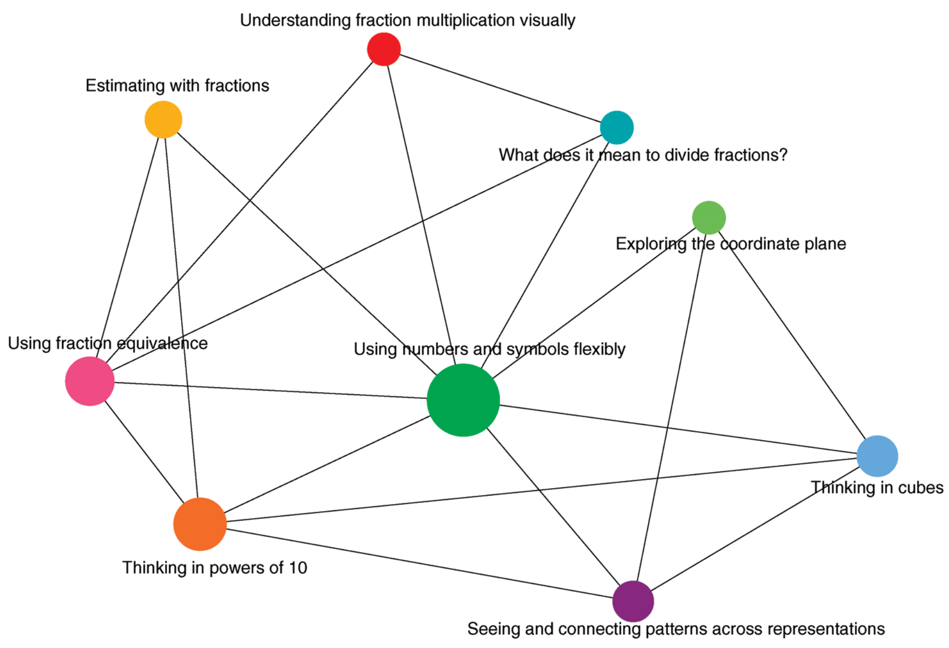

Figure I.4

Structure of the Book

Visualize. Play. Investigate. These three words provide the structure for each book in the series. They also pave the way for open student thinking, for powerful brain connections, for engagement, and for deep understanding. How do they do that? And why is this book so different from other mathematics curriculum books?

Visualize

For the past few years, I have been working with a neuroscience group at Stanford, under the direction of Vinod Menon, which specializes in mathematics learning. We have been working together to think about the ways that findings from brain science can be used to help learners of mathematics. One of the exciting discoveries that has been emerging over the last few years is the importance of visualizing for the brain and our learning of mathematics. Brain scientists now know that when we work on mathematics, even when we perform a bare number calculation, five areas of the brain are involved, as shown in Figure I.5.

Two of the five brain pathways—the dorsal and ventral pathways—are visual. The dorsal visual pathway is the main brain region for representing quantity. This may seem

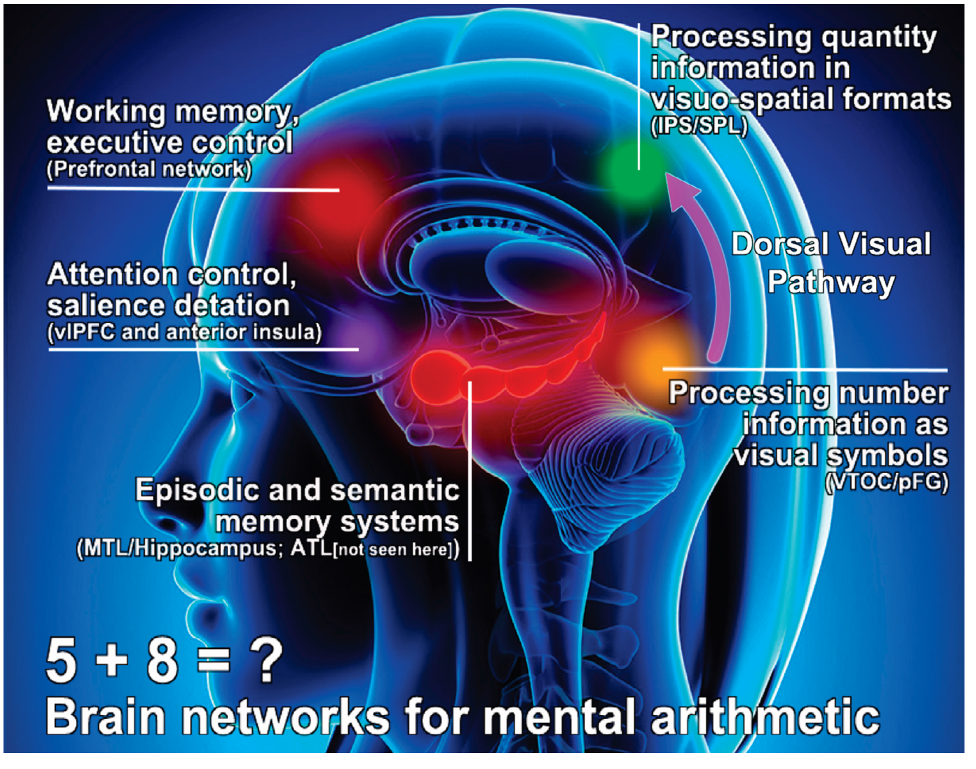

Figure I.5

surprising, as so many of us have sat through hundreds of hours of mathematics classes working with numbers, while barely ever engaging visually with mathematics. Now brain scientists know that our brains "see" fingers when we calculate, and knowing fingers well—what they call finger perception—is critical for the development of an understanding of number. If you would like to read more about the importance of finger work in mathematics, look at the visual mathematics section of youcubed.org. Number lines are really helpful, as they provide the brain with a visual representation of number order. In one study, a mere four 15-minute sessions of students playing with a number line completely eradicated the differences between students from low-income and middle-income backgrounds coming into school (Siegler & Ramani, 2008).

Our brain wants to think visually about mathematics, yet few curriculum materials engage students in visual thinking. Some mathematics books show pictures, but they rarely ever invite students to do their own visualizing and drawing. The neuroscientists' research shows the importance not only of visual thinking but also of students' connecting different areas of their brains as they work on mathematics. The scientists now know that as children learn and develop, they increase the connections between different parts of the brain, and they particularly develop connections between symbolic and visual representations of numbers. Increased mathematics achievement comes about when students are developing those connections. For so long, our emphasis in mathematics education has been on symbolic representations of numbers, with students developing one area of the brain that is concerned with symbolic number representation. A more productive and engaging approach is to develop all areas of the brain that are involved in mathematical thinking, and visual connections are critical to this development.

In addition to the brain development that occurs when students think visually, we have found that visual activities are really engaging for students. Even students who think they are "not visual learners" (an incorrect idea) become fascinated and think deeply about mathematics that is shown visually—such as the visual representations of the calculation 18×5 shown in Figure I.6.

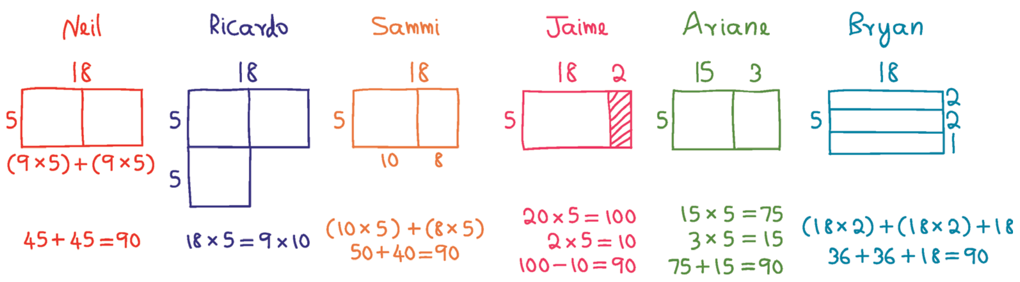

Figure I.6

In our Youcubed teaching of summer school to sixth- and seventh-grade students and in our trialing of Youcubed's WIM materials, we have found that students are inspired by the creativity that is possible when mathematics is visual. When we were trialing the materials in a local middle school one day, a parent stopped me and asked what we had been doing. She said that her daughter had always said she hated and couldn't do math, but after working on our tasks, she came home saying she could see a future for herself in mathematics. We had been working on the number visuals that we use throughout these teaching materials, shown in Figure I.7.

The parent reported that when her daughter had seen the creativity possible in mathematics, everything had changed for her. I strongly believe that we can give these insights and inspirations to many more learners with the sort of creative, open mathematics tasks that fill this book.

We have also found that when we present visual activities to students, the status differences that often get in the way of good mathematics teaching disappear. I was visiting a first-grade classroom recently, and the teacher had set up four different stations around the room. In all of them, the students were working on arithmetic. In one, the teacher engaged students in a mini number talk; in another, a teaching

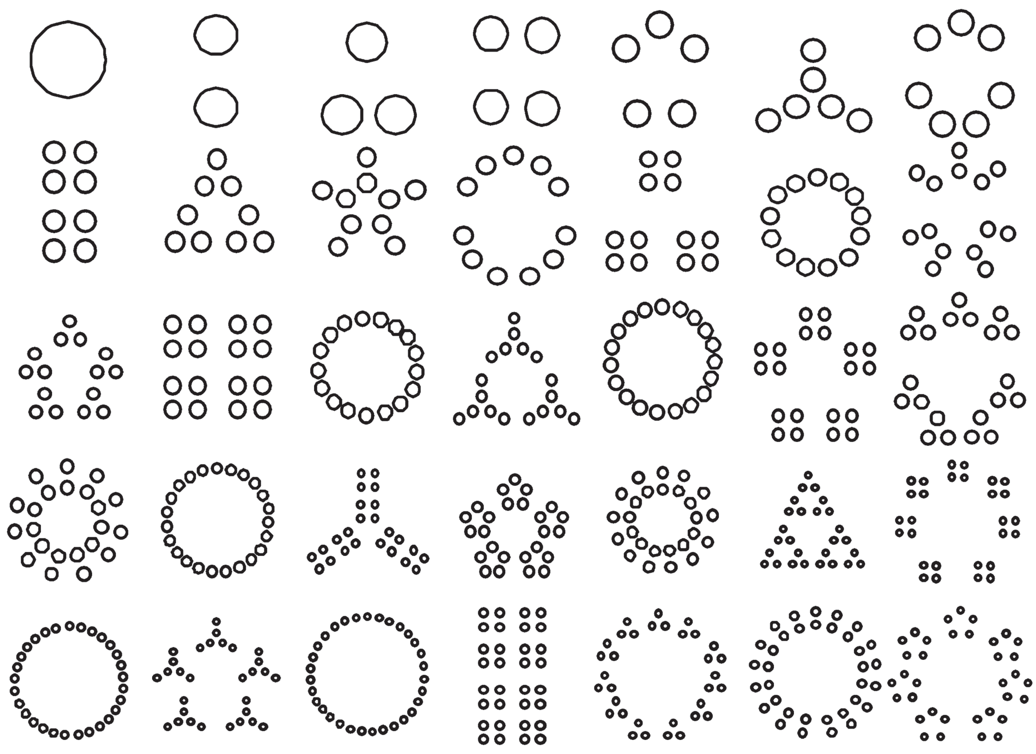

Figure I.7

assistant worked on an activity with coins; in the third, the students played a board game; and in the fourth, they worked on a number worksheet. In each of the first three stations, the students collaborated and worked really well, but as soon as students went to the worksheet station, conversations changed, and in every group I heard statements like "This is easy," " I've finished," "I can't do this," and "Haven't you finished yet?" These status comments are unfortunate and off-putting for many students. I now try to present mathematical tasks without numbers as often as possible, or I take out the calculation part of a task, as it is the numerical and calculational aspects that often cause students to feel less sure of themselves. This doesn't mean that students cannot have a wonderful and productive relationship with numbers, as we hope to promote in this book, but sometimes the key mathematical idea can be arrived at without any numbers at all.

Almost all the tasks in our book invite students to think visually about mathematics and to connect visual and numerical representations. This encourages important brain connections as well as deep student engagement.

Play

The key to reducing status differences in mathematics classrooms, in my view, comes from *opening* mathematics. When we teach students that we can see or approach any mathematical idea in different ways, they start to respect the different thinking of all students. Opening mathematics involves inviting students to see ideas differently, explore with ideas, and ask their own questions. Students can gain access to the same mathematical ideas and methods through creativity and exploration that they can by being taught methods that they practice. As well as reducing or removing status differences, open mathematics is more engaging for students. This is why we are inviting students, through these mathematics materials, to play with mathematics. Albert Einstein famously once said that "play is the highest form of research." This is because play is an opportunity for ideas to be used and developed in the service of something enjoyable. In the Play activities of our materials, students are invited to work with an important idea in a free space where they can enjoy the freedom of mathematical play. This does not mean that the activities do not teach essential mathematical content and practices—they do, as they invite students to work with the ideas. We have designed the Play activities to downplay competition and instead invite students to work with each other, building understanding together.

Investigate

Our Investigate activities add something very important: they give students opportunities to take ideas to the sky. They also have a playful element, but the difference is that they pose questions that students can explore and take to very high levels. As I mentioned earlier, all of our tasks are designed to be as low floor and high ceiling as possible, as these provide the best conditions for engaging all students, whatever their prior knowledge. Any student can access them, and students can take the ideas to high levels. We should always be open to being surprised by what our learners can do, and always provide all students with opportunities to take work to high levels and to be challenged.

A crucial finding from neuroscience is the importance of students struggling and making mistakes—these are the times when brains grow the most. In one of my meetings with a leading neuroscientist, he stated it very clearly: if students are not struggling, they are not learning. We want to put students into situations where they feel that work is hard, but within their reach. Do not worry if students ask questions that you don't know the answer to; that is a good thing. One of the damaging ideas that teachers and students share in education is that teachers of mathematics know everything. This gives students the idea that mathematics people are those who know a lot and never make mistakes, which is an incorrect and harmful message. It is good to say to your students, "That is a great question that we can all think about" or "I have never thought about that idea; let's investigate it together." It is even good to make mistakes in front of students, as it shows them that mistakes are an important part of mathematical work. As they investigate, they should be going to places you have never thought about—taking ideas in new directions and exploring uncharted territory. Model for students what it means to be a curious mathematics learner, always open to learning new ideas and being challenged yourself.

* * *

We have designed activities to take at least a class period, but some of them could go longer, especially if students ask deep questions or start an investigation into a cool idea. If you can be flexible about students' time on activities, that is ideal, or you may wish to suggest that students continue activities at home. In our teaching of these activities, we have found that students are so excited by the ideas that they take them home to their families and continue working on them, which is wonderful. At all times, celebrate deep thinking over speed, as that is the nature of real mathematical thought. Ask students to come up with creative representations of their ideas; celebrate their drawing, modeling, and any form of creativity. Invite your students into a journey of mathematical curiosity and take that journey with them, walking by their side as they experience the wonder of open, mindset mathematics.

References

Lockhart, P. (2012). *Measurement.* Cambridge, MA: Harvard University Press.

Siegler, R. S., & Ramani, G. B. (2008). Playing linear numerical board games promotes low income children's numerical development. *Developmental Science, 11*(5), 655–661. doi:10.1111/j.1467-7687.2008.00714.x

Thurston, W. (1990). Mathematical education. *Notices of the American Mathematical Society, 37*(7), 844–850.

Activities for Building Norms

Encouraging Good Group Work

We always use this activity before students work on math together, as it helps improve group interactions. Teachers who have tried this activity have been pleased by students' thoughtful responses and found the students' thoughts and words helpful in creating a positive and supportive environment. The first thing to do is to ask students, in groups, to reflect on things they don't like people to say or do in a group when they are working on math together. Students come up with quite a few important ideas, such as not liking people to give away the answer, to rush through the work, or to ignore other people's ideas. When students have had enough time in groups brainstorming, collect the ideas. We usually do this by making a What We Don't Like list or poster and asking each group to contribute one idea, moving around the room until a few good ideas have been shared (usually about 10). Then we do the same for the What We Do Like list or poster. It can be good to present the final posters to the class as the agreed-on classroom norms that you and they can reflect back on over the year. If any student shares a negative comment, such as "I don't like waiting for slow people," do not put it on the poster; instead use it as a chance to discuss the issue. This rarely happens, and students are usually very thoughtful and respectful in the ideas they share.

Activity	Time	Description/Prompt	Materials
Launch	5 min	Explain to students that working in groups is an important part of what mathematicians do. Mathematicians discuss their ideas and work together to solve challenging problems. It's important to work together, and we need to discuss what helps us work well together.	
Explore	10 min	Assign a group facilitator to make sure that all students get to share their thoughts on points 1 and 2. Groups should record every group member's ideas and then decide which they will share during the whole-class discussion. In your groups . . . 1. Reflect on the things you do not like people to say or do when you are working on math together in a group. 2. Reflect on the things you do like people to say or do when you are working on math together in a group.	• Paper • Pencil or pen
Discuss	10 min	Ask each group to share their findings. Condense their responses and make a poster so that the student ideas are visible and you can refer to them during the class.	Two to four pieces of large poster paper to collect the students' ideas

Paper Folding: Learning to Reason, Convince, and Be Skeptical

Connection to CCSS
5.G.3
5.G.4

One of the most important topics in mathematics is reasoning. Whereas scientists prove or disprove ideas by finding cases, mathematicians prove their ideas by reasoning—making logical connections between ideas. This activity gives students an opportunity to learn to reason well by having to convince others who are being skeptical.

Before beginning the activity, explain to students that their role is to be convincing. The easiest person to convince is yourself. A higher level of being convincing is to convince a friend, and the highest level of all is to convince a skeptic. In this activity, the students learn to reason to the extent that they can convince a skeptic. Students should work in pairs and take turns to be the one convincing and the one being a skeptic.

Give each student a square piece of paper. If you already have 8.5×11 paper, you can ask them to make the square first.

The first challenge is for one of the students to fold the paper to make a scalene triangle that does not include any of the edges of the paper. She should convince her partner that it is a scalene triangle, using what she knows about triangles to be convincing. The skeptic partner should ask lots of skeptical questions, such as "How do you know that all sides are a different length?" and not accept that they are because it looks like they are.

The partners should then switch roles, and the other student folds the paper into a trapezoid that does not include any of the edges of the paper. His partner should be skeptical and push for high levels of reasoning.

The partners should then switch again, and the challenge is to fold the paper to make a rhombus, again not using the edges of the paper.

The fourth challenge is to make a different rhombus. For each challenge, partners must reason and be skeptical.

When the task is complete, facilitate a whole-class discussion in which students discuss the following questions:

- Which was the most challenging task? Why?
- What was hard about reasoning and being convincing?
- What was hard about being a skeptic?

Activity	Time	Description/Prompt	Materials
Launch	5 min	Tell students that their role for the day is to be convincing and to be a skeptic. Ask students to fold a piece of paper into a square. Choose a student and model being a skeptic.	
Explore	10 min	Show students the task and explain that in each round, they are to solve the folding problem. In pairs, students alternate folding and reasoning and being the skeptic. After students convince themselves they have solved each problem, they switch roles and fold the next challenge. Give students square paper or ask them to start by making a square. The convincing challenges are as follows: 1. Fold your paper into a scalene triangle that does not include any edges of the paper. 2. Fold your paper into a trapezoid that does not include any edges of the paper. 3. Fold your paper into an rhombus that does not include any edges of the paper. 4. Fold your paper into a different rhombus that does not include any edges of the paper.	• One piece of 8.5″ × 11″ paper per student • Paper Folding worksheet for each student
Discuss	10 min	Discuss the activity as a class. Make sure to discuss the roles of convincer and skeptic.	

Paper Folding: Learning to Reason, Convince, and Be a Skeptic

1. Fold your paper into a scalene triangle that does not include any edges of the paper. Convince a skeptic that it is a triangle.
   Reflection:

   Switch roles

2. Fold your paper into a trapezoid that does not include any edges of the paper. Convince a skeptic that it is a trapezoid.
   Reflection:

   Switch roles

3. Fold your paper into a rhombus that does not include any edges of the paper. Convince a skeptic that it is a rhombus.
   Reflection:

   Switch roles

4. Fold your paper into a different rhombus that does not include any edges of the paper. Convince a skeptic that it is a rhombus.
   Reflection: