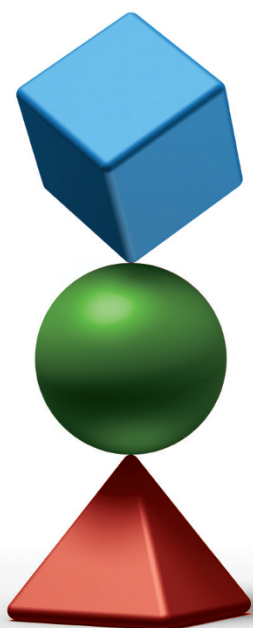


LEARNING MADE EASY



Geometry Essentials

for
dummies[®]
A Wiley Brand



Critical theorems
for geometry proofs

The principles and formulas
you need to know

Key concepts in quick,
focused lessons

Mark Ryan

Author of *Calculus For Dummies*



Geometry Essentials

by Mark Ryan

for
dummies[®]
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Geometry Essentials For Dummies®

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Introduction

Geometry is a subject full of mathematical richness and beauty. The ancient Greeks were into it big time, and it's been a mainstay in secondary education for centuries. Today, no education is complete without at least some familiarity with the fundamental principles of geometry.

But geometry is also a subject that bewilders many students because it's so unlike the math that they've done before. Geometry requires you to use deductive logic in formal proofs. This process involves a special type of verbal and mathematical reasoning that's new to many students. The subject also involves working with two- and three-dimensional shapes. The spatial reasoning required for this is another thing that makes geometry different and challenging.

Geometry Essentials For Dummies can be a big help to you if you've hit the geometry wall. Or if you're a first-time student of geometry, it can prevent you from hitting the wall in the first place. When the world of geometry opens up to you and things start to click, you may come to really appreciate this topic, which has fascinated people for millennia.

About This Book

Geometry Essentials For Dummies covers all the principles and formulas you need to analyze two- and three-dimensional shapes, and it gives you the skills and strategies you need to write geometry proofs.

My approach throughout is to explain geometry in plain English with a minimum of technical jargon. Plain English suffices for geometry because its principles, for the most part, are accessible with your common sense. I see no reason to obscure geometry concepts behind a lot of fancy-pants mathematical mumbo-jumbo. I prefer a street-smart approach.

This book, like all *For Dummies* books, is a reference, not a tutorial. The basic idea is that the chapters stand on their own as much as possible. So you don't have to read this book cover to cover — although, of course, you might want to.

Conventions Used in This Book

Geometry Essentials For Dummies follows certain conventions that keep the text consistent:

- » Variables and names of points are in *italics*.
- » Important math terms are often in *italics* and are defined when necessary. Italics are also sometimes used for emphasis.
- » Important terms may be **bolded** when they appear as keywords within a bulleted list. I also use bold for the instructions in many-step processes.
- » As in most geometry books, figures are not necessarily drawn to scale — though most of them are.

Foolish Assumptions

As I wrote this book, here's what I assumed about you:

- » You're a high school student (or perhaps a junior high student) currently taking a standard high school-level geometry course, or . . .
- » You're a parent of a geometry student, and you'd like to understand the fundamentals of geometry so you can help your child do his or her homework and prepare for quizzes and tests, or . . .
- » You're anyone who wants to refresh your recollection of the geometry you studied years ago or wants to explore geometry for the first time.
- » You remember some basic algebra. The good news is that you need very little algebra for doing geometry — but you do need some. In the problems that do involve algebra, I try to lay out all the solutions step by step.

Icons Used in This Book



REMEMBER

Next to this icon are definitions of geometry terms, explanations of geometry principles, and a few other things you should remember as you work through the book.



TIP

This icon highlights shortcuts, memory devices, strategies, and so on.



WARNING

Ignore these icons, and you may end up doing lots of extra work or getting the wrong answer or both. Read carefully when you see the bomb with the burning fuse!



THEOREMS &
POSTULATES

This icon identifies the theorems and postulates — little mathematical truths — that you use to form the logical arguments in geometry proofs.

Where to Go from Here

If you're a geometry beginner, you should probably start with Chapter 1 and work your way through the book in order, but if you already know a fair amount of the subject, feel free to skip around. For instance, if you need to know about quadrilaterals, check out Chapter 6. Or if you already have a good handle on geometry proof basics, you may want to dive into the more advanced proofs in Chapter 5.

And from there, naturally, you can go

- »» To the head of the class
- »» To Go to collect \$200
- »» To chill out
- »» To explore strange new worlds, to seek out new life and new civilizations, to boldly go where no man (or woman) has gone before

If you're still reading this, what are you waiting for? Go take your first steps into the wonderful world of geometry!

IN THIS CHAPTER

- » Surveying the geometric landscape: Shapes and proofs
- » Understanding points, lines, rays, segments, angles, and planes
- » Cutting segments and angles in two or three congruent pieces

Chapter **1**

An Overview of Geometry

Studying geometry is sort of a Dr. Jekyll-and-Mr. Hyde thing. You have the ordinary geometry of shapes (the Dr. Jekyll part) and the strange world of geometry proofs (the Mr. Hyde part).

Every day, you see various shapes all around you (triangles, rectangles, boxes, circles, balls, and so on), and you're probably already familiar with some of their properties: area, perimeter, and volume, for example. In this book, you discover much more about these basic properties and then explore more advanced geometric ideas about shapes.

Geometry proofs are an entirely different sort of animal. They involve shapes, but instead of doing something straightforward like calculating the area of a shape, you have to come up with a mathematical argument that proves something about a shape. This process requires not only mathematical skills but verbal skills and logical deduction skills as well, and for this reason, proofs trip up many, many students. If you're one of these people and have already started singing the geometry-proof blues, you might even describe proofs — like Mr. Hyde — as monstrous. But I'm confident that, with the help of this book, you'll have no trouble taming them.

The Geometry of Shapes

Have you ever reflected on the fact that you're literally surrounded by shapes? Look around. The rays of the sun are — what else? — rays. The book in your hands has a shape, every table and chair has a shape, every wall has an area, and every container has a shape and a volume; most picture frames are rectangles, DVDs are circles, soup cans are cylinders, and so on.

One-dimensional shapes

There aren't many shapes you can make if you're limited to one dimension. You've got your lines, your segments, and your rays. That's about it. On to something more interesting.

Two-dimensional shapes

As you probably know, two-dimensional shapes are flat things like triangles, circles, squares, rectangles, and pentagons. The two most common characteristics you study about 2-D shapes are their area and perimeter. I devote many chapters in this book to triangles and *quadrilaterals* (shapes with four sides); I give less space to shapes that have more sides, like pentagons and hexagons. Then there are the shapes with curved sides: The only curved 2-D shape I discuss is the circle.

Three-dimensional shapes

In this book, you work with prisms (a box is one example), cylinders, pyramids, cones, and spheres. The two major characteristics of these 3-D shapes are their *surface area* and *volume*. These two concepts come up frequently in the real world; examples include the amount of wrapping paper you need to wrap a gift box (a surface area problem) and the volume of water in a backyard pool (a volume problem).

Geometry Proofs

A *geometry proof* — like any mathematical proof — is an argument that begins with known facts, proceeds from there through a series of logical deductions, and ends with the thing you're trying to prove. Here's a very simple example using the line segments in Figure 1-1.

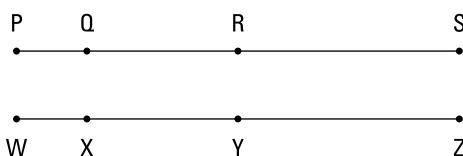


FIGURE 1-1: \overline{PS} and \overline{WZ} , each made up of three pieces.

For this proof, you're told that segment \overline{PS} is congruent to (the same length as) segment \overline{WZ} , that \overline{PQ} is congruent to \overline{WX} , and that \overline{QR} is congruent to \overline{XY} . You have to prove that \overline{RS} is congruent to \overline{YZ} .

Now, you may be thinking, "That's obvious — if \overline{PS} is the same length as \overline{WZ} and both segments contain these equal short pieces and the equal medium pieces, then the longer third pieces have to be equal as well." And you'd be right. But that's not how the proof game is played. You have to spell out every little step in your thinking. Here's the whole chain of logical deductions:

1. $\overline{PS} \cong \overline{WZ}$ (this is given).
2. $\overline{PQ} \cong \overline{WX}$ and $\overline{QR} \cong \overline{XY}$ (these facts are also given).
3. Therefore, $\overline{PR} \cong \overline{WY}$ (because if you add equal things to equal things, you get equal totals).
4. Therefore, $\overline{RS} \cong \overline{YZ}$ (because if you start with equal segments, the whole segments \overline{PS} and \overline{WZ} , and take away equal parts of them, \overline{PR} and \overline{WY} , the parts that are left must be equal).

Am I Ever Going to Use This?

You'll likely have plenty of opportunities to use your knowledge about the geometry of shapes. What about geometry proofs? Not so much.

When you'll use your knowledge of shapes

Shapes are everywhere, so every educated person should have a working knowledge of shapes and their properties. If you have to buy fertilizer or grass seed for your lawn, you should know something about area. You might want to understand the volume

measurements in cooking recipes, or you may want to help a child with an art or science project that involves geometry. You certainly need to understand something about geometry to build some shelves or a backyard deck. And after finishing your work, you might be hungry — a grasp of how area works can come in handy when you're ordering pizza: a 20-inch pizza is four, not two, times as big as a 10-incher. There's no end to the list of geometry problems that come up in everyday life.

When you'll use your knowledge of proofs

Will you ever use your knowledge of geometry proofs? I'll give you a politically correct answer and a politically incorrect one. Take your pick.

First, the politically correct answer (which is also *actually* correct). Granted, it's extremely unlikely that you'll ever have occasion to do a single geometry proof outside of a high school math course. However, doing geometry proofs teaches you important lessons that you can apply to nonmathematical arguments. Proofs teach you . . .

- » Not to assume things are true just because they seem true
- » To carefully explain each step in an argument even if you think it should be obvious to everyone
- » To search for holes in your arguments
- » Not to jump to conclusions

In general, proofs teach you to be disciplined and rigorous in your thinking and in communicating your thoughts.

If you don't buy that PC stuff, I'm sure you'll get this politically incorrect answer: Okay, so you're never going to use geometry proofs, but you want to get a decent grade in geometry, right? So you might as well pay attention in class (what else is there to do, anyway?), do your homework, and use the hints, tips, and strategies I give you in this book. They'll make your life much easier. Promise.