Reasoning in Science and Mathematics

Essays on Logic as
The Art of Reasoning Well

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CAUSE and EFFECT, CONDITIONALS, EXPLANATIONS

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- The Directedness of Emotions
- Conditionals
- Explanations

PRESCRIPTIVE REASONING

- Reasoning with Prescriptive Claims
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The FUNDAMENTALS of ARGUMENT ANALYSIS (2013)

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Preface

This series of books is meant to present the fundamentals of reasoning well in a clear enough manner to be accessible to both scholars and students. The body of each essay gives the main development of the subject, while the footnotes and appendices place the research within a larger scholarly context.

The topic of this volume is the nature and evaluation of reasoning in science and mathematics. Science and mathematics can both be understood as proceeding by a method of abstraction from experience. Mathematics is distinguished from other sciences only in its greater abstraction and its demand for necessity in its inferences.

That methodology of abstraction is the main focus here. In a companion volume, *Cause and Effect, Conditionals, Explanations*, the roles of laws and explanations in science are discussed more fully.

The study of these subjects is not just of academic interest. If we can be clear about the methods and evaluation of science and mathematics, we can clarify our ideas and do better work as scientists and mathematicians. We have a duty to understand what we are doing so that we can not only produce good research but explain that work to others. First comes clear thinking, then comes clear research and clear writing.

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Background material

In order to make the essays suitable to be read independently, there is some repetition in them, and brief introductions to some background material on claims, inferences, arguments, and explanations are included. Those are filled out in the first essay "Background." That in turn is only a brief sketch of the ideas which are developed in *The Fundamentals of Argument Analysis* and the essay "Explanations" in *Cause and Effect, Conditionals, Explanations*, both in this series of books.

Acknowledgments

Many people have helped me over the many years I have been working on the material in this volume. William S. Robinson and Fred Kroon, in particular, have given much of their time and thought to suggestions that have improved the work. The late Benson Mates was a major inspiration for much of the effort to clarify my ideas.

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Much that is good in this book comes from the generous help of these people, to whom I am most grateful. The mistakes are mine, all mine.

Publishing history of the essays

"Models and Theories" is a revision of "On Models and Theories, with Applications To Economics," *The Bulletin of Advanced Reasoning and Knowledge*, vol. 2, 2004, pp. 79–100, parts of it having appeared previously in *The Guide to Critical Thinking in Economics*, Southwestern, 2004, and *Science Workbook for Critical Thinking*, Wadsworth, 2002.

An earlier version of "Mathematics as the Art of Abstraction" appeared in *Computability*: *Computable Functions*, *Logic*, *and the Foundations of Mathematics*, 3rd ed., Advanced Reasoning Forum, 2008. A later revision appeared in *The Argument of Mathematics*, eds. Andrew Aberdein and Ian Dove, Springer, 2012.

"Experiments" draws on material that first appeared in *Science Workbook for Critical Thinking*, Wadsworth, 2002 and in *Science Reasoning Supplement to the Pocket Guide to Critical Thinking*, Advanced Reasoning Forum, 2011.

The material in the background essay is revised from work in *Critical Thinking*, Wadsworth, 3rd ed., 2005, and *Five Ways of Saying* "*Therefore*", Wadsworth, 2001.

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Background

This essay presents the essential material about inferences, arguments, and explanations needed for the essays that follow. This is itself only a sketch, drawing on the material in *The Fundamentals of Argument Analysis* and *Cause and Effect, Conditionals, Explanations*.

Claims

Claims A *claim* is a written or uttered part of speech that we agree to view as true or false, but not both.

The word "uttered" is meant to include silent uttering to oneself, what we might call a linguistic thought.

We do not need to make a judgment about whether a sentence is true or whether it is false to classify it as a claim. A claim need not be an *assertion*: a piece of language put forward as true by someone.

Some say that claims only represent things that are true or false: abstract propositions or thoughts. But utterances are what we use in reasoning together, and we can focus on those, as representatives, if you like, of abstract propositions or thoughts.

The word "agree" in the definition of "claim" suggests that it is a matter of convention whether we take a sentence to be a claim. But almost all our conventions, agreements, and assumptions are implicit. Our agreements may be due to many different reasons or causes, including perhaps that there are abstract propositions.

Often when we reason we identify one utterance with another, as when Dick says "Ralph is not a dog" and later, when Suzy thinks about it, she says "I agree. Ralph is not a dog." We do so when we believe that the utterances are equivalent for all our purposes in reasoning.

Equivalent claims To say that wo claims are equivalent for our purposes in reasoning means that no matter how the world could be, the one is true if and only if the other is true.

I will often assume such equivalences without explicitly saying so.

Often what people say is *too vague* to take as a claim: there's no single obvious way to understand the words, as when someone says "This is a free country." Yet, since everything we say is somewhat vague, it isn't whether a sentence is vague but whether it's too vague, given the context, for us to agree to view it as true or false. In an auditorium lit by a single candle, some parts are clearly lit and some are clearly dark, even if we can't draw a precise line where it stops being light and starts being dark. The *drawing the line fallacy* is to argue that if you can't make the difference precise, there's no difference.

Inferences

We reason in order to discern whether certain claims are true. But we also reason to discern whether a particular claim follows from one or more other claims. We might not know whether those other claims are true. But were they true, would the truth of this other claim follow?

Inferences An *inference* is a collection of claims, one of which is designated the ***conclusion*** and the others the ***premises***, which is intended by the person who sets it out either to show that the conclusion follows from the premises or to investigate whether that is the case.

In order to investigate the idea of a conclusion following from the premises of an inference, we make some definitions.

Valid, strong, and weak inferences An inference is ***valid*** if it is impossible for the premises to be true and conclusion false at the same time and in the same way.

An inference is ***strong*** if it is possible but unlikely for the premises to be true and conclusion false at the same time and in the same way. An inference is ***weak*** if it is neither valid nor strong.

The classification of invalid inferences is on a scale from the strongest to the weakest, as we deem the possibilities we consider in which the premises are true and conclusion false to be more or less likely.

The following, for example, is a valid inference:

Maria is a widow.
So Maria was married.

We do not know if the premise is true, but if it is, then the conclusion is not false. In this case the conclusion surely follows from the premise.

The following is valid, too:

All dogs bark.
Spot is a dog.
So Spot barks.

Here we know that the first premise is false: Basenjis can't bark, and some dogs have had their vocal cords cut. It's not the truth or falsity of the premises and conclusion that determines whether an inference is valid; rather, it is the ways in which the premises and conclusion could be true or false. Were the premises of this inference true, the conclusion would be also; the inference is valid.

In contrast, the following inference is strong:

Almost all dogs bark.
Ralph is a dog.
So Ralph barks.

If we know no more about Ralph than that he is a dog, then any way in which the premises could be true and conclusion false is unlikely, for we know how rare those are. In this case, too, we say that the conclusion follows from the premises, though there is no certainty, no "must" in that. It is only that, relative to what we know, it seems to us very unlikely that the premises could be true and conclusion false.

The following, however, is weak:

Louise is a student.
So Louise isn't married.

There lots of ways the premise could be true and conclusion false: for all we know, Louise might be forty years old with a husband and child.

Our evaluation of the strength of an inference is relative to what we believe. "Likely" means "likely to us." But typically the scale from strong to weak is not so completely relative to a particular person that there is no hope we can agree on the strength of inferences. Suppose we disagree. I find a particular inference strong, and you find it weak. If we wish to reason together, you should describe to me a way the premises could be true and the conclusion false that you think is not unlikely. That may depend on knowledge you have of how the premises could be true that I do not have, but once you've made that explicit, we can agree or disagree that there is such a possibility. The only issue, then, would be whether we agree that the possibility is likely. Sometimes we can't come to a clear determination, but further examination will leave us with a clearer understanding of what our differences in evaluation are, based on more than just whim. When the beliefs involved in determining the strength of an inference are made explicit, determining the inference to be strong or weak is far more likely to be a shared judgment.

In sum, we say that the conclusion of an inference ***follows from*** the premises if the inference is valid or strong.

Arguments

The paradigmatic use of inferences is attempts to convince someone that a claim is true.

Arguments An *argument* is an inference that is intended by the person who sets it out to convince someone that the conclusion is true.

Arguments are attempts to convince, whether someone tries to convince you, or you try to convince someone else, or you try to convince yourself. But that does not mean that the criterion for whether an argument is good is whether the argument actually does convince. If your friend is drunk, you may give him an excellent argument that his driving home is dangerous; though he remains unconvinced, the argument is no worse. A politician may make a bad argument that you should vote for her, but though you may be convinced that does not mean the argument is good. Perhaps other ways to convince, such as entreaties, exhortations, sermons, or advertisements can be judged by

how well they convince, but that is not a criterion for judging attempts to establish the truth of a claim. A *good argument* is one that gives us good reason to believe the conclusion. But what does "good reason" mean?

If an argument is to give us good reason to believe its conclusion, we should have good reason to believe its premises, for from a false claim we can reason as easily to a false conclusion as a true one.

The Prime Minister of England is a dog. All dogs have fur.
So the Prime Minister of England has fur. (false conclusion)

The Prime Minister of England is a dog. All dogs have a liver.
So the Prime Minister of England has a liver. (true conclusion)

It seems, then, that a good argument should have true premises. But consider:

There are an even number of stars in the sky.
So the number of stars in the sky can be divided by 2.

There are an odd number of stars in the sky.
So the number of stars in the sky cannot be divided by 2.

One of these has a true premise, but we cannot tell which. A standard that gives us no way to evaluate arguments is not part of the art of reasoning well. Rather, for an argument to be good we must have good reason to believe its premises and recognize that we have good reason to believe them, and as well actually believe them, for otherwise what convincing is done has no basis in our beliefs.

Plausible claims A claim is *plausible* to a particular person at a particular time if:

- The person has good reason to believe it.
- The person recognizes that he or she has good reason to believe it.
- The person believes it.

A claim that is not plausible is *implausible or dubious*.

The classification of claims as plausible or implausible is on a scale from the most plausible, ones we recognize as true, to the least plausible, ones we recognize as false. Though we do not have precise measures of plausibility, we can often compare the plausibility of claims; and by being explicit about our background we can usually