Probability FOR DUMMIES®

by Deborah Rumsey, PhD

Wiley Publishing, Inc.

Probability FOR DUMMIES®

Probability FOR DUMMIES®

by Deborah Rumsey, PhD

Wiley Publishing, Inc.

Probability For Dummies®
Published by
Wiley Publishing, Inc.
111 River St.
Hoboken, NJ 07030-5774
www.wiley.com

Copyright © 2006 by Wiley Publishing, Inc., Indianapolis, Indiana

Published simultaneously in Canada

No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning, or otherwise, except as permitted under Sections 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, 222 Rosewood Drive, Danvers, MA 01923, 978-750-8400, fax 978-646-8600. Requests to the Publisher for permission should be addressed to the Legal Department, Wiley Publishing, Inc., 10475 Crosspoint Blvd., Indianapolis, IN 46256, 317-572-3447, fax 317-572-4355, or online at http://www.wiley.com/go/permissions.

Trademarks: Wiley, the Wiley Publishing logo, For Dummies, the Dummies Man logo, A Reference for the Rest of Us!, The Dummies Way, Dummies Daily, The Fun and Easy Way, Dummies.com, and related trade dress are trademarks or registered trademarks of John Wiley & Sons, Inc., and/or its affiliates in the United States and other countries, and may not be used without written permission. All other trademarks are the property of their respective owners. Wiley Publishing, Inc., is not associated with any product or vendor mentioned in this book.

LIMIT OF LIABILITY/DISCLAIMER OF WARRANTY: THE PUBLISHER AND THE AUTHOR MAKE NO REPRESENTATIONS OR WARRANTIES WITH RESPECT TO THE ACCURACY OR COMPLETENESS OF THE CONTENTS OF THIS WORK AND SPECIFICALLY DISCLAIM ALL WARRANTIES, INCLUDING WITHOUT LIMITATION WARRANTIES OF FITNESS FOR A PARTICULAR PURPOSE. NO WARRANTY MAY BE CREATED OR EXTENDED BY SALES OR PROMOTIONAL MATERIALS. THE ADVICE AND STRATEGIES CONTAINED HEREIN MAY NOT BE SUITABLE FOR EVERY SITUATION. THIS WORK IS SOLD WITH THE UNDERSTANDING THAT THE PUBLISHER IS NOT ENGAGED IN RENDERING LEGAL, ACCOUNTING, OR OTHER PROFESSIONAL SERVICES. IF PROFESSIONAL ASSISTANCE IS REQUIRED, THE SERVICES OF A COMPETENT PROFESSIONAL PERSON SHOULD BE SOUGHT. NEITHER THE PUBLISHER NOR THE AUTHOR SHALL BE LIABLE FOR DAMAGES ARISING HEREFROM. THE FACT THAT AN ORGANIZATION OR WEBSITE IS REFERRED TO IN THIS WORK AS A CITATION AND/OR A POTENTIAL SOURCE OF FURTHER INFORMATION DOES NOT MEAN THAT THE AUTHOR OR THE PUBLISHER ENDORSES THE INFORMATION THE ORGANIZATION OR WEBSITE MAY PROVIDE OR RECOMMENDATIONS IT MAY MAKE. FURTHER, READERS SHOULD BE AWARE THAT INTERNET WEBSITES LISTED IN THIS WORK MAY HAVE CHANGED OR DISAPPEARED BETWEEN WHEN THIS WORK WAS WRITTEN AND WHEN IT IS READ.

For general information on our other products and services, please contact our Customer Care Department within the U.S. at 800-762-2974, outside the U.S. at 317-572-3993, or fax 317-572-4002.

For technical support, please visit www.wiley.com/techsupport.

Wiley also publishes its books in a variety of electronic formats. Some content that appears in print may not be available in electronic books.

Library of Congress Control Number: 2005938252

ISBN-13: 978-0-471-75141-0

ISBN-10: 0-471-75141-3

Manufactured in the United States of America

10 9 8 7 6 5 4 3 2 1

1B/RV/QT/QW/IN

About the Author

Deborah Rumsey has a PhD in Statistics from The Ohio State University (1993). Upon graduating, she joined the faculty in the Department of Statistics at Kansas State University, where she won the distinguished Presidential Teaching Award and earned tenure and promotion in 1998. In 2000, she returned to Ohio State and is now a Statistics Education Specialist/Auxiliary Faculty Member for the Department of Statistics. Dr. Rumsey has served on the American Statistical Association's Statistics Education Executive Committee and is the Editor of the Teaching Bits section of the *Journal of Statistics Education.* She's the author of the books *Statistics For Dummies* and *Statistics Workbook For Dummies* (Wiley). She also has published many papers and given many professional presentations on the subject of Statistics Education. Her particular research interests are curriculum materials development, teacher training and support, and immersive learning environments. Her passions, besides teaching, include her family, fishing, bird watching, driving a new Kubota tractor on the family "farm," and Ohio State Buckeye football (not necessarily in that order).

Dedication

To my husband Eric: Thanks for rolling the dice and taking a chance on me.
To my son Clint Eric: Your smile always brings me good luck.

Author's Acknowledgments

Thanks again to Kathy Cox for believing in me and signing me up to write this book; to Chrissy Guthrie for her continued excellence and for being a wonderful source of support as my project editor; and to Dr. Marjorie Bond, Monmouth College, for another invaluable technical review. Thanks to Josh Dials for his editing that kept things light. Thanks to Kythrie Silva for believing in me; to Peg Steigerwald for her constant support and friendship; and to my family, especially my parents, for loving me through it all. I also wish to thank all the students I have had the privilege of teaching; you are the inspiration for all of my work.

Publisher's Acknowledgments

We're proud of this book; please send us your comments through our Dummies online registration form located at www.dummies.com/register/.

Some of the people who helped bring this book to market include the following:

Acquisitions, Editorial, and Media Development

Project Editor: Christina Guthrie

Acquisitions Editor: Kathy Cox

Copy Editor: Josh Dials

Editorial Program Coordinator: Hanna K. Scott

Technical Editor: Marjorie Bond, PhD

Editorial Manager: Christine Meloy Beck

Editorial Assistants: Erin Calligan, Nadine Bell, David Lutton

Cover Photos: © Medioimages/Getty Images

Cartoons: Rich Tennant (www.the5thwave.com)

Composition Services

Project Coordinator: Adrienne Martinez

Layout and Graphics: Andrea Dahl, Carrie A. Foster, Lauren Goddard, Denny Hager, Heather Ryan

Proofreaders: Dwight Ramsey, Evelyn W. Still

Indexer: Broccoli Information Management

Special Help

Neil Johnson

Publishing and Editorial for Consumer Dummies

Diane Graves Steele, Vice President and Publisher, Consumer Dummies

Joyce Pepple, Acquisitions Director, Consumer Dummies

Kristin A. Cocks, Product Development Director, Consumer Dummies

Michael Spring, Vice President and Publisher, Travel

Kelly Regan, Editorial Director, Travel

Publishing for Technology Dummies

Andy Cummings, Vice President and Publisher, Dummies Technology/General User

Composition Services

Gerry Fahey, Vice President of Production Services

Debbie Stailey, Director of Composition Services

Contents at a Glance

Introduction 1

Part 1: The Certainty of Uncertainty: Probability Basics 7

Chapter 1: The Probability in Everyday Life 9
Chapter 2: Coming to Terms with Probability 19
Chapter 3: Picturing Probability: Venn Diagrams, Tree Diagrams, and Bayes' Theorem 39

Part 11: Counting on Probability and Betting to Win 65

Chapter 4: Setting the Contingency Table with Probabilities 67
Chapter 5: Applying Counting Rules with Combinations and Permutations 77
Chapter 6: Against All Odds: Probability in Gaming 103

Part 111: From A to Binomial: Basic Probability Models 129

Chapter 7: Probability Distribution Basics 131
Chapter 8: Juggling Success and Failure with the Binomial Distribution 151
Chapter 9: The Normal (but Never Dull) Distribution 167
Chapter 10: Approximating a Binomial with a Normal Distribution 187
Chapter 11: Sampling Distributions and the Central Limit Theorem 201
Chapter 12: Investigating and Making Decisions with Probability 221

Part 1V: Taking It Up a Notch: Advanced Probability Models 233

Chapter 13: Working with the Poisson (a Nonpoisonous) Distribution 235
Chapter 14: Covering All the Angles of the Geometric Distribution 251
Chapter 15: Making a Positive out of the Negative Binomial Distribution 261
Chapter 16: Remaining Calm about the Hypergeometric Distribution 273

Part V: For the Hotshots: Continuous Probability Models 283

Chapter 17: Staying in Line with the Continuous Uniform Distribution 285
Chapter 18: The Exponential (and Its Relationship to Poisson) Exposed 299

Part VI: The Part of Tens 311
Chapter 19: Ten Steps to a Better Probability Grade.....313
Chapter 20: Top Ten (Plus One) Probability Mistakes 323
Appendix: Tables for Your Reference 333
Index 343

Table of Contents

Introduction 1

About This Book 1
Conventions Used in This Book 2
What You're Not to Read 2
Foolish Assumptions 3
How This Book Is Organized 3
Part I: The Certainty of Uncertainty: Probability Basics 3
Part II: Counting on Probability and Betting to Win 4
Part III: From A to Binomial: Basic Probability Models 4
Part IV: Taking It Up a Notch: Advanced Probability Models 4
Part V: For the Hotshots: Continuous Probability Models 4
Part VI: The Part of Tens 5
Appendix 5
Icons Used in This Book 5
Where to Go from Here 6

Part 1: The Certainty of Uncertainty: Probability Basics 7

Chapter 1: The Probability in Everyday Life 9

Figuring Out what Probability Means 9
Understanding the concept of chance 10
Interpreting probabilities: Thinking large and long-term 10
Seeing probability in everyday life 11
Coming Up with Probabilities 12
Be subjective 13
Take a classical approach 13
Find relative frequencies 14
Use simulations 15
Probability Misconceptions to Avoid 17
Thinking in 50-50 terms when you have two outcomes 17
Thinking that patterns can't occur 18

Chapter 2: Coming to Terms with Probability 19

A Set Notation Overview 19
Noting outcomes: Sample spaces 19
Noting subsets of sample spaces: Events 21

Noting a void in the set: Empty sets 22
Putting sets together: Unions, intersections, and complements 22
Probabilities of Events Involving A and/or B 24
Probability notation 24
Marginal probabilities 25
Union probabilities 26
Intersection (joint) probabilities 26
Complement probabilities 26
Conditional probabilities 27
Understanding and Applying the Rules of Probability 29
The complement rule (for opposites, not for flattering a date) 29
The multiplication rule (for intersections, not for rabbits) 30
The addition rule (for unions of the nonmarital nature) 31
Recognizing Independence in Multiple Events 32
Checking independence for two events with the definition 32
Utilizing the multiplication rule for independent events 33
Including Mutually Exclusive Events 34
Recognizing mutually exclusive events 34
Simplifying the addition rule with mutually exclusive events 35
Distinguishing Independent and Mutually Exclusive Events 36
Comparing and contrasting independence and exclusivity 36
Checking for independence or exclusivity in a 52-card deck 37

Chapter 3: Picturing Probability: Venn Diagrams, Tree Diagrams, and Bayes' Theorem 39

Diagramming Probabilities with Venn Diagrams 40
Utilizing Venn diagrams to find probabilities beyond those given 40
Using Venn diagrams to organize and visualize relationships 41
Proving intermediate rules about sets, Using Venn diagrams 42
Exploring the limitations of Venn diagrams 44
Finding probabilities for complex problems with Venn diagrams 45
Mapping Out Probabilities with Tree Diagrams 47
Showing multi-stage outcomes with a tree diagram 49
Organizing conditional probabilities with a tree diagram 51
Reviewing the limitations of tree diagrams 54
Drawing a tree diagram to find probabilities for complex events 54
The Law of Total Probability and Bayes' Theorem 56
Finding a marginal probability using the Law of Total Probability 57
Finding the posterior probability with Bayes' Theorem 60

Part II: Counting on Probability and Betting to Win.....65

Chapter 4: Setting the Contingency Table with Probabilities 67

Organizing a Contingency Table.....67
Defining the sample space 68
Setting up the rows and columns.....69
Inserting the data 69
Adding the row, column, and grand totals.....70
Finding and Interpreting Probabilities within a Contingency Table.....70
Figuring joint probabilities.....71
Calculating marginal probabilities 71
Identifying conditional probabilities 72
Checking for Independence of Two Events.....74

Chapter 5: Applying Counting Rules with Combinations and Permutations 77

Counting on Permutations.....78
Unraveling a permutation 78
Permutation problems with added restrictions: Are we having fun yet?.....82
Finding probabilities involving permutations 86
Counting Combinations.....88
Solving combination problems.....89
Combinations and Pascal's Triangle.....90
Probability problems involving combinations 91
Studying more complex combinations through poker hands 93
Finding probabilities involving combinations.....100

Chapter 6: Against All Odds: Probability in Gaming 103

Knowing Your Chances: Probability, Odds, and Expected Value.....104
Playing the Lottery 105
Mulling the probability of winning the lottery 105
Figuring the odds.....107
Finding the expected value of a lottery ticket.....107
Hitting the Slot Machines.....111
Understanding average payout 111
Unraveling slot machine myths.....113
Implementing a simple strategy for slots.....114
Spinning the Roulette Wheel 116
Covering roulette wheel basics 116
Making outside and inside bets.....117
Developing a roulette strategy 120

Getting Your Chance to Yell "BINGO!" 121
Ways to win at BINGO 121
The probability of getting BINGO — more complicated than you may think....123
Knowing What You're Up Against: Gambler's Ruin....125
The Famous Birthday Problem 126

Part III: From A to Binomial: Basic Probability Models 129

Chapter 7: Probability Distribution Basics 131

The Probability Distribution of a Discrete Random Variable 131
Defining a random variable 132
Finding and using the probability distribution 133
Finding and Using the Cumulative Distribution Function (cdf) 138
Interpreting the cdf 139
Graphing the cdf....140
Finding probabilities with the cdf 141
Determining the pmf given the cdf....143
Expected Value, Variance, and Standard Deviation of a Discrete Random Variable....144
Finding the expected value of X 145
Calculating the variance of X 147
Finding the standard deviation of X....148
Outlining the Discrete Uniform Distribution 148
The pmf of the discrete uniform....149
The cdf of the discrete uniform 149
The expected value of the discrete uniform 150
The variance and standard deviation of the discrete uniform....150

Chapter 8: Juggling Success and Failure with the Binomial Distribution 151

Recognizing the Binomial Model....151
Checking the binomial conditions step by step 152
Spotting a variable that isn't binomial 153
Finding Probabilities for the Binomial....155
Finding binomial probabilities with the pmf....155
Finding binomial probabilities with the cdf....160
Formulating the Expected Value and Variance of the Binomial 165
The expected value of the binomial....165
The variance and standard deviation of the binomial 166

Chapter 9: The Normal (but Never Dull) Distribution 167

Charting the Basics of the Normal Distribution.....167
The shape, center, and spread.....168
The standard normal (Z) distribution 170
Finding and Using Probabilities for a Normal Distribution 172
Getting the picture 173
Translating a problem into probability notation.....173
Using the Z-formula.....174
Utilizing the Z table to find the probability.....176
Handling Backwards Normal Problems.....180
Setting up a backwards normal problem 181
Using the Z table backward.....183
Returning to X units, using the Z-formula solved for X 185

Chapter 10: Approximating a Binomial with a Normal Distribution 187

Identifying When You Need to Approximate Binomials 187
Why the Normal Approximation Works when n Is Large Enough.....188
Symmetric situations: When p is close to 0.50.....189
Skewed situations: When p is close to zero or one.....190
Understanding the Normal Approximation to the Binomial 192
Determining if n is large enough.....192
Finding the mean and standard deviation to put in the Z-formula 193
Making the continuity correction.....194
Approximating a Binomial Probability with the Normal: A Coin Example.....197

Chapter 11: Sampling Distributions and the Central Limit Theorem 201

Surveying a Sampling Distribution 202
Setting up your sample statistic.....202
Lining up possibilities with the sampling distribution.....202
Saved by the Central Limit Theorem 204
Gaining Access to Your Statistics through the Central Limit Theorem (CLT).....205
The main result of the CLT 205
Why the CLT works 206
The Sampling Distribution of the Sample Total (t).....210
The CLT applied to the sample total.....211
Finding probabilities for t with the CLT.....211
The Sampling Distribution of the Sample Mean, $\overline{X}$.....214
The CLT applied to the sample mean 215
Finding probabilities for $\overline{X}$ with the CLT.....216

The Sampling Distribution of the Sample Proportion, $\hat{p}$ 217
The CLT applied to the sample proportion.....217
Finding probabilities for $\hat{p}$ with the CLT 218

Chapter 12: Investigating and Making Decisions with Probability 221

Confidence Intervals and Probability.....221
Guesstimating a probability.....222
Assessing the cost of probably (hopefully?) being right 224
Interpreting a confidence interval with probability 225
Probability and Hypothesis Testing 226
Testing a probability.....226
Putting the p in probability with p-values.....228
Accepting the probability of making the wrong decision 229
Putting the lid on data snoopers.....230
Probability in Quality Control 231

Part IV: Taking It Up a Notch: Advanced Probability Models 233

Chapter 13: Working with the Poisson (a Nonpoisonous) Distribution 235

Counting On Arrivals with the Poisson Model.....236
Meeting conditions for the Poisson model 236
Pitting Poisson versus binomial.....237
Determining Probabilities for the Poisson.....237
The pmf of the Poisson.....238
The cdf of the Poisson 240
Identifying the Expected Value and Variance of the Poisson 243
Changing Units Over Time or Space: The Poisson Process.....244
Approximating a Poisson with a Normal.....245
Satisfying conditions for using the normal approximation 246
Completing steps to approximate the Poisson with a normal 248

Chapter 14: Covering All the Angles of the Geometric Distribution 251

Shaping Up the Geometric Distribution 252
Meeting the conditions for a geometric distribution 252
Choosing the geometric distribution over the binomial and Poisson 253
Finding Probabilities for the Geometric by Using the pmf 254
Building the pmf for the geometric.....255
Applying geometric probabilities.....256

Uncovering the Expected Value and Variance of the Geometric.....258
The expected value of the geometric 258
The variance and standard deviation of the geometric 259

Chapter 15: Making a Positive out of the Negative Binomial Distribution 261

Recognizing the Negative Binomial Model 261
Checking off the conditions for a negative binomial model 262
Comparing and contrasting the negative binomial, geometric, and binomial models 262
Formulating Probabilities for the Negative Binomial 264
Developing the negative binomial probability formula.....264
Applying the negative binomial pmf 265
Exploring the Expected Value and Variance of the Negative Binomial 269
The expected value of the negative binomial.....269
The variance and standard deviation of the negative binomial...270
Applying the expected value and variance formulas 271

Chapter 16: Remaining Calm about the Hypergeometric Distribution 273

Zooming In on the Conditions for the Hypergeometric Model 274
Finding Probabilities for the Hypergeometric Model.....275
Setting up the hypergeometric pmf 275
Breaking down the boundary conditions for X 277
Finding and using the pmf to calculate probabilities 279
Measuring the Expected Value and Variance of the Hypergeometric 281
The expected value of the hypergeometric 281
The variance and standard deviation of the hypergeometric.....281

Part V: For the Hotshots: Continuous Probability Models 283

Chapter 17: Staying in Line with the Continuous Uniform Distribution 285

Understanding the Continuous Uniform Distribution.....286
Determining the Density Function for the Continuous Uniform Distribution.....287
Building the general form of f(x) 287
Finding f(x) given a and b.....288
Finding the value of b given f(x).....289

Drawing Up Probabilities for the Continuous Uniform Distribution 290
Finding less-than probabilities 291
Finding greater-than probabilities.....292
Finding probabilities between two values 293
Corralling Cumulative Probabilities, Using F(x).....294
Figuring the Expected Value and Variance of the Continuous Uniform.....296
The expected value of the continuous uniform 297
The variance and standard deviation of the continuous uniform.....297

Chapter 18: The Exponential (and Its Relationship to Poisson) Exposed 299

Identifying the Density Function for the Exponential 300
Determining Probabilities for the Exponential.....302
Finding a less-than probability for an exponential 302
Finding a greater-than probability for an exponential.....304
Finding a between-values probability for an exponential 305
Figuring Formulas for the Expected Value and Variance of the Exponential.....307
The expected value of the exponential 307
The variance and standard deviation of the exponential 308
Relating the Poisson and Exponential Distributions.....309

Part VI: The Part of Tens.....311

Chapter 19: Ten Steps to a Better Probability Grade 313

Get Into the Problem.....314
Understand the Question.....314
Organize the Information 315
Write Down the Formulas You Need.....316
Check the Conditions 317
Calculate with Confidence 318
Show Your Work 319
Check Your Answer.....319
Interpret Your Results 321
Make a Review Sheet 321

Chapter 20: Top Ten (Plus One) Probability Mistakes 323

Forgetting a Probability Must Be Between Zero and One.....323
Misinterpreting Small Probabilities 324
Using Probability for Short-Term Predictions 325
Thinking That 1-2-3-4-5-6 Can't Win.....325

"Keep 'em Coming . . . I'm on a Roll!" 326
Giving Every Situation a 50-50 Chance 326
Switching Conditional Probabilities Around 327
Applying the Wrong Probability Distribution 328
Leaving Probability Model Conditions Unchecked 329
Confusing Permutations and Combinations 330
Assuming Independence 331

Appendix: Tables for Your Reference 333

Binomial Table 333
Normal Table 337
Poisson Table 340

Index 343

Introduction

Probability is all around you every day — in every decision you make and in everything that happens to you — yet it can't ever give you a guarantee, which forces you to carry your umbrella and get a flu shot every year "just in case." A probability question can be so easy to ask, yet so hard to answer. I suppose that's the beauty as well as the curse of probability. You're walking through an airport three states away from your home, and you see someone you knew from high school and say, "What are the odds of that happening?" Or you hear about someone who won the lottery not once, but twice, and you wonder if you could have the same luck. Or maybe you just heard your teacher say that the chance of two people in the class having the same birthday is 80 percent, and you think, "No way can that be true — he must be crazy!" Well, before you send your professor to the loony bin, know this: Probability and intuition don't mix. But don't worry — this book is here to help.

About This Book

The main goal of this book is to cut down the amount of time you spend spinning your wheels to figure out a probability. The design of this book allows you to quickly find out how to solve the probability questions you're asking (or that you have to answer).

This book gives you the tools to read, set up, and solve a wide range of probability problems. Because all probability problems tend to look different, I build strategies that help you identify what type of problem you're working with, what tools you need to pull out to solve it, and what calculations get you the correct answer. You also gain practice interpreting probability and discovering what misconceptions and common errors you should avoid.

Along the way, you find some interesting surprises and a bird's eye view of how probability pulls on the strings of the real world. I also include tips and strategies for playing games of chance, so if you do win the lottery, you can write about this book in your travel journal on the way to Fiji!

This book is different from other probability books in many ways:

- I focus on material that instructors cover in probability and/or statistics courses, in addition to real-world probability topics. Most probability books out there help you win casino games but don't help you out much with the probability problems you see in a probability and/or statistics course.

- I provide an extensive number of examples to cover the many different types of problems you face.
- You see plenty of tips, strategies, and warnings based on my vast experience with students of all backgrounds and learning styles (and my experiences with grading their papers).
- I focus on building strong problem-solving skills to help you develop a similar problem-solving strategy when you take exams.
- My nonlinear approach allows you to skip around in the book and still have easy access and understanding of any given topic.
- The conversational narrative comes from a student's point of view.
- I use understandable language to help you comprehend, remember, and put into practice probability definitions, techniques, and processes.
- I concentrate on clear and concise step-by-step procedures that intuitively explain how to work through probability problems and remember how to do them later on.

Conventions Used in This Book

In this book, I use the following conventions:

- When I introduce and define a new probability-related term, I *italicize* it.
- The following symbol indicates multiplication: $*$.

What You're Not to Read

It pains me to tell you that any part of my book is skippable, but I have to be honest: You can pass right over any paragraphs that I mark with the Technical Stuff icon, if you're so inclined, and be no worse for wear.

Also, throughout the book you'll find sidebars (the gray boxes) that contain fun and interesting, yet skippable, tidbits. I often use these sidebars to illustrate how people put probability to use in everyday life. Taking a moment to read the sidebars will enhance your understanding and appreciation of probability, but if you're pressed for time or simply uninterested, you won't miss out on any essential information.

Foolish Assumptions

I wrote this book for anyone who wants and/or needs to know about probability with little or no experience necessary. For students, you may be taking a course just in probability, and you're interested in getting help with counting rules, permutations, combinations, and some of the more advanced probability distributions such as the geometric and negative binomial.

Or you may be taking a probability *and* statistics class, which involves an equal treatment of both probability and statistics. This book helps you with the probability part (and *Statistics For Dummies,* also by yours truly [Wiley], helps you with the statistics). But it also helps you see how statistics fits into the area of probability, and vice versa. (If you're taking a straight statistics course, you're likely to run into more probability than you may have bargained for. If so, this book is for you as well.)

Perhaps you're interested in probability from an everyday point of view. If so, you can find plenty of real-world information in this book that you'll find helpful, such as how to find basic probability, win the lottery, become rich and famous, and the like.

How This Book Is Organized

This book is organized into five major parts that explore the main topic areas in probability. I also include a part that offers a couple quick top-ten references for you to use. Each part contains chapters that break down each major objective into understandable pieces.

Part 1: The Certainty of Uncertainty: Probability Basics

This part gives you the fundamentals of probability, along with strageties for setting up and solving the most common probability problems in the introductory course. It starts by introducing probability as a topic that has an impact on all of us every day and underscores the point that probability often goes against our intuition. You discover the basic definitions, terms, notation, and rules for probability, and you get answers to those all-important (and often frustrating) questions that perplex students of probability, such as, "What's the real difference between independent and mutually exclusive events?" You also see different methods for organizing the information given to you, including Venn diagrams, tree diagrams, and tables. Finally, you discover good strategies for solving more complex probability problems involving the Law of Total Probability and Bayes' Theorem.

Part II: Counting on Probability and Betting to Win

In this part, you get down to the nitty gritty of probability, solving problems that involve two-way tables, permutations and combinations, and games of chance. The bottom line in this part? Probability and intuition don't always mix!

Part III: From A to Binomial: Basic Probability Models

In this part, you build an important foundation for creating, using, and evaluating probability models. You discover all the ins and outs of a probability distribution; the basic concepts and rules for defining probability distributions; and how to find probabilities, means, and variances. You work with the binomial and normal distributions, and you find out how probability ties in to the major results from statistics: the Central Limit Theorem, hypothesis testing, and overall decision making in the real world.

Part IV: Taking It Up a Notch: Advanced Probability Models

In this part, you work with more intermediate probability models that count and try to predict the number of arrivals, successes, or the number of trials needed to achieve a certain goal. The probability distributions I focus on are the Poisson, negative binomial, geometric, and hypergeometric. You find out how many customers you expect to come into a bank (Poisson distribution); the number of poker hands you need to draw before you get four of a kind (geometric distribution); the number of frames you need to bowl before getting your third strike (the negative binomial distribution); and the probability of getting a hand in poker (hypergeometric distribution).

Part V: For the Hotshots: Continuous Probability Models

In this part, you look at some of the models you find in probability and statistics courses that have calculus as a prerequisite — mainly the uniform (continuous case) distribution, exponential distribution, and other user-defined probability density functions. You see how to find probabilities and the expected value, variance, and standard deviation of continuous probability models. And you apply the models to situations such as the time between

arrivals of customers at the bank, time to complete a task, or the length of a phone call. ***Note:*** Calculus is useful but not required for this part. I introduce the methods that use calculus, but I also provide formulas and other methods of solution that don't use calculus for the uniform and exponential.

Part VI: The Part of Tens

In this part, you find my top tens lists: ten steps to a better probability grade and ten probability misconceptions and how to avoid them. This information is based on my years of experience teaching, answering questions, writing questions, and grading homework. This part will help you pinpoint the most important ideas in probability and the most common errors that are made. It also serves as a quick and condensed resource as you are studying for exams.

Appendix

I also include an appendix that contains three handy tables for your reference. These tables help you find probabilities for the binomial distribution, the normal distribution, and the Poisson distribution.

Icons Used in This Book

I use various icons in this book to draw your attention to certain features that occur on a regular basis. Think of the icons as road signs you encounter on a trip. Some signs tell you about shortcuts, and others offer more information that you may need; some signs alert you to possible warnings, and others leave you with something to remember.

I use this icon to point out exciting and perhaps surprising situations where people use probability in the real world, from actuarial science to manufacturing (and casinos, of course).

These I save for particular ideas that I hope you'll remember long after you read this book. They mainly refer to actions you can take to help you determine which technique to use in a given probability problem.

Feel free to skip over the paragraphs that feature this icon if you're in an introductory level course. The info is either ancillary or more advanced than is necessary for an introductory probability course. However, if you're interested in the gory details, or if you have to be for your more advanced level course, go for it!

Tips refer to helpful hints, ideas, or shortcuts that you can use to save time. They may also give you alternative ways to think about a particular concept.

Warning icons alert you to specific ways that you may get tripped up working a certain kind of problem. I also reserve this icon to discuss common misconceptions about probability that can get you into trouble.

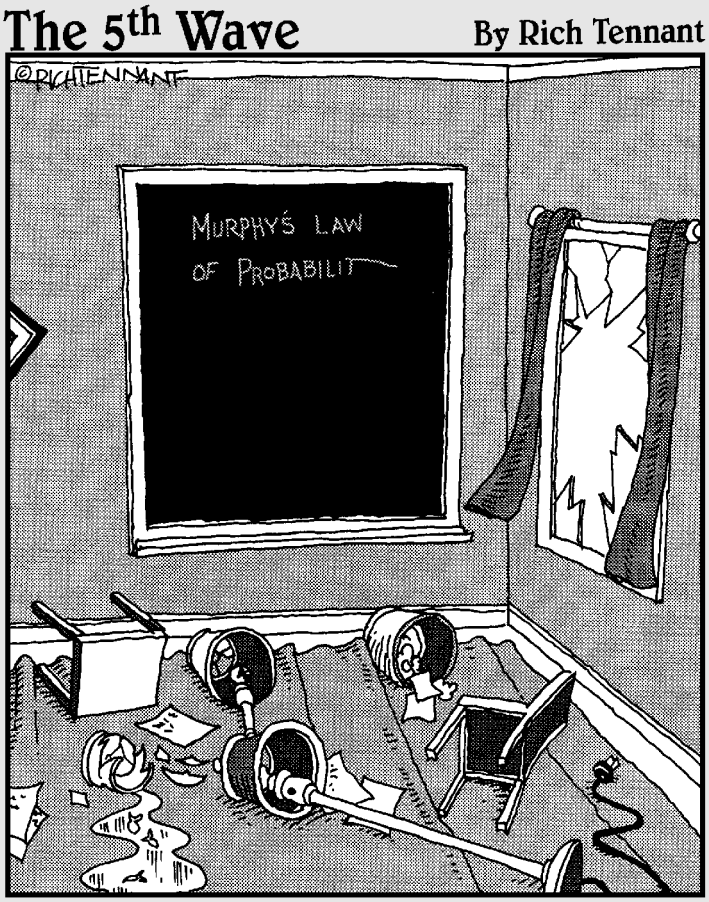
Where to Go from Here

I wrote this book in a modular way, meaning you can start anywhere and still understand what's happening. However, I can make some recommendations to people who are unsure about where to start:

- If you're taking a probability or statistics class based in algebra, I recommend starting with Part I to build a basic foundation for probability and how to set up problems.
- If you're taking a probability class based in calculus, you may want to start with Part IV and work your way to Part V. In Part V, you have a chance to see your calculus at work as you find probabilities as areas under a curve.
- If you're taking a statistics and/or probability course that focuses heavily on counting rules, combinations, and permutations, head to Chapter 5. There you'll find examples of counting problems under every scenario I could think of to help you build a strong set of strategies so each problem doesn't look different.
- If you're interested in games of chance, head to Chapters 5 and 6. You'll find some ideas on what your expected winnings are with various games, and you'll discover how to calculate your odds of winning.

Part I

The Certainty of Uncertainty: Probability Basics



In this part . . .

In Part I, you get started with the basics of probability — the terminology, the basic ideas of finding a probability, and, perhaps most importantly, how to organize and set up all the information you have in order to successfully calculate a probability. You also discover ways in which people use probability in the real world.

But let's be honest. When it comes to a class that involves probability, is there truly a *real* world? Maybe, maybe not. Counting the number of ways to pick three green balls and four red balls from an urn that contains twenty green balls and thirty red balls doesn't sound all that relevant — and it isn't. That's why you won't see a single "urn problem" anywhere in this part. However, if you do run across an "urn problem" in your life, you'll know how to answer it, using the techniques from Part I.